Workshop:
Quantum Contextuality in Quantum Mechanics and Beyond

held in Prague on May 18 and 19, 2019

Abstracts (alphabetic order)

Sivert Aasnæss

University of Oxford, UK

Contextuality as a Resource for Shallow Circuits

Bravyi, Gosset, and König recently showed that bounded depth (shallow) quantum circuits are more powerful than their classical analogues. Their principle contribution can be described as an upper bound on the ability of any shallow classical circuit to reproduce the output statistics of a certain shallow quantum circuit.

We allow the classical circuit to sample a contextual resource, and derive their bound as a special case of an inequality introduced by Abramsky, Barbosa, and Mansfield

$$\tilde{p}_F \geq \mathrm{NCF}(e) \cdot \frac{n-k}{n}$$

Here $\tilde{p}_F$ denotes the average failure probability of a no-communication strategy e for a non-local game, $\mathrm{NCF}(e)$ is the non-contextual fraction of the strategy, and $\frac{n-k}{n}$ is a measure of the hardness of the game.

Samson Abramsky

University of Oxford, UK

Simulations of quantum resources and the degrees of contextuality

(joint work with Rui Soares Barbosa, Martti Karvonen and Shane Mansfield)

We describe a notion of simulation between quantum resources. This will also be discussed in the talk by Martti Karvonen. Here we provide a wider perspective, and discuss some directions of ongoing research. The notion of simulation is expressed as a morphism of empirical models, in a form which allows the behaviour of one set of correlations to be simulated in terms of another using classical processing and shared randomization. Mathematically, this is expressed as coKleisli maps for a comonad of "measurement protocols" on the category of empirical models. This setting is expressive, and allows for a number of variations, e.g. grading the simulation by the number of copies of the simulating resource, and also allows for a natural relaxation to approximate simulation. As with classical notions of reduciibility in computability and complexity theory, the existence of simulation maps allows us to compare different contextual behaviours in a fine-grained, mathematically robust way. We can define a degree of contextuality as an equivalence class of empirical models under two-way simulability. These degrees are then partially ordered by the existence of simulations between representatives. Some existing results from the study of non-locality can be interpreted as showing the richness of this order, and there are many natural further questions which arise. Since non-contextuality itself can be expressed as the existence of a certain special form of simulation, there are natural generalizations of existing results on contextuality. In particular, we can formulate a "generalized Vorob'ev conjecture", relating to the well-known Vorob'ev theorem.

Rui Soares Barbosa

University of Oxford, UK

Acyclicity and Vorob'ev's theorem

A measurement scenario is represented by an (abstract) simplicial complex, whose vertices are measurements and whose faces are contexts, i.e. sets of measurements that can be jointly performed. An empirical model specifies a probability distribution on joint outcomes for each context. The generalised no-signalling condition is captured by compatibility of marginals, a form of *local consistency*. Non-contextuality means that these empirical probability distributions arise as the marginals of a global distribution on joint outcomes for all the measurements, a form of *global consistency*. Contextuality thus corresponds to having *local consistency* but *global inconsistency*.

A classical result due to Vorob'ev [1], originally motivated by a problem in game theory, can be read as providing an answer to the following question:

> *for which measurement scenarios is it the case that any no-signalling empirical model is non-contextual?*

Vorob'ev derived a necessary and sufficient condition on simplicial complexes for this to be the case. This condition turns out to be equivalent to the well-studied notion of acyclic database schema, which arose in the context of relational database theory [2, Chapter 13] to address an analogous concern:

> *for which database schemata does pairwise projection consistency imply total consistency, i.e. existence of a universal relation instance?*

Note that this fits neatly with the connection presented in [3] between the study of contextuality and of relational databases. In both instances, the same condition − acyclicity − characterises exactly the situations for which local consistency implies global consistency (i.e. for which 'contextuality' cannot arise). In this talk, we discuss the notion of acyclicity under various guises, including a characterisation related to topological properties of the simplicial complexes. Moreover, we identify conditions under which an analogue of Vorob'ev's theorem can be obtained, generalising the two instances above.

This talk is based on results from [4].

[1] N. N. Vorob'ev, Consistent families of measures and their extensions, Theory of Probability and its Applications (Teoriya Veroyatnostei i ee Primeneniya) 7(2): 147–163 (English: N. Greenleaf, trans.), 153–159 (Russian), 1962.

[2] D. Maier, The theory of relational databases, Computer Science Press, Rockville, MD, 1983.

[3] S. Abramsky, Relational databases and Bell's theorem, In search of elegance in the theory and practice of computation: Essays dedicated to Peter Buneman (Tannen, Wong, Libkin, Fan, Tan, Fourman, eds.), Lecture Notes in Computer Science 8000: 13–35, 2013.

[4] R. S. Barbosa, Contextuality in quantum mechanics and beyond, DPhil thesis, University of Oxford, 2015.

Adán Cabello

University of Seville, Spain

Device-independent quantum information based on thermodynamics

We show that simulating the predictions of quantum theory for an experiment in which quantum questions taken from a big suitably chosen Kochen-Specker set are randomly asked of a black box that actually contains an isolated finite-state machine would make the black box to emit heat not related to the measurements, but to the fact that the finite-state machine has to erase its memory from time to time in order to make room for the new information needed to keep simulating the quantum predictions. Then, Landauer's principle requires that the black box emits heat. We propose using this result for a new paradigm of device-independent quantum information based on thermodynamics (rather than on Bell nonlocality) in which (instead of assuring spatial separation and testing Bell nonlocality) we assure that the black box does not emit heat and we test the quantum predictions for a big suitably chosen Kochen-Specker set. We present an example that can certify that the answers of the black box cannot be simulated by any machine built with the current technology for making classical memories.

Giovanni Carú

University of Oxford, UK

Contextuality and generic inference: a theory of disagreement

(joint work with Samson Abramsky)

The goal of this talk is to establish a strong link between two apparently un- related topics: the study of conflicting information in the formal framework of valuation algebras, and the phenomena of non-locality and contextuality. In particular, we show that these peculiar features of quantum physics are mathematically equivalent to a general notion of disagreement. This result generalises previously observed connections between contextuality, relational databases and constraint satisfaction problems, and further proves that contextuality is not a phenomenon limited to quantum mechanics, but pervades various domains of mathematics and computer science. The connection allows to translate theorems, methods and algorithms from one field to the other. We take advantage of this strong interaction to develop new algorithms of generic inference for the detection of non-locality and contextuality, which outperform the current methods.

Víctor Hernando Cervantes

Purdue University, USA

Using probabilistic couplings in data analysis

(joint work with Ehtibar N. Dzhafarov)

It is common in statistics to treat stochastically unrelated random variables as if they were stochastically independent. There is no logical justification for this, and we investigated a more principled approach: using all possible couplings and choosing one that is optimal in accordance with certain criteria. We applied the maximum likelihood meaning of optimality to the very basic task of identifying and comparing two probabilities from two stochastically unrelated sets of binary events. It can be shown that optimal couplings are readily identifiable, and the independent coupling is rarely optimal. This research may lead to rethinking of the most basic procedures of statistical analysis.

Marcelo Terra Cunha

University of Campinas, Brazil

Fibre Bundle Approach to Contextuality

Last year, in the Purdue-Winer Memorial Conference, I introduced the Fibre Bundle Approach to Contextuality [1]. In this talk I want to come back to the central ideas and results of the theme, as well as to give an update to the community of the works in progress.

[1] Marcelo Terra Cunha, On Measures and Measurements: a Fibre Bundle approach to Contextuality, arXiv:1903.0abcd (submitted to the special number of Phil. Trans. Roy. Soc. A on Contextuality.

Nadish de Silva

University College London, UK

Contextuality and classical simulation of quantum circuits

We will present results connecting contextuality with the inability for a classical computer to efficiently simulate outputs of quantum circuits.

Ehtibar Dzhafarov

Purdue University, USA

Measures of contextuality and noncontextuality

(joint work with Janne V. Kujala)

I discuss three measures of contextuality for systems of dichotomous random variables. They are developed within the framework of the Contextuality-by-Default (CbD) theory, and apply to inconsistently connected systems (those with "disturbance" allowed). For one of these measures of contextuality, presented in Ref. [1] for the first time, we construct a corresponding measure of noncontextuality. The other two CbD-based measures do not suggest ways in which degree of noncontextuality of a system can be quantified. The same is true for the contextual fraction measure developed by Abramsky, Barbosa, and Mansfield. This measure of contextuality is confined to consistently connected systems, but it can be generalized to arbitrary systems.

[1] J.V. Kujala & E.N. Dzhafarov (2019). Measures of contextuality and noncontextuality. arXiv:1903.07170.

Federico Holik

Institute of Physics La Plata - CONICET, Argentine

Why algebras of operators are so important in physics?

In this talk, we propose a new approach to the problem of explaining why algebras of operators play a key role in physics and other probasbilistic theories as well. Instead of trying to explain the singularities of Hilbert-space quantum mechanics, we assume that quantum theory is one among a huge family of models of contextual probabilistic theories. This move is motivated by the fact that, in the last decades, many models of non-Kolmogorovian probability have been considered outside the quantum domain, and found applications in biology, psychology, social sciences, economics and linguistics. While many of these approaches use Hilbert spaces, it has became evident that there is no reason to expect that the Hilbert space description will be the optimal one for every model of interest. Here, it is assumed that any system (understood as a domain of phenomena in which we are interested) can be represented by a propositional structure that acquires meaning in an empirical/operational way. We will assume that the propositional structures can be represented by bounded orthomodular lattices. The generality of this assumption is justified by the operational approach to physical theories, but also by the fact that, most important theories of physics fall into this setting: states of classical, quantum, quantum relativistic and quantum statistical theories can be described as measures over bounded orthomodular lattices. Our contribution to the problem of explaining why algebras of operators are so important is as follows: assuming that the propositional structure associated to our system of interest can be represented by an orthomodular lattice, we show that an algebra of operators acting on a suitably defined space can be constructed in a natural way. It turns out that, if the lattices are Boolean (distributive), the algebra of operators will be Abelian, while it will fail to be Abelian for non-distributive lattices. Thus, one of the most important features of quantum theory, namely, that observables must be represented by non-commutative algebras of operators, is not as strange as its seems: it is a common feature of many contextual probabilistic models. When models are non-contextual, the algebras are still there, but since they are Abelian, they find a natural representation as functions over a suitably defined phase space.

Paweł Horodecki

University of Gdansk, Poland

Randomness amplification against no-signaling adversary - from Hardy paradox to contextuality

(joint work with K. Horodecki, M. Horodecki, S. Pironio and R. Ramanathan)

The concept of randomness amplification against no-signaling eavesdropper will be discussed including the scheme based on local contextuality [1]. A new scheme of randomness amplification [2] combining the Hardy paradox [3] and the so called measurement dependent locality [4]. The security of the scheme is based on more fundamental assumptions (no-signaling principle) than those of standard quantum mechanics and much simpler from the experimental point of view than any previous protocol of this type (see [1], [5]). Finally the idea [1] how to construct Hardy paradoxes basing directly on the local contextuality proofs will be presented.

[1] R. Ramanathan, F. G. S. L. Brandao, K. Horodecki, M. Horodecki, P. Horodecki, H. Wojewódka, Phys. Rev. Lett. 117, 230501 (2016)

[2] R. Ramanathan, M. Horodecki, S. Pironio, K. Horodecki, P. Horodecki, "Generic randomness amplification schemes using Hardy paradoxes" , arXiv:1810.11648.

[3] L. Hardy, Phys. Rev. Lett. 71, 1665 (1993).

[4] G. Putz, D. Rosset, T. J. Barnea, Y.-C. Liang, and N. Gisin, Phys. Rev. Lett. 113, 190402 (2014).

[5] R. Colbeck and R. Renner, Nature Physics 8, 450 (2012); R. Arnon-Friedman and A. Ta-Shma, Physical Review A 86, 062333 (2012); R. Gallego, L. Masanes, G. De La Torre, C. Dhara, L. Aolita, A. Acín, Nature Communications 4, 2654 (2013); F. G. S. L. Brandao, R. Ramanathan, A. Grudka, K. Horodecki, M. Horodecki, P. Horodecki, T. Szarek, H. Wojewódka, Nature Communications 7, 11345 (2016); M. N. Bera, A. Acín, M. Kuś, M. Mitchell, M. Lewenstein Rep. Prog. Phys. 80, 124001 (2017).

Matt Jones

University of Colorado, USA

Causal and Probabilistic Approaches to Contextuality

A primary goal in recent research on contextuality has been to extend this concept to cases of inconsistent connectedness, where observables have different distributions in different con- texts. I propose a solution within the framework of probabilistic causal models [1,2], which extend hidden-variables theories, and then demonstrate an equivalence to the contextuality- by-default (CbD) framework [3,4]. CbD distinguishes contextuality from direct influences of context on observables, which it defines purely in terms of probability distributions. Here I take a causal view of direct influences, defining direct influence within any causal model as the probability of all hidden or latent states of the system in which a change of context changes the outcome of a measurement. Model-based contextuality (M-contextuality) is then defined as the necessity of stronger direct influences to model a full system than to model each observable individually. For consistently connected systems, M-contextuality agrees with traditional contextuality. For general systems, it is proved that M-contextuality is equivalent to the property that any model of a system must contain "hidden influences", meaning direct influences that go in opposite directions for different latent states, or equivalently signaling between observers that carries no information [5]. This criterion can be taken as formalizing the "no-conspiracy" principle that has been proposed in connection with CbD [6]. M-contextuality is then proved to be equivalent to CbD-contextuality, thus providing a new interpretation of CbD-contextuality as the non-existence of a model for a system without hidden direct influences.

[1] Pearl, J. (2000). Causality: Models, reasoning and inference. Cambridge, England: Cambridge University Press.

[2] Cavalcanti, E. G. (2018). Classical causal models for Bell and Kochen-Specker inequality violations require fine-tuning. Physical Review X, 8, 021018.

[3] Kujala, J.V., Dzhafarov, E.N., & Larsson, J.-A. (2015). Necessary and sufficient con- ditions for extended noncontextuality in a broad class of quantum mechanical systems. Physical Review Letters, 115, 150401.

[4] Dzhafarov, E.N., & Kujala, J.V. (2017). Contextuality-by-Default 2.0: Systems with binary random variables. In J.A. de Barros, B. Coecke, & E. Pothos (Eds.), Lecture Notes in Computer Science 10106, 16-32.

[5] Atmanspacher, H., & Filk, T. (in press). Contextuality revisited – Signaling may differ from communicating. Synthese.

[6] Cervantes, V.H., & Dzhafarov, E.N. (2018). Snow Queen is evil and beautiful: Experimental evidence for probabilistic contextuality in human choices. Decision, 5, 193-204.

Marcin Karczewski

Adam Mickiewicz University, Poland

Extracting entanglement from the indistinguishability of particles

(joint work with S.Y. Lee, J. Ryu, Z. Lasmar, D. Kaszlikowski, and P. Kurzyński))

The states of a system of identical particles are required to be symmetric for bosons and antisymmetric for fermions. Are these inherent correlations just a mathematical artifact or can they be treated as a resource? In the talk I will support the latter point of view by showing how operationally accessible entanglement can be extracted form the indistinguishability of particles. I will focus on a recently proposed method [1] based on tailored single-photon subtractions from multiboson states.

[1] Karczewski, Marcin, et al. Sculpting out quantum correlations with bosonic subtraction. arXiv preprint arXiv:1902.08159 (2019).

Dagomir Kaszlikowski

National University of Singapore, Singapore

On negative probabilities - a new perspective

In this rather risky talk, I will share some preliminary results on a possible use of negative probabilities in quantum theory, specifically in the problems of quantum contextuality and the so called quantum non-locality. I am prepared for a crucifixion after the talk.

Martti Karvonen

University of Edinburgh, UK

Simulations between contextual resources

(joint work with Samson Abramsky, Rui Soares Barbosa and Shane Mansfield)

We describe a resource theory for contexuality in two ways and prove them to be equivalent. The two viewpoints are given by (i) starting with a very simple notion of an empirical model simulating another one and then extending it to a more useful notion by passing over to a co-Kleisli category, and (ii) considering free algebraic operations that transform empirical models to other empirical models. These viewpoints capture various notions of intraconversions between correlations that have been studied in the literature on non-locality. An advantage of having a rigorous theory of such intraconversions is in new techniques suggested by the approach: a case in point is a no-cloning/no-broadcasting theorem which states that an empirical model can simulate two independent copies of itself iff it is non-contextual. If time permits, we will discuss ongoing work and further questions suggested by the framework.

Paweł Kurzyński

Adam Mickiewicz University, Poland

Contextuality and physics – Can spin magnitude be conserved in hidden variable models?

(joint work with W. Laskowski, A. Kołodziejski, K. F. Pál, J. Ryu, T. Vértesi)

The squares of the three components of the spin-s operators sum up to $s(s+1)$. However, a similar relation is rarely satisfied by the set of possible spin projections onto mutually orthogonal directions. This has fundamental consequences if one tries to construct a hidden variable (HV) theory describing measurements of spin projections. We propose a test of local HV models in which spin magnitudes are conserved. These additional constraints imply that the corresponding inequalities are violated within quantum theory by larger classes of correlations than in the case of standard Bell inequalities. We conclude that in any HV theory pertaining to measurements on a spin one can find situations in which either HV assignments do not represent a physical reality of a spin vector, but rather provide a deterministic algorithm for prediction of the measurement out- comes, or HV assignments represent a physical reality, but the spin cannot be considered as a vector of fixed length.

[1] Communications Physics 2, 15 (2019)

Jan-Åke Larsson

Linköping University, Linköping, Sweden

The Stabilizer formalism, Spekkens' toy theory, and Contextuality

(joint work with Felix Huber and Niklas Johansson)

The stabilizer formalism is motivated within quantum information theory mainly from its use in quantum error-correction, through discretization of errors and stabilizer codes. We will here look into what quantum stabilizers are, and in particular, what the stabilizer generators correspond to from a formal perspective. A parallel structure emerges in Spekkens' toy theory, linking the epistemic states of the theory to a similar structure constituting what could be called Spekkens' stabilizers. The two structures will be compared, in particular with respect to contextuality, and we will also discuss stabilizer codes and the possibility to perform error correction with and without contextuality.

Shane Mansfield

Sorbonne University, France

Circuit contextuality

Interest in contextuality has grown in recent years due to its links with quantum- over-classical advantages in physical implementations of informatic tasks. The intuition is clear: quantum-over-classical advantage can only be due to some non-classical behaviour of the quantum systems used in the implementation— and in many cases the behaviour is contextuality. However, contextuality, in its traditional sense which relates to measurement statistics, is not the only non-classical behaviour of quantum systems, nor is it responsible for all known quantum-over-classical advantages. I will consider some examples and propose a broader notion of circuit contextuality that may be general enough to subsume other candidate non-classical behaviours, including contextuality in its traditional sense, non-locality, instances of Spekkens' contextuality, the notion of dynamic contextuality that I introduced in joint work with Elham Kashefi, and others. This aims to lay the groundwork for a broad foundational approach to investigating non-classical behaviours, as well as their relationships to quantum-over-classical computational advantages.

Oľga Nánásiová

Slovak University of Technology in Bratislava, Slovakia

Causality and contextuality

(joint work with Mária Bohdalová, Martin Kalina, Karla Čipková)

In our contribution, we deal with the issues of the stochastic causality and the contextuality. Of course, the difference between these two concepts depends on the specifically chosen definitions and on the axiomatic system used by the model. In our approach, we use the model of non-compatible random variables (or, more specifically, non-compatible observable) introduced by Birkhoff and von Neumann [1]. Paper [1] has initiated a lively discussion not only among physicians but also among mathematicians and it has so far inspired many scientists. Let us cite [1]:

"It asserts that even a complete mathematical description of a physical system Θ *does not in general enable one to predict with certainty the result of an experiment on* Θ*, and that in particular one can never predict with certainty both the position and the momentum of* Θ *(Heisenberg's Uncertainty Principle). It further asserts that most pairs of observations are incompatible, and cannot be made on* Θ *simultaneously (Principle of Non-commutativity of observations)."*

It is obvious that each mathematical problem that uses some axiomatic system must neglect some properties of the reality. In our contribution, we introduce the concept of stochastic causality and contextuality on quantum logic model. Our aim is to show the relationships between these two concepts. The paper follows papers [2-5].

[1] Birkhoff, G., von Neumann, J. (1936). The Logic of Quantum Mechanics. Annals of Mathematics, 37(4), second series, 823–843.

[2] Nánásiová, O. (2003): Map for simultanous measuremants for a quantum logic. Int. J. of Theoretical Physics, **42**, 1889–1903.

[3] Nánásiová, O., Kalina, M. (2015): Calculus for non-compatible observables, construction through conditional states. *Int. J. Theor. Phys.*, **54**, 506–518.

[4] Nánásiová O., Valášková, Ľ. (2010): Maps on a quantum logic. Soft Computing, **14**, Issue 10, 1047–1052

[5] Bohdalová, M., Kalina, M., Nánásiová, O. (2016) Granger causality from a different viewpoint. Informační bulletin České statistické společnosti, **27**, No. 2, 23–28.

Cihan Okay

University of British Columbia, Canada

What homotopy tells us about contextuality

(joint work with Robert Raussendorf)

Quantum assignments that are not classically realizable can be seen as proofs of contextuality generalizing the Mermin-Peres square. A quantum assignment consists of a collection of operators, labeled by the vertices of an hypergraph, satisfying a relation for each edge (context). If the set of relations cannot be satisfied by the eigenvalues of the operators then the assignment is called magic, also known as a proof of contextuality. When each vertex is contained in exactly two edges and the operators has eigenvalues ± 1 Arkhipov [1] proved that the assignment is magic if and only if the intersection graph (dual of the given hypergraph) is not planar. The goal of this work is to generalize this result by removing the restrictions on the hypergraph and the eigenvalues. Our approach is to use the topology of two-dimensional cell complexes to study the operator relations, and use topological criteria to decide whether the assignment is magic. A two-dimensional cell complex X, called a topological realization, can be constructed from the operator relations. Then the fundamental group of X, denoted by $\pi_1(X)$, can be used to detect contextuality generalizing one direction of Arkhipov's result. More precisely, we show that if the assignment is magic then $\pi_1(X)$, is non-trivial for any topological realization of the operator relations.

[1] Arkhipov, Alex. "Extending and characterizing quantum magic games." arXiv preprint arXiv:1209.3819 (2012).

Ravishankar Ramanathan

The University of Hong Kong, Hong Kong

Gadget structures in proofs of the Kochen-Specker theorem

(joint work with Monika Rosicka, Karol Horodecki, Stefano Pironio, Michał Horodecki, Paweł Horodecki)

The Kochen-Specker theorem is a fundamental result in quantum foundations that has spawned massive interest since its inception. We show (in [1]) that within every Kochen-Specker graph, there exist interesting subgraphs which we term 01-gadgets [2], that capture the essential contradiction necessary to prove the Kochen-Specker theorem, i.e., every Kochen-Specker graph contains a 01-gadget and from every 01-gadget one can construct a proof of the Kochen-Specker theorem. Moreover, we show that the 01-gadgets form a fundamental primitive that can be used to formulate state-independent and state-dependent statistical Kochen-Specker arguments as well as to give simple constructive proofs of an "extended" Kochen-Specker theorem first considered by Pitowsky [3]. We conclude with some recent developments on the topic.

[1] R. Ramanathan, M. Rosicka, K. Horodecki, S. Pironio, M. Horodecki and P. Horodecki. Gadget structures in proofs of the Kochen-Specker theorem. arXiv: 1807.00113 (2018).

[2] R. K. Clifton. Getting Contextual and Nonlocal Elements of Reality the Easy Way. American Journal of Physics, 61: 443 (1993).

[3] I. Pitowsky. Infinite and finite Gleason's theorems and the logic of indeterminacy. Journal of Mathematical Physics 39, 218 (1998).

Karl Svozil

Vienna University of Technology, Austria

Kolmogorov-type conditional probabilities among distinct context

We suggest to apply Kolomogorov's axioms of probability theory to conditional probabilities among distinct (but not necessarily disjoint and non-intertwining) contexts. Formally, this amounts to bistochastic matrices whose entries characterize the conditional probability to find some observable in one context, given an observable in another context. As the respective probabilities need not (but, depending on the physical/model realization, can) be of the Born rule type, this generalizes similar approaches to quantum probabilities by Aufféves and Grangier, which in turn are inspired by Gleason's theorem.