Workshop:
Quantum Contextuality in Quantum Mechanics and Beyond

held in Prague on May 19 and 20, 2018

Abstracts

Samson Abramsky

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Cohomology for everyone: using cohomology to detect contextuality

Cohomology is a powerful tool of modern mathematics, with a huge range of applications. Recently, cohomology has been applied to contextuality: a cohomology “obstruction” is defined for some class of contextuality scenarios, which vanishes when there is a non-contextual description. Thus one replaces impossibility proofs by contextuality witnesses, which in many cases can be computed.

We shall give an introductory, fairly concrete account of these ideas, which should be accessible with minimal prerequisites.

Bárbara Lopes Amaral

Universidade Federal de São João Del Rey - CAP, Brasil

Noncontextual wirings

Contextuality is a fundamental feature of quantum theory necessary for certain models of quantum computation and communication. Serious steps have therefore been taken towards a formal framework for contextuality as an operational resource. However, the main ingredient of a resource theory - a concrete, explicit form of free operations of contextuality– was still missing. Here we provide such a component by introducing noncontextual wirings: a class of contextuality-free operations with a clear operational interpretation and a friendly parametrization. We characterize them completely for general black-box measurement devices with arbitrarily many inputs and outputs. As applications, we show that the relative entropy of contextuality is a contextuality monotone and that maximally contextual boxes that serve as contextuality bits exist for a broad class of scenarios. Our results complete a unified resource-theoretic framework for contextuality and Bell nonlocality.

Guido Bacciagaluppi

Utrecht University, The Netherlands

Settings-source dependence and signalling

(Joint work with Ronnie Hermens and Gijs Leegwater)

It is well-known that hidden variables theories violating parameter independence allow for signalling in cases of disequilibrium (i.e. should it be possible to prepare or discover ensembles in which the distribution of hidden variables deviates from the quantum distribution). The Colbeck-Renner theorem recently made fully rigorous by Leegwater shows further that hidden variables theories possibly violating outcome independence have to violate also parameter independence in order to improve on the predictions of quantum mechanics when in disequilibrium. These results, however, assume settings-source independence (the distribution of the hidden variables is independent of the measurement settings), which may be violated specifically in retrocausal or superdeterminist theories. In this talk, we shall show that even in a theory with settings-source dependence, a disequilibrium distribution allowing for improved predcitions generically leads to signalling.

Rui Soares Barbosa

University of Oxford, UK

Contextuality and advantage in informatics tasks

Contextuality is a fundamental phenomenon of quantum mechanics setting it apart from classical physical theories. Recent results have also established its role as a source of advantage in informatic tasks, including the additional computational power in some specific schemes for quantum computation. We consider contextuality from the point of view of resource theory and present some results showing quantum advantages afforded by the presence of contextuality, with applications to a wide variety of areas in classical computation such as constraint satisfaction or graph theory. This also highlights the relationship between non-locality and state-independent contextuality. Moreover, it suggests some avenues for further fundamental research aiming for a quantum version of finite model theory and descriptive complexity.

Costantino Budroni

Institute for Quantum Optics and Quantum Information, Austria

Memory cost of temporal correlations

We investigate a possible notion of nonclassicality for single systems, based on the notion of memory cost of simulating observed probabilities obtained after a sequence of measurements. More precisely, we investigate the set of probabilities achievable with finite-state machines, i.e., machines with a finite number of perfectly-distinguishable states, output probabilities, and state-transition probabilistic rules. We discuss such models in the framework of classical, quantum, and generalized probabilistic theories (GPTs). The finite number of perfectly-distinguishable states is interpreted as a memory resource available to the machine and needed to produce correlations. With this restriction, we show how our notion allow us to distinguish classical, quantum, and GPTs correlations already in the simplest nontrivial scenario, namely, that of two inputs, two outputs, and sequences of length two. Moreover, we investigate the behavior of such models in more complex scenarios and in the asymptotic limit of measurement sequences of infinite length. Finally, we comment on the relation with other notions of nonclassicality for single systems such as contextuality and non-macrorealism.

Adán Cabello

University of Sevilla, Spain

A simple explanation of Born's rule

Identifying the property of the world that enforces the Born rule is a longstanding problem in physics. We show that, for any physical theory that assigns probabilities to the outcomes of ideal measurements, the largest possible set of probability assignments for any given structure of exclusivity is exactly the one that satisfies the Born rule. No probability assignment outside this set is logically consistent. This suggests that the extraordinary agreement between quantum theory and experiments simply reflects a one-to-one correspondence between the probability assignments that are logically possible when there are no restrictions and the state preparations and transformations that are physically realizable. Specifically, the Born rule would be a consequence of the absence of physical laws determining the probabilities of the outcomes of quantum measurements.

Giovanni Carù

University of Oxford, UK

Towards a cohomology invariant for non-locality and contextuality

The sheaf theoretic description of non-locality and contextuality by Abramsky and Brandenburger sets the ground for a topological study of these peculiar features of quantum mechanics. This viewpoint has been recently developed thanks to sheaf cohomology, which provides a sufficient condition for the contextuality of empirical models in quantum mechanics and beyond. Subsequently, a number of studies proposed methods to detect contextuality based on different cohomology theories. However, none of these cohomological descriptions succeeds in giving a full invariant for contextuality. In the present work, we introduce a cohomology invariant for possibilistic and strong contextuality which is applicable to the vast majority of empirical models.

Víctor Hernando Cervantes

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Two behavioral experiments revealing contextuality

(joint work with Ehtibar Dzhafarov)

Behavioral data are inconsistently connected (i.e. violate the condition variously called "no-signaling," "no-disturbance," "non-invasiveness," etc.). This created a difficulty for the previous search for quantum-like contextuality in behavioral data: one had either to declare all behavioral data trivially contextual (by taking inconsistent connectedness as a form of contextuality), or to declare the notion of contextuality inapplicable, as it was only defined for consistently connected systems [1]. The situation has changed with the introduction of the Contextuality-by-Default (CbD) theory that allows one to detect and measure contextuality under inconsistent connectedness. We present experimental evidence for contextuality in two experiments. In the first, crowdsourcing experiment, the respondents were asked to make two conjoint choices related to characters in a H.C. Andersen's story [2]. Contextuality here was established by the application of a mathematical criterion developed in CbD for cyclic systems of dichotomic variables. In the second, psychophysical experiment, extending one described in Ref. [3], participants were asked to judge the location of two dots with respect to the centers of two respective circles presented side-by-side. Contextuality here was established by means of a recently developed necessary condition for contextuality in systems of categorical random variables with more than two possible values [4].

[1] Dzhafarov, E.N., Zhang, R., & Kujala, J.V. (2016). Is there contextuality in behavioral and social systems? Philosophical Transactions of the Royal Society A 374: 20150099.

[2] Cervantes, V.H., & Dzhafarov, E.N. (2018). Snow Queen is evil and beautiful: Experimental evidence for probabilistic contextuality in human choices. To be published in *Decision*.

[3] Cervantes, V.H., & Dzhafarov, E.N. (2017). Advanced analysis of quantum contextuality in a psychophysical double-detection experiment. *Journal of Mathematical Psychology* 79, 77-84.

[4] Dzhafarov, E.N., Cervantes, V.H., Kujala, J.V. (2017). Contextuality in canonical systems of random variables. *Philosophical Transactions of the Royal Society A* 375: 20160389.

Marcelo Terra Cunha

Universidade Estadual de Campinas, Brazil

Nonlocality tests including compatible measurements

(joint work with Tassius Temistocles and Rafael Rabelo)

Noncompatibility of observables, or measurements, is one of the key features of quantum mechanics, related, among others, to Heisenberg's uncertainty relations and Bell nonlocality. In this manuscript we show, however, that even though noncompatible measurements are necessary for the violation of any Bell inequality, novel, relevant Bell-like inequalities may be obtained if compatibility relations are assumed between some of the local measurements of one (or more) parties. Hence, compatibility of measurements is not necessarily a drawback and may, on the other hand, be useful for the detection of Bell nonlocality.

Nadish de Silva

University College London, UK

Contextuality and quantum gate injection

(Joint work with Abramsky, Barbosa, Caru, Kishida, Mansfield)

We analyse the minimum quantum resources needed to realise strong non-locality, as exemplified e.g. by the classical GHZ construction. It was already known that no two-qubit system, with any finite number of local measurements, can realise strong non-locality. For three-qubit systems, we show that strong non-locality can only be realised in the GHZ SLOCC class, and with equatorial measurements. However, we show that in this class there is an infinite family of states which are pairwise non-LU-equivalent that realise strong non-locality with finitely many measurements. These states have decreasing entanglement between one qubit and the other two, necessitating an increasing number of local measurements on the latter.

Ehtibar Dzhafarov

Purdue University, USA

Contextuality analysis: Directions of development

The Contextuality-by-Default (CbD) theory deals with arbitrary systems of double-indexed (by properties being measured and by contexts of measurements) random variables [1]. CbD allows one to determine whether the system is contextual and to measure the degree of its contextuality in a principled way. It generalizes the traditional understanding of contextuality in that it allows the random variables measuring the same property in different contexts to be differently distributed [2]. CbD was initially fully developed for cyclic system of binary random variables, where each property is being measured in precisely two contexts [3]. Recently CbD was extended to arbitrary variables, by representing each of them by a set of its dichotomizations, and to arbitrary numbers of random variables sharing a measured property, by replacing the notion of a maximal coupling with that of a multimaximal coupling [4]. One can even replace multimaximal couplings with other uniquely determinable couplings, each type of a coupling defining a specific meaning of contextuality that might be of interest in a specific area outside quantum physics [5]. A very recent, tentative direction of development extends double-indexing from random variables representing measurement outcomes to the quantum observables generating these random variables [6].

[1] Dzhafarov E.N., Kujala J.V. (2017). Probabilistic foundations of contextuality. *Fortschritte der Physik - Progress of Physics* 65, 1600040 (1-11).

[2] Dzhafarov E.N., Kujala J.V., Larsson J.-A. (2015). Contextuality in three types of quantum-mechanical systems. *Foundations of Physics* 7, 762-782.

[3] Kujala J.V., Dzhafarov E.N., Larsson J.-A. (2015). Necessary and sufficient conditions for extended noncontextuality in a broad class of quantum mechanical systems. *Physical Review Letters* 115, 150401.

[4] Dzhafarov E.N., Cervantes V.H., Kujala J.V. (2017). Contextuality in canonical systems of random variables. *Philosophical Transactions of the Royal Society A* 375: 20160389.

[5] Dzhafarov, E.N. Replacing nothing with something special: Contextuality-by-Default and dummy measurements. In A. Khrennikov & T. Bourama (Eds) *Quantum Foundations, Probability and Information.* To be published by Springer.

[6] Dzhafarov E.N (2018). Quantum mechanics with contextually labeled observables. *arXiv:1802.08685.*

Chris Fields

Caunes-Minervois, France

Contextuality and system identification

Observers identify "systems", including their experimental apparatus, records of previous observations, and colleagues, by making finite observations at finite resolution. These observations are physical interactions with a universe having no a priori or "preferred" state-space decomposition, i.e. a universe characterized by an associative decomposition operator. Each such set of observational outcomes must satisfy a finitely-specified set of memory-resident identification criteria to pick out a "known" system from the background or "environment" in which it is embedded. Arbitrarily many distinct configurations of the observed universe with distinct causal consequences in principle satisfy each of these sets of criteria; hence system identification is arbitrarily ambiguous in every case. Quantum theory arises naturally in this setting. Quantum contextuality can, in this setting, be viewed as a consequence of the interleaving of observations that identify "systems" with observations that record values of "pointer variables" associated with those systems.

Matthias Kleinmann

University of Siegen, Germany

Theory-independent proof of stronger-than-binary correlations

Quantum theory surely is a well tested theory and with the advent of loophole-free Bell-tests it is established that nature has stronger correlations than any local classical theory would allow. Another way to understand these tests is to read them as theory-independent proofs of the existence of measurements with two genuine nonclassical outcomes. We show that the predictions of quantum theory go far beyond this statement and that according to quantum theory there exist n-outcome measurements, which produce correlations that cannot be explained by any theory with only binary measurements, not even by Popescu-Rohrlich boxes. In this talk I will summarize our current understanding of genuine n-ary correlations and detail on the first experimental demonstration of stronger-than-binary correlations.

Maria Kon

Purdue University, USA

Universal Applicability of Classical Probability with Contextually Labeled Random Variables

(joint work with Ehtibar Dzhafarov, *to be presented as a poster*)

The use of quantum probability is often justified by the inadequacies of classical (Kolmogorovian) probability theory. Three widespread statements about the inadequacy of classical probability theory are:

1. Classical probability requires that particular inequalities (e.g., Bell-type) hold for certain systems of random variables, but quantum mechanics and behavioral experiments indicate that these inequalities may be violated.
2. In classical probability, the joint occurrence of two events is commutative, but quantum mechanics and behavioral experiments indicate that the order of two events generally matters for their joint probability.
3. Classical probability is additive, but quantum mechanics and behavioral experiments indicate that this additivity can be violated.

Classical probability theory has been recently extended by the Contextuality-by-Default theory. It identifies random variables by: (i) content, i.e., what is being measured, e.g., a particle goes through the right slit, the response to question B, and (ii) context, i.e., properties of the experiment, e.g., both slits are open, question A is asked before question B. We apply this double indexing of random variables to quantum and behavioral phenomena used to support the three statements. We argue that these statements are untenable and based on the misidentification of random variables.

Shane Mansfield

Sorbonne University, France

Contextuality for transformations

I will introduce a notion of contextuality for transformations in sequential contexts, distinct from the Bell-Kochen-Specker and Spekkens notions of contextuality, which can arise even in a single qubit system. Within a transformation-based model for quantum computation it will be shown that strong sequential transformation contextuality is necessary for deterministic computation of non-linear functions. For probabilistic computation, sequential transformation contextuality is necessary for obtaining advantage over classical implementations and the degree of advantage quantifiably relates to the degree of contextuality.

Marcin Markiewicz

Jagiellonian University, Poland

Creating entanglement of particles from independent sources without ever touching

Quantum entanglement is often thought of as a phenomenon arising from some kind of interaction between particles. On the other hand it was formerly known that indistinguishable particles from independent sources can be used to create two-qubit Bell states and three-qubit GHZ-type states. In this talk a general scheme for creating entanglement from indistinguishable particles arising from independent sources will be presented. The scheme is based on the 'no-touch' paradigm, which means the particles being processed may neither interact nor even meet during the entire procedure. Within presented scenario one can produce all genuinely three-partite entangled three-qubit states, including the family of W states. What is more interesting, the entire scheme does not depend on the quantum statistics of the particles (bosonic, fermionic or anyonic), but only demands indistinguishability of them.

Yoshihiro Maruyama

Kyoto University, Japan

A structural perspective on quantum cognition: From Penrose's argument to no-go theorems in cognitive science

We first revisit Penrose's argument on the quantum nature of cognition, which is highly controversial in its material interpretation, and yet can be made more acceptable in its structural interpretation. From this structural perspective, then, we discuss cognitive counterparts of No-Go theorems in quantum physics, including contextuality, on both theoretical and experimental bases. We argue, inter alia, that certain No-Go theorems hold in cognitive science even if cognition violates the marginal selectivity or no-signalling condition.

Olga Nanasiova

Slovak University of Technology, Slovakia

Quantum probability and Boolean functions

Our contribution deals with problems of probability measures and boolean functions. We will show that it is possible to introduce a boolean function also for incompatible random events, via probability measure. This work is a continuation of [1, 2, 3]. (The author would like to thank for the support of the VEGA grant agency by means of grant VEGA 1/0710/15.)

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[2] Nanasiova, O., Valaskova´, L.: Maps on a quantum logic. Soft Computing 14 (10), 1047-1052 (2010)

[3] Pykacz, J., Frackiewicz, P.: The Problem of Conjunction and Disjunction in Quantum Logics. International Journal of Theoretical Physics, 56(12), pp. 3963-3970 (2017)

Karl Svozil

Vienna University of Technology, Austria

Value (in)definiteness and contextuality

Contextuality might be traced back to the observation by Bohr that "the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear." And in view of the impossibility to simultaneously define certain (finite) collections of observables in a consistent manner, the realist Bell suggested “the result of an observation may reasonably depend ... on the complete disposition of the apparatus” -- that is, on the measurement context.

Our contemporary understanding of contextuality is ambivalent, and is often a confusing canvas; an incoherent mix of Bell’s realistic conceptions, of what Pitowsky referred to as “value indefiniteness,” or simply of the violation of certain (generalized) Boolean “conditions of possible [[classical]] experience.”

I will discuss some aspects of the present situation, and give reasons for the emphasis on value indefiniteness.