PSY 201: Statistics in Psychology

Lecture 07
Normal distribution
Describing everyone's height.

Greg Francis

Purdue University

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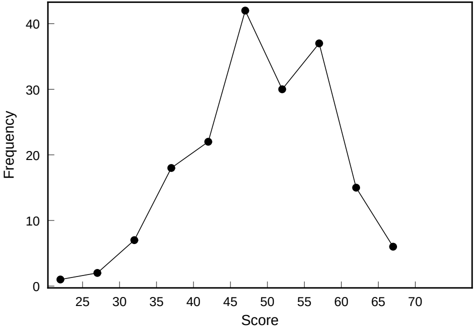
DISTRIBUTION

- frequency of scores plotted against score



- frequency → likelihood, probability

GOAL

- describe (summarize) distributions
  - shape: unimodal, bimodal, skew,...
  - central tendency: mode, median, mean
  - variation: range, variance, standard deviation
- summarizing forces you to lose information
- some **theoretical** distributions are special!
  - a few numbers completely specify the distribution

NORMAL DISTRIBUTION

$$Y = \frac{1}{\sigma\sqrt{2\pi}} e^{-(X-\mu)^2/2\sigma^2}$$

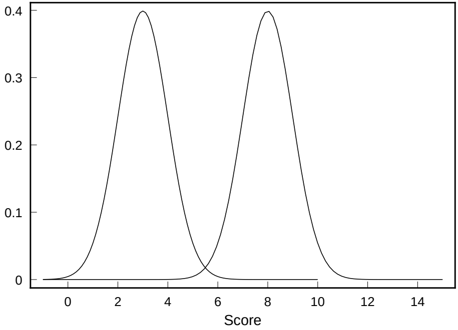
- Y height of the curve for any given value of X in the distribution of scores
- π mathematical value of the ratio of the circumference of a circle to its diameter. A constant (3.14159.....)
- e base of the system of natural logarithms. A constant (2.7183...)
- μ mean of the distribution of scores
- σ standard deviation of a distribution of scores

sometimes written as

$$Y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-(X-\mu)^2/2\sigma^2\right]$$

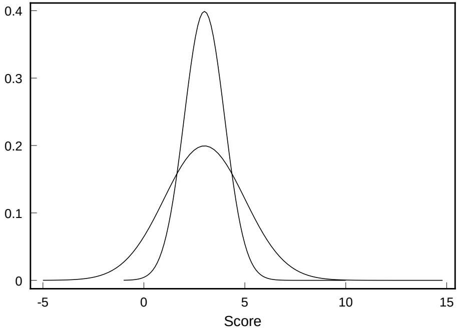
PARAMETERS

- a **family** of distributions
- member of the family is designated by the mean μ and standard deviation σ
- changing μ shifts the curve to the left or the right
  - shape remains the same



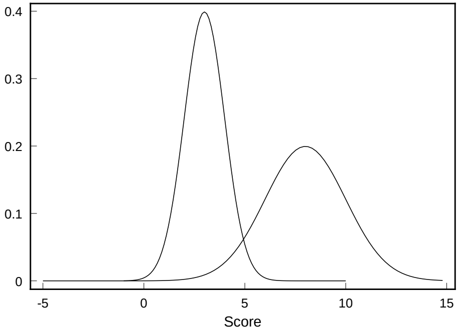
PARAMETERS

- changing σ changes the **spread** of the curve
- compare normal distributions for $\sigma = 1$ and $\sigma = 2$, both with $\mu = 3$



PARAMETERS

- changing μ and σ together produces predictable results



PROPERTIES

- all normal distributions have the following in common
  - Unimodal, symmetrical, bell shaped, maximum height at the mean.
  - A normal distribution is continuous. X must be a **continuous** variable, and there is a corresponding value of Y for each X value.
  - A normal distribution asymptotically approaches the X axis.

STANDARD NORMAL

- remember z-scores:
  - 0 mean
  - 1 standard deviation
- if the z-scores are normally distributed

$$Y = \frac{1}{\sigma\sqrt{2\pi}} e^{-(X-\mu)^2/2\sigma^2}$$

- becomes

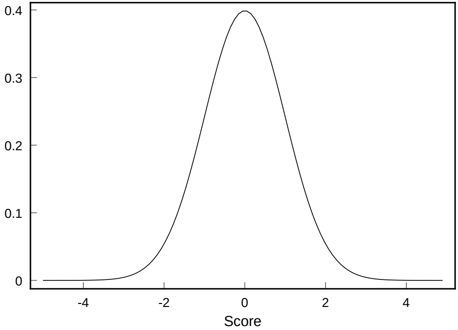
$$Y = \frac{1}{1\sqrt{2\pi}} e^{-(z-0)^2/2(1^2)}$$

- or

$$Y = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

STANDARD NORMAL

- looks like



SIGNIFICANCE

- It turns out that lots of frequency distributions can be described as a normal distribution
- for example, an estimate of height

SIGNIFICANCE

- It turns out that lots of frequency distributions can be described as a normal distribution
  - intelligence scores
  - weight
  - reaction times
  - judgment of distance
  - rating of personality
  - ...
- almost any situation where small independent components come together

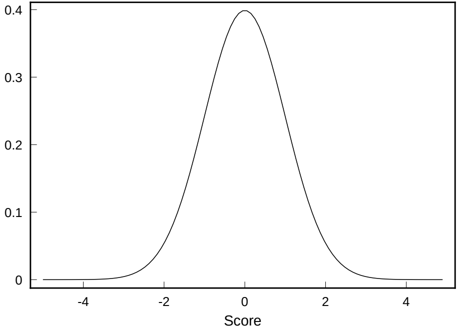
SIGNIFICANCE

- when the distribution is a normal distribution, we can describe the distribution by just specifying
  - Mean: $\overline{X}$
  - Standard deviation: s
  - Noting it is a normal distribution
- that's all we need!
- That's part of our goal: describe distributions

STANDARD NORMAL

- assume you have a standard normal distribution (don't worry about where it came from)

$$Y = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$



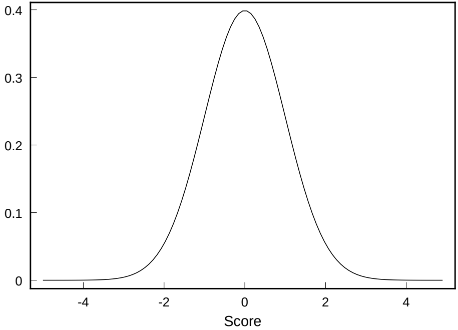
- if your distribution is normal, you can create a standard normal by converting to z-scores

USE

- same as all other distributions
  - identify key aspects of the data
  - percentiles
  - percentile rank
  - proportion of scores within a range
  - ...
- make it easier to interpret data significance!

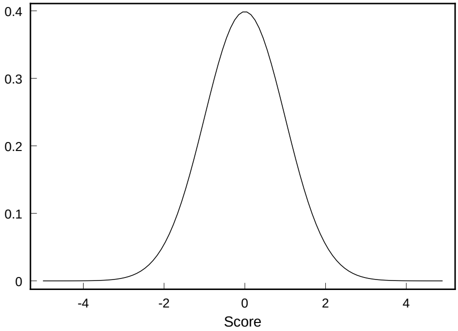
STANDARD NORMAL

- total area under the curve **always** equals 1.0
- area under the curve from the mean (0) to one tail equals 0.5



STANDARD NORMAL

- area under the curve one standard deviation away from the mean is approximately 0.3413
- area under the curve two standard deviations away from the mean is approximately 0.4772
- area under the curve three standard deviations away from the mean is approximately 0.4987



CONCLUSIONS

- normal distribution
  - equations
  - properties
  - standard normal equations

NEXT TIME

- area under the curve
- proportions
- percentiles
- percentile ranks

Business decisions.