Two behavioral experiments revealing contextuality

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Quantum Contextuality in Quantum Mechanics and Beyond
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Overview

1. Introduction
   - Contextuality-by-Default
2. Experiments
   - Crowdsourcing experiment
   - Psychophysical experiment
3. Closing remarks

Introduction

Contextuality-by-Default

Double-indexing

- The identity of a random variable is determined by its content, (the property it measures) and its context (conditions under which this property is measured): R_q^c

Introduction

Contextuality-by-Default

Principles

- Within a context, random variables are jointly distributed.

$$(R_q^c, R_{q'}^c, R_{q''}^c)$$

- Otherwise, they are *stochastically unrelated*.

Introduction

Contextuality-by-Default

R_1^1	R_2^1	$\cdot$	$\cdot$	c_1
$\cdot$	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	$\cdot$	R_3^3	R_4^3	c_3
R_1^4	$\cdot$	$\cdot$	R_4^4	c_4
R_1^5	$\cdot$	R_3^5	$\cdot$	c_5
q_1	q_2	q_3	q_4	$\boxed{\mathcal{R}}$

Contextuality in Contextuality-by-Default theory

Contextuality

A system of random variables is said to be *noncontextual* (with respect to multimaximality property) if R if it has a coupling S in which any two content-sharing random variables are equal to each other with the maximal possible propbability.

Otherwise R is said to be *contextual* (with respect to multimaximality).

Contextuality in Contextuality-by-Default theory

- In CbD, for systems of categorical random variables, we use multimaximal couplings of sets of dichotomizations of their category values.
- For consistently connected systems, multimaximality reduces to identity couplings. Then we have the traditional understanding of contextuality.

Experiments

Crowdsourcing

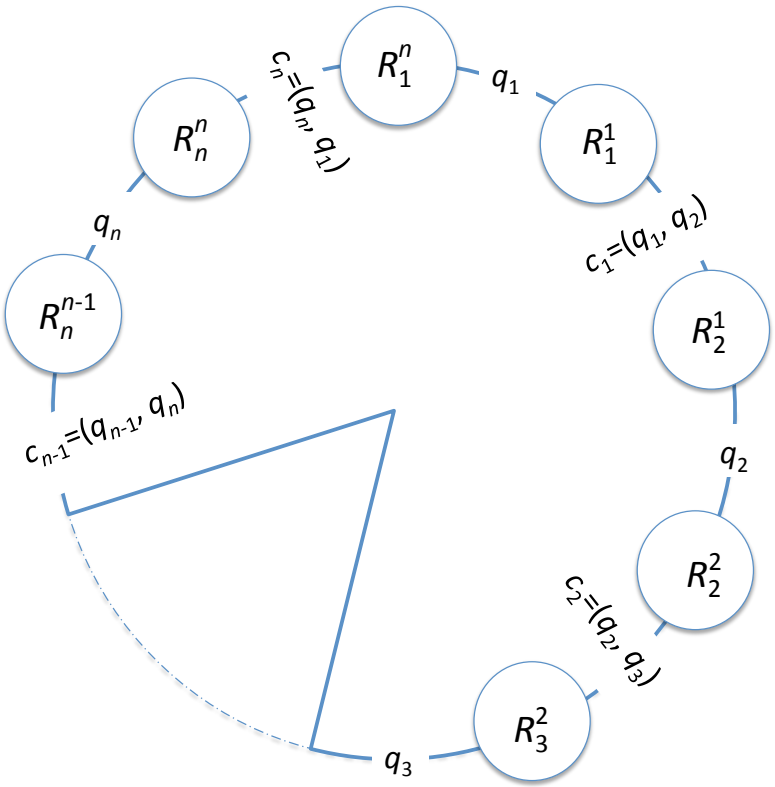
- Four randomly assigned conditions.
- Two choices:
  - One of a character from a given pair of characters,
  - and of a suitable characteristic of this character from a given pair of characteristics.

Psychophysical

- Joint identification of two stimuli locations.
- Five locations per stimulus.

Experiments

Cyclic system of random variables



R_1^1	R_2^1	·	·	c_1
·	R_2^2	R_3^2	·	c_2
·	·	R_3^3	R_4^3	c_3
R_1^4	·	·	R_4^4	c_4
q_1	q_2	q_3	q_4	$\mathcal{R}$

Contextuality in Contextuality-by-Default theory

Criterion for cyclic systems (Kujala & Dzhafarov, 2016)

A cyclic system of binary random variables taking values ± 1 is noncontextual if and only if

$$\max_{\text{odd \# of } -'s} \sum_{i=1}^{n} \pm \left\langle R_i^i R_{i\oplus 1}^i \right\rangle - \sum_{i=1}^{n} \left| \left\langle R_i^{i\ominus 1} \right\rangle - \left\langle R_i^i \right\rangle \right| - n + 2 \leqslant 0,$$

where $\langle X \rangle$ denotes the expected value of X.

A necessary condition for noncontextuality of a system of categorical random variables

Contextuality in Contextuality-by-Default theory

Criterion for cyclic systems

A necessary condition for noncontextuality of a system of categorical random variables (Dzhafarov, Cervantes, & Kujala, 2017)

Nominal dominance. If the system of all possible dichotomizations of a system of categorical random variables is noncontextual, then for each pair of categorical random variables in the same connection

$$\Pr\left[R_q^c = x\right] < \Pr\left[R_q^{c'} = x\right]$$

holds for no more than one value x.

Contextuality in Contextuality-by-Default theory

Nominal dominance - Example

Let X, Y, Z have (marginal) distributions given by

	$\Pr(X = x)$	$\Pr(Y = y)$	$\Pr(Z = z)$
1	.7	.1	.30
2	.1	.7	.30
3	.1	.1	.25
4	.1	.1	.05

Nominal dominance holds (mutually) for X and Y.

Contextuality in Contextuality-by-Default theory

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Nominal dominance holds (mutually) for X and Y.
It does not for Z and either X or Y.

Snow Queen experiment

The Snow Queen by Elena Ringo.

Hans Christian Andersens The Snow Queen story involves the following characters with the following characteristics:

- Snow Queen is Beautiful and Evil.
- Gerda is Beautiful and Kind.
- The Troll is Unattractive and Evil.
- The Old Finn Woman is Unattractive and Kind.

Snow Queen experiment

R_1^1	R_2^1	·	·
·	R_2^2	R_3^2	·
·	·	R_3^3	R_4^3
R_1^4	·	·	R_4^4
q_1	q_2	q_3	q_4

$c_1 = (q_1, q_2)$

$c_2 = (q_2, q_3)$

$c_3 = (q_3, q_4)$

$c_4 = (q_1, q_4)$

$\boxed{\mathcal{R}}$

Choices:

- q_1 Gerda / Troll
- q_2 Beautiful / Unattractive
- q_3 Snow Queen / Old Finn woman
- q_4 Kind / Evil

Snow Queen experiment

	Character choice	Characteristic choice	N total (correct)
Context 1	⋆ Gerda ⋆ Troll	⋆ Beautiful ⋆ Unattractive	447 (425)
Context 2	⋆ Snow Queen ⋆ Old Finn Woman	⋆ Beautiful ⋆ Unattractive	446 (410)
Context 3	⋆ Snow Queen ⋆ Old Finn Woman	⋆ Kind ⋆ Evil	453 (388)
Context 4	⋆ Gerda ⋆ Troll	⋆ Kind ⋆ Evil	453 (429)

Snow Queen experiment

Context 1	Beautiful	Ugly	Mar. Character
Gerda	0.887	0	0.887
Troll	0	0.113	0.113
Mar. Characteristic	0.887	0.113	1 (equality)

Context 2	Beautiful	Ugly	Mar. Character
Snow Queen	0.837	0	0.837
Old Finn woman	0	0.163	0.163
Mar. Characteristic	0.837	0.163	1 (equality)

Context 4	Kind	Evil	Mar. Character
Gerda	0.841	0	0.841
Troll	0	0.159	0.159
Mar. Characteristic	0.841	0.159	1 (equality)

Context 3	Kind	Evil	Mar. Character
Snow Queen	0	0.626	0.626
Old Finn woman	0.374	0	0.374
Mar. Characteristic	0.374	0.627	0 (equality)

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Prominently inconsistent connectedness ("signaling") — the main difficulty for contextuality analysis of behavioral/social data.

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Snow Queen experiment

Context 1	Beautiful	Ugly	Mar. Character
Gerda	0.843	0.020	0.864
Troll	0.029	0.107	0.136
Mar. Characteristic	0.872	0.128	0.951 (equality)

Context 2	Beautiful	Ugly	Mar. Character
Snow Queen	0.769	0.011	0.780
Old Finn woman	0.070	0.150	0.220
Mar. Characteristic	0.839	0.161	0.919 (equality)

Context 4	Kind	Evil	Mar. Character
Gerda	0.797	0.035	0.832
Troll	0.018	0.150	0.168
Mar. Characteristic	0.815	0.185	0.947 (equality)

Context 3	Kind	Evil	Mar. Character
Snow Queen	0.135	0.537	0.672
Old Finn woman	0.320	0.008	0.328
Mar. Characteristic	0.455	0.545	0.143 (equality)

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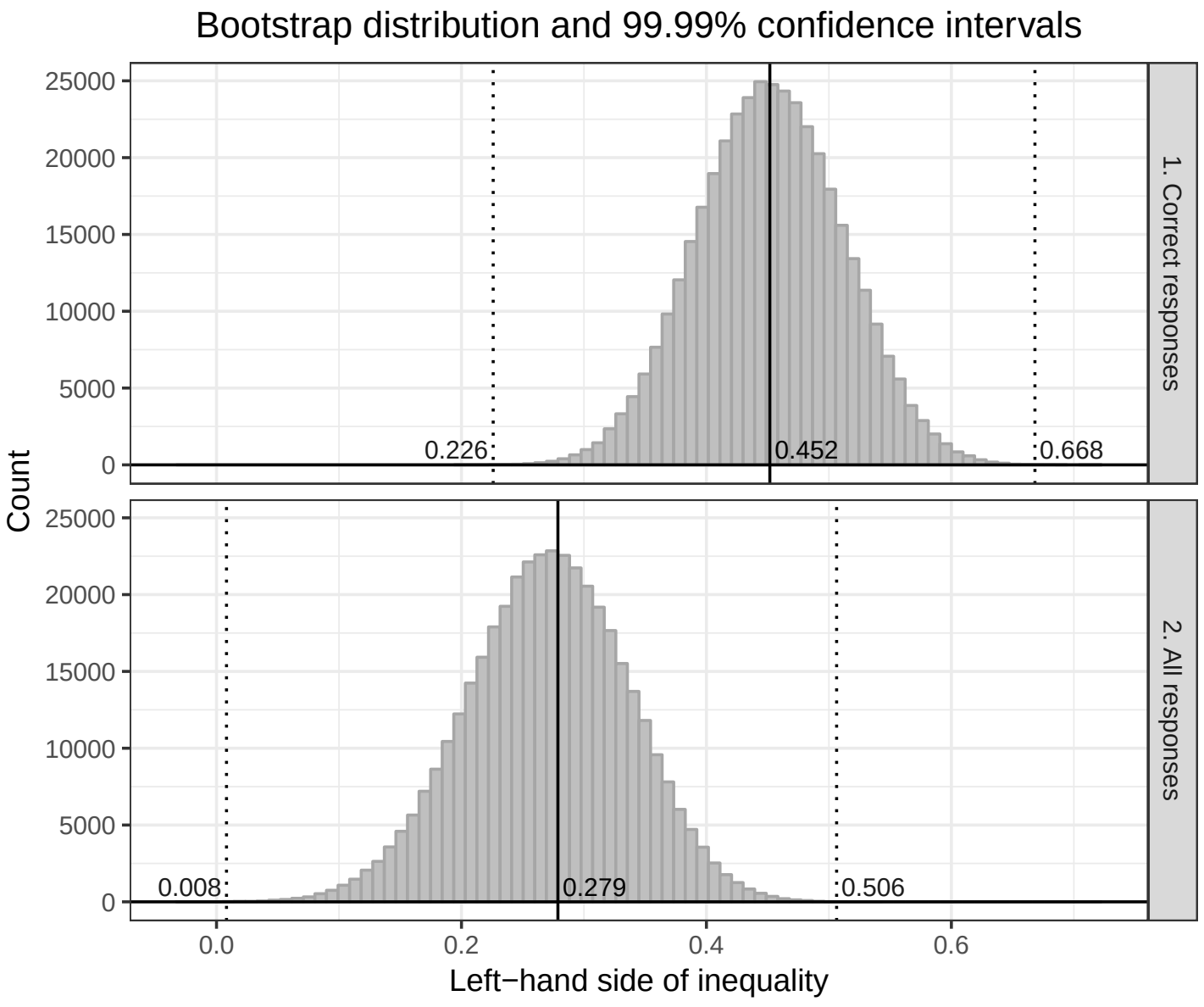
$$\max_{\text{odd \# of } -'\text{s}} \sum_{i=1}^{n} \pm \left\langle R_i^i R_{i\oplus 1}^i \right\rangle - \sum_{i=1}^{n} \left| \left\langle R_i^{i\ominus 1} \right\rangle - \left\langle R_i^i \right\rangle \right| - n + 2 = 0.280 > 0$$

Snow Queen experiment



Double identification experiment

Participants

Three volunteers, one female and two males, graduate students at Purdue University, served as participants for the experiment

Procedure

- $20 - 23$ sessions of about 30 min each.
- Approximately 380 trials per session.
- At least 8000 total trials.

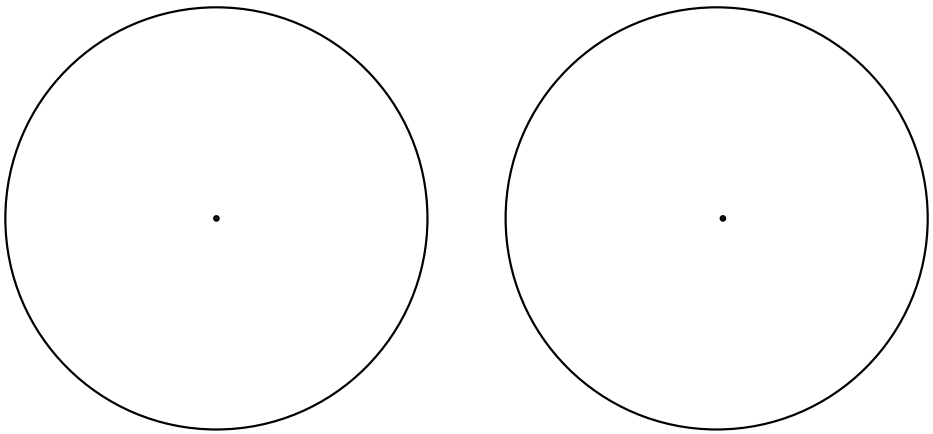
Double identification experiment

Procedure

- $20 - 23$ sessions of about 30 min each.
- Approximately 380 trials per session.
- At least 8000 total trials.
- Five locations in each circle:
  - Center
  - Left
  - Right
  - Up
  - Down

Double identification experiment

Stimulus example



Double identification experiment

Properties (q_i) and contexts $c_j = (q_i, q_{i'})$:

		Right circle				
		(-c)	(-l)	(-r)	(-u)	(-d)
Left circle	Center (c-)	cc	cl	cr	cu	cd
	Left (l-)	lc	ll	lr	lu	ld
	Right (r-)	rc	rl	rr	ru	rd
	Up (u-)	uc	ul	ur	uu	ud
	Down (d-)	dc	dl	dr	du	dd

For each of the contexts, each participant responded to at least 320 trials.

Double identification experiment

	c-	-c	l-	-l	r-	-r	u-	-u	d-	-d
cc	R^{cc}_{c-}	R^{cc}_{-c}								
cl	R^{cl}_{c-}			R^{cl}_{-l}						
cr	$\star$					$\star$				
cu	$\star$							$\star$		
cd	$\star$									$\star$
lc		$\star$	$\star$							
ll			$\star$	$\star$						
lr			$\star$			$\star$				
lu			$\star$					$\star$		
ld			$\star$							$\star$
$\vdots$					$\vdots$					$\vdots$

Double identification experiment

P1	Response to content c-				
Context	Center	Left	Right	Up	Down
cc	0.335	0.497	0.000	0.162	0.006
cl	0.231	0.631	0.000	0.137	0.000
cr	0.403	0.415	0.000	0.156	0.026
cd	0.276	0.554	0.000	0.153	0.017

P3	Response to content -c				
Context	Center	Left	Right	Up	Down
cc	0.634	0.149	0.015	0.188	0.015
lc	0.700	0.104	0.039	0.151	0.006
uc	0.699	0.134	0.021	0.137	0.009
dc	0.618	0.137	0.009	0.230	0.006

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Inconsistent connectedness ("signaling") — the main difficulty for contextuality analysis of behavioral/social data.

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NOTE:

Nominal dominance for some pairs

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NOTE:

Nominal dominance for some pairs BUT not all of them.

Double identification experiment

- For each participant, the nominal dominance condition fails for at least one pair of content-sharing random variables.

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- Out of 100000 bootstrap resamples, in none of them, for none of the participants, was the condition satisfied.

Double identification experiment

- For each participant, the nominal dominance condition fails for at least one pair of content-sharing random variables.
- Out of 100000 bootstrap resamples, in none of them, for none of the participants, was the condition satisfied.
- This means that the probability with which the observed violations of this necessary condition could occur by chance is smaller than 0.00021.

Closing remarks

- Based on the CbD analysis of many published experiments, it had been hypothesized that all behavioral systems are noncontextual (Dzhafarov, Kujala, Cervantes, Zhang, & Jones, 2016; Dzhafarov, Zhang, & Kujala, 2015).

Closing remarks

- Based on the CbD analysis of many published experiments, it had been hypothesized that all behavioral systems are noncontextual (Dzhafarov et al., 2016, 2015).
- These experiments show this hypothesis to be false.

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- These experiments show this hypothesis to be false.
- We are now able to explore and find contextuality on human behavior (current work Basieva, Cervantes, Dzhafarov, & Khrennikov).

Bibliography I

Cervantes, V. H., & Dzhafarov, E. N. (2017). Advanced analysis of quantum contextuality in a psychophysical double-detection experiment. *Journal of Mathematical Psychology*, *79*, 77–84. doi: 10.1016/j.jmp.2017.03.003

Cervantes, V. H., & Dzhafarov, E. N. (in press). Snow queen is evil and beautiful: Experimental evidence for probabilistic contextuality in human choices. *Decision*.

Dzhafarov, E. N., Cervantes, V. H., & Kujala, J. V. (2017). Contextuality in canonical systems of random variables. *Philosophical Transactions of the Royal Society A*, *375*, 20160389. doi: 10.1098/rsta.2016.0389

Dzhafarov, E. N., Kujala, J. V., Cervantes, V. H., Zhang, R., & Jones, M. (2016). On Contextuality in Behavioral Data. *Philosophical Transactions of the Royal Society A*, *374*, 20150234. doi: 10.1098/rsta.2015.0234

Bibliography II

Dzhafarov, E. N., Zhang, R., & Kujala, J. V. (2015). Is there contextuality in behavioural and social systems? *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *374*(2058), 20150099. doi: 10.1098/rsta.2015.0099

Kujala, J. V., & Dzhafarov, E. N. (2016). Proof of a Conjecture on Contextuality in Cyclic Systems with Binary Variables. *Foundations of Physics*, *46*, 282–299.