PSY 201: Statistics in Psychology

Lecture 23

Hypothesis tests for a proportion

Can you read my mind? Part II

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HYPOTHESIS TESTING

- four steps
  1. State the hypothesis and the criterion
  2. Compute the test statistic.
  3. Compute the p-value.
  4. Make a decision

HYPOTHESIS TESTING

- we need to know the properties of the sampling distribution
- for the mean, the central limit theorem tells us that the sampling distribution is normal, and specifies the mean and standard deviation (standard error)
- area under the curve of the sampling distribution gives probability of getting that sampled value, or values more extreme (p-value)
- for other types of statistics, the sampling distribution is different
  - area under the curve of sampling distribution still gives probability of getting that sampled value, or values more extreme
- proportion

HYPOTHESIS TESTING

- the approach is still basically the same
- we compute

$$\text{Test statistic} = \frac{\text{statistic - parameter}}{\text{standard error of the statistic}}$$

- and use it to compute a *p*-value, which we compare to α

PROPORTION

- many times we want to know what proportion (P) of a population has a certain trait
  - Own a phone.
  - Are a democrat.
  - Are a republican.
  - Own a computer.
  - ...
- dichotomous population (have trait or do not)
- percentages

PROPORTION

- we can take a random sample and calculate a sample proportion p
- we can test hypotheses about the population parameter P
  e.g.

$$H_0 : P = 0.5$$

$$H_a : P \neq 0.5$$

- the sampling distribution of p is the binomial distribution
- for large samples it is very close to the normal distribution

STANDARD ERROR

- an estimate of the standard error of the sampling distribution (standard error of the sample proportion) is:

$$s_p = \sqrt{\frac{PQ}{n}}$$

  - P = population proportion possessing characteristic
  - $Q = 1 - P$ = population proportion not possessing characteristic
  - n = sample size
- now we can apply the techniques of hypothesis testing!

PEPSI CHALLENGE

- several years ago Pepsi sponsored the **Pepsi Challenge** where you sampled Coke and Pepsi and decided which tasted better
- after testing hundreds of people, they found that more than half the Coke drinkers preferred Pepsi (63%)
- how would we test to see if the proportion of people who preferred Pepsi over Coke was a significant proportion (different from chance)?

HYPOTHESIS

- Step 1. State the hypothesis and criterion. By chance we would expect the proportion of people that preferred Pepsi would be 50%

$$H_0 : P = 0.5$$

$$H_a : P \neq 0.5$$

- Let's set our level of significance at $\alpha = 0.05$, two-tailed test

CRITERION

- Step 2. Compute the test statistic. Suppose the sample proportion is

$$p = \frac{189}{300} = 0.63$$

- Let's suppose $n = 300$ people were tested, and so the standard error of the sample proportion is:

$$s_p = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{(0.5)(0.5)}{300}} = 0.02886$$

TEST STATISTIC

- the test statistic is:

$$z = \frac{p - P}{s_p} = \frac{0.63 - 0.5}{0.02886} = 4.50$$

- Step 3. Compute the p-value. We use the Normal Distribution Calculator to compute

$$p \approx 0$$

- Step 4. Make a decision. Since $p < \alpha = 0.05$, we can reject H_0!
  - If $P = 0.5$, the probability of getting $p = 0.63$, or an even bigger difference from $P = 0.5$, from a random sample of 300 people is less than 0.05.
  - The observed difference is a significant difference.

CONFIDENCE INTERVALS

- Let's construct a confidence interval with level of confidence $1-\alpha = 0.95$
- The critical value z_{cv} is found from the Inverse Normal Distribution Calculator

$$z_{cv} = 1.96$$

- so

$$CI_{95} = p \pm (1.96)(s_p)$$

- For the confidence interval, we recompute the standard error by using the estimate from the sample

$$s_p = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.63)(0.37)}{300}} = 0.0279$$

$$CI_{95} = 0.63 \pm (1.96)(0.0279)$$

$$CI_{95} = (0.57, 0.68)$$

- which does not include the chance level $P = 0.5$

MIND READING

- I am going to pick one of the following words as a “special” word
- You try to read my mind as to which one is “special”
- write it down on a sheet of paper. I’ll write down my chosen word on a sheet of paper
    - COMPUTER
    - STEREO
    - BICYCLE
    - STAPLER
    - BOOKCASE
    - DESK

MIND READING

- Now, I tell you my special word, and we find out how many of you were correct. We are measuring p, the sample proportion
- we can test whether you can read my mind
- (1) State the hypothesis and the criterion
  - the null hypothesis is that you cannot read my mind, so we say that

$$H_0 : P = \frac{1}{6} = 0.167$$

$$H_a : P \neq 0.167$$

  - where 0.167 is what you would get just by guessing
- $\alpha = 0.10$

MIND READING

- (2) Compute the test statistic

$$s_p = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{(0.167)(0.833)}{n}} = \sqrt{\frac{0.1391}{n}} =$$

$$z = \frac{p - P}{s_p} =$$

- (3) Which we plug in to the Normal Distribution Calculator to find the *p*-value
- (4) Make a decision
- We can do it all with the One Sample Proportion Test Calculator in the textbook

POWER

- How would we design a good experiment to test Mind Reading abilities?
- How big a sample do we need to have a 90% chance of rejecting the H_0?
- Conceptually, this is the same issue as estimating power or sample size for a hypothesis test of means
- We just need to use the sampling distribution for a sample proportion instead of the sampling distribution for a sample mean

POWER

- We have to set the specific proportion for the alternative hypothesis
- Suppose we plan to test

$$H_0 : P = 0.167, H_a : P \neq 0.167$$

- and we set the specific alternative as

$$H_a : P_a = 0.2$$

- What is the probability that a random sample of $n = 25$ will reject the H_0?
- The on-line calculator does all the work!

POWER

Specify the population characteristics:

$H_0 : P_0 =$ 0.167

$H_a : P_a =$ 0.2

Specify the properties of the test:

Type of test: Two-tails

Type I error rate, $\alpha =$ 0.05

Power= 0.090999

Sample size, $n =$ 25

Calculate minimum sample size

Calculate power

- Less than 10% chance of rejecting the null hypothesis
- What sample size do we need to have 90% power?

Specify the population characteristics:

$H_0 : P_0 =$ 0.167

$H_a : P_a =$ 0.2

Specify the properties of the test:

Type of test: Two-tails

Type I error rate, $\alpha =$ 0.05

Power= 0.9

Sample size, $n =$ 1421

Calculate minimum sample size

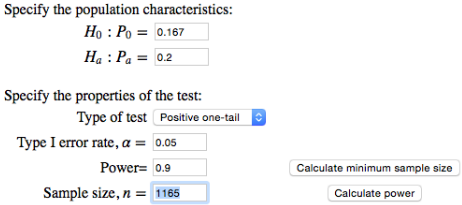
Calculate power

POWER

- Suppose we plan to test

$$H_0 : P = 0.167, H_a : P > 0.167$$

- What sample size do we need to have 90% power?



POWER

- Let's use the proportion we found for the class as the specific alternative value
  - Power?
  - Sample size for 90% power?

CONCLUSIONS

- testing significance of proportions
- confidence intervals for proportions
- power for tests of proportions

NEXT TIME

- hypothesis testing of correlations
- Fisher z transform
- another t test
- confidence interval
- power

Is there a correlation between homework and exam scores?