The 4th Workshop:
Quantum Contextuality in Quantum Mechanics and Beyond

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Abstracts (alphabetic order)

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Contextuality for Circuits

It has been proposed that a quantum circuit should be seen as contextual when it lacks a classical probabilistic interpretation that preserves the compositional structure of the gates. We show that this is consistent with the traditional definition of contextuality in the sense that a circuit is contextual precisely when it cannot be translated into a non-contextual measurement based quantum computer of the same shape.

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Partial boolean algebras and the logical exclusivity principle

(joint work with Rui Soares Barbosa)

Kochen and Specker's seminal work on contextuality used the formalism of partial boolean algebras. Unlike quantum logic in the sense of Birkhoff – von Neumann, partial boolean algebras only admit physically meaningful operations. We describe a refinement of current approaches to contextuality, in particular the sheaf-theoretic and graph-theoretic approaches, to incorporate partial boolean algebras. We discuss some striking and little-known results of Conway and Kochen (*not* the so-called "Free Will Theorem"!) in relation to this. We introduce a new axiom for partial boolean algebras, the Logical Exclusivity Axiom, and show that all probability models based on partial boolean algebras satisfying this axiom obey Specker's Exclusivity Principle.

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From Vorob'ev's theorem to monogamy of non-locality and local macroscopic averages

Vorob'ev's theorem characterises those configurations of contexts (i.e. measurement scenarios) which are inherently classical in the sense that the no-signalling or no-disturbance condition is enough to ensure locality or non-contextuality. Even if at first glance these scenarios may look a bit boring from the point of view of contextuality, we will discuss how Vorob'ev's result can be applied to provide an elegant structural explanation for two related phenomena: (1) monogamy of non-locality, which establishes a trade-off between strength of non-locality shared between a party and multiple others, and (2) locality of average macroscopic behaviour, regardless of the non-classicality present in the microscopic state of a system. Since Vorob'ev's theorem depends solely on the compatibility structure of measurements, these results hold not just for quantum theory, but for any empirical behaviours satisfying only the no-signalling condition.

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Local certification of programmable quantum devices of arbitrary high dimensionality

(joint work with Maharshi Ray, Antonios Varvitsiotis, Adán Cabello and Leong-Chuan Kwek)

The onset of the era of fully-programmable error-corrected quantum computers will be marked by major breakthroughs in all areas of science and engineering. These devices promise to have significant technological and societal impact, notable examples being the analysis of big data through better machine learning algorithms and the design of new materials. Nevertheless, the capacity of quantum computers to faithfully implement quantum algorithms relies crucially on their ability to prepare specific high-dimensional and high-purity quantum states, together with suitable quantum measurements. Thus, the unambiguous certification of these requirements without assumptions on the inner workings of the quantum computer is critical to the development of trusted quantum processors. One of the most important approaches for benchmarking quantum devices is through the mechanism of self-testing that requires a pair of entangled non-communicating quantum devices. Nevertheless, although computation typically happens in a localized fashion, no local self-testing scheme is known to benchmark high dimensional states and measurements. Here, we show that the quantum self-testing paradigm can be employed to an individual quantum computer that is modelled as a programmable black box by introducing a noise-tolerant certification scheme. We substantiate the applicability of our scheme by providing a family of outcome statistics whose observation certifies that the computer is producing specific high-dimensional quantum states and implementing specific measurements.

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Measuring locality vs free choice in Bell experiments

(joint work with E. M. Pothos, Y. M. Yearsley, C. Gallus and E. Borsuk)

Locality and free choice are the two assumptions in derivation of Bell inequalities which lead to nontrivial constraints on the strength of correlations in a simple experimental scenario. Violation of the inequalities means that at least one of them has to fail if the causal picture is to be maintained. In this talk, I will discuss the extent to which one of the assumptions needs to be relaxed for the other one to hold at all costs.

It is crucial to observe that the failure of locality or freedom of choice does not have to be the case on every experimental trial and the correlations still could be explained (e.g., by occasional communication or control over the settings). This lends itself to a good measure defined as the minimal rate at which given assumption must fail to allow for simulation of a given experimental statistics. I will consider both cases separately, measure of locality and measure of free choice, and show that for any non-signalling statistics in the Bell scenario it is directly related to the amount of violation of the Clauser-Horne-Shimony-Holt inequalities.

I will also comment on the particular case of the quantum-mechanical predictions, where for a maximally entangled state both measures tend to zero if the number of settings goes to infinity (it means non-locality or no free choice on every experimental trial).

[1] Blasiak, P., Pothos, E. M., Yearsley, J. M., and Gallus, C.
"Measuring non-locality in Bell experiments" (to be published) (2020)

[2] Blasiak, P., Pothos, E. M., Yearsley, J. M., and Gallus, C.
"Measuring free choice in Bell experiments" (to be published) (2020)

[3] Blasiak, P., Borsuk, E., and Pothos, E. M.
"No free choice under locality assumption" (to be published) (2020)

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Quantum Contextuality: from logical contradictions to experimental tests

Contextuality refers to the impossibility of reproducing the outcome statistics of freely and independently chosen measurements, under the assumption that ideal measurements reveal pre-existing values that are independent of the context each measurement belongs to.

Originated in the work of Kochen and Specker in the 1960s, this notion has been further developed in the subsequent fifty years by several authors in different directions. In particular, several approaches has been proposed for the problem of performing experimental tests of contextuality.

In this contribution, we provide an overview of some of the main approaches to quantum contextuality and its experimental tests; we address the different definitions of noncontextual hidden variable theories and noncontextuality inequalities, the physical assumptions on measurement operations involved in experimental tests of contextuality, and so on.

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Tsirelson's problem and contextuality

Ji *et al.* (arXiv:2001.04383) have recently shown that there exists bipartite Bell scenarios for which the closure of the set of quantum tensor product correlations is strictly contained in the set of quantum commuting correlations. Here, we show that there is a one-to-one correspondence between relativistic n-partite Bell nonlocal correlations and nonrelativistic Kochen-Specker (KS) contextual correlations for KS scenarios with n-partite graphs of compatibility. As a consequence, the result of Ji *et al.* implies that, in the nonrelativistic regime, KS contextuality is strictly more powerful than Bell nonlocality as there are correlations that can be produced by sequences of compatible ideal measurements on an indivisible quantum system that cannot be obtained with measurements on parts of a composite quantum system. This in turn implies that it can be experimentally tested that a quantum computer consisting of an indivisible quantum system is strictly more powerful than one based on a composite quantum system.

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Noncontextuality Inequalities from Antidistinguishability

(joint work with: Matthew S. Leifer)

Noncontextuality inequalities are usually derived from distinguishability properties of objects. In particular, when considering quantum states, it is customary to use orthogonality for distinguishing them. In this talk, I will try to convince you that antidistinguishability can also be used to derive noncontextuality inequalities. The Yu-Oh 13 ray contextuality inequality can be rederived and generalized as an instance of this antidistinguishability method. Remarkably, for some sets of states, the antidistinguishability method gives tighter bounds on noncontextual models than just considering orthogonality, and the Hadamard states provide an example of this. Antidistiguishability-based inequalities were initially discovered as overlap bounds for the reality of the quantum state. The main contribution I want to put out is that they are also noncontextuality inequalities..

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All About Cyclic Systems

(joint work with Janne V. Kujala and Víctor H. Cervantes)

Cyclic systems of dichotomous random variables have played a prominent role in contextuality research, describing such experimental paradigms as the KCBS, EPR/Bell, and Leggett-Garg ones. We present a theoretical analysis of the degree of contextuality in cyclic systems (if they are contextual) and the degree of noncontextuality in them (if they are not). The Contextuality-by-Default (CbD) theory allows us to drop the commonly made assumption that systems of random variables are consistently connected (i.e., we allow for all possible forms of "disturbance" or "signaling" in them). By contrast, all previously proposed measures of contextuality are confined to consistently connected systems, and most of them cannot be extended to measures of noncontextuality. Our measures of contextuality and noncontextuality are defined by the L_1-distance between a point representing a cyclic system and the surface of the polytope representing all possible noncontextual cyclic systems with the same single-variable marginals. We completely characterize this polytope, and establish that, in relation to the maximally tight Bell-type CbD inequality for cyclic systems, the measure of contextuality is proportional to the absolute value of the difference between its two sides. For noncontextual cyclic systems, the measure of noncontextuality is shown to be proportional to the smaller of the same difference and the L_1-distance to the surface of the hyperbox circumscribing the noncontextuality polytope. These simple relations, however, do not generally hold beyond the class of cyclic systems, and noncontextuality of a system does not follow from noncontextuality of its cyclic subsystems. We also compute the volumes of the noncontextuality polytope and the circumscribing hyperbox to answer the following question: if one chooses a cyclic system "at random" (i.e., uniformly within the hyperbox), what are the odds that it will be (non)contextual? We find that the odds of contextuality rapidly tend to zero as the size of the system increases.

[1] Dzhafarov, E.N., Kujala, J.V., & Cervantes, V.H. Contextuality and noncontextuality measures and generalized Bell inequalities for cyclic systems. Physical Review A, in press (arXiv:1907.03328, 2020).

[2] Dzhafarov, E.N., Kujala, J.V., & Cervantes, V.H. Epistemic odds of contextuality in cyclic systems. arXiv:2002.07755, 2020.

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Bounds on overlaps give us coherence, contextuality and nonlocality inequalities

(joint work with Daniel J. Brod and Rui S. Barbosa)

A SWAP test is a quantum circuit that measures the overlaps $r_{\rho\sigma} = Tr(\rho\sigma)$ between two states ρ, σ. A simple photonic implementation of a SWAP test consists of a Hong-Ou-Mandel interferometry test between two single photons. If we consider a set of N quantum states, different bounds on two-state overlaps result when we either have i) diagonal, coherence-free states, or ii) general quantum states. The difference between i) and ii) allowed us to propose novel basis-independent coherence witnesses [1]. I will also show how the bounds i) (for coherence-free states) also give us noncontextuality and locality inequalities, which suggests a unified framework for resource theories of coherence, contextuality and nonlocality.

[1] Ernesto F. Galvão and Daniel J. Brod, "Quantum and classical bounds for unknown two-state overlaps". https://arxiv.org/abs/1902.11039

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Proof of the Peres conjecture for contextuality

(joint work with Zhen-Peng Xu and Jing-Ling Chen)

A central result in the foundations of quantum mechanics is the Kochen-Specker theorem. In short, it states that quantum mechanics cannot be reconciled with classical models that are noncontextual for compatible observables. The first explicit derivation by Kochen and Specker was rather complex, but considerable simplifications have been achieved thereafter. We propose a systematic approach to find Hardy-type and Greenberger-Horne-Zeilinger-type (GHZ-type) proofs of the Kochen-Specker theorem, these are characterized by the fact that the predictions of classical models are opposite to the predictions of quantum mechanics. With this approach, we find the provably minimal GHZ-type proof. Based on our results, we show that the Kochen-Specker set with 18 vectors from Cabello et al. [1] is the minimal set for any dimension, verifying a long-standing conjecture by Peres. Our results allow to identify minimal contextuality scenarios and to study their usefulness for information processing.

[1] A. Cabello, J. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

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Simple Communication Complexity Separation from Quantum State Antidistinguishability

(joint work with Jonathan Barrett)

A set of n pure quantum states is called antididistinguishable if there exists an n-outcome measurement that never outputs the outcome 'k' on the k-th quantum state. We describe sets of quantum states for which any subset of three states is antidistinguishable and use this to produce a two-player communication task that can be solved with $\log d$ qubits, but requires one-way communication of at least $\log(4/3)(d-1) - 1 \approx 0.415(d-1) - 1$ classical bits. The advantages of the approach are that the proof is simple and self-contained – not needing, for example, to rely on hard-to-establish prior results in combinatorics – and that with slight modifications, non-trivial bounds can be established in any dimension ≥ 3. The task can be framed in terms of the separated parties solving a relation. We show, however, that for this particular task, the separation disappears if two-way classical communication is allowed, or if we only require a bounded error solution. We then state a conjecture regarding antidistinguishability of sets of states, and provide supporting numerical evidence. If the conjecture holds, then there is a two-player communication task that can be solved with $\log d$ qubits, but requires one-way communication of $\Omega(d \log d)$ classical bits.

Finally, I will discuss if it is possible to make the above complexity separation noise-robust by using a class of non-contextuality inequalities of Leifer and Duarte [1].

[1] M. Leifer, C. Duarte, "Noncontextuality Inequalities from Antidistinguishability". arXiv:2001.11485

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On global states of collections of random variables

One of the characteristic traits of quantum theory is that the description of a quantum system involves a collection of incompatible measurement contexts. Each context can be seen as a classical random variable, defined by a complete set of commuting observables. But it turns out that contexts are intertwined: quantum probabilistic models can be described as very specific pastings of Boolean algebras, which are globally non-Boolean. States are represented by density operators that define global states, and give place to classical probabilities when restricted to the maximal Boolean subalgebras associated to measurement contexts. The characterization of the peculiar pasting occurring in the quantum domain has been a topic of much research, and is related to the understanding of quantum contextuality. In this talk we discuss different techniques for combining collections of (possibly non-compatible) random variables in such a way that one obtains -as in the quantum case- a global state that yields classical probabilities when restricted to the local Boolean subalgebras. After commenting different approaches related to the possibility of using negative probabilities, we address the well known problem of pasting families of Boolean algebras. We discuss some of our findings with regard to the problem of defining global objects representing states of contextual probabilistic theories.

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General Causal-Model Characterization of Contextuality

This work extends my previous results [1] characterizing contextuality in terms of probabilistic causal models, paralleling [2] but applicable to cases of inconsistent connectedness (i.e., violation of no-disturbance). There, I proved the equivalence of three definitions for contextuality of any measurement system M: (1) there exists no causal model for M that simultaneously minimizes all direct influences of context on measurement outcomes, (2) any causal model for M contains hidden influences (influences that go in opposite directions for different latent states, or equivalently signaling that carries no information), and (3) contextuality as defined in the Contextuality-by-Default (CbD) theory [3]. These previous results were limited to a class of causal models having a particular canonical structure. Here I generalize the results to arbitrary causal models.

[1] Jones, M. (2019). Relating causal and probabilistic approaches to contextuality. Philosophical Transactions of the Royal Society A, 377, 20190133. (Presented at QCQMB19)

[2] Cavalcanti, E. G. (2018). Classical causal models for Bell and Kochen-Specker inequality violations require fine-tuning. Physical Review X, 8, 021018.

[3] Dzhafarov, E.N., & Kujala, J.V. (2017). Contextuality-by-Default 2.0: Systems with binary random variables. In J.A. de Barros, B. Coecke, & E. Pothos (Eds.), Lecture Notes in Computer Science 10106, 16-32.

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A little bit of classical magic to simulate quantum behaviour

(joint work with Pawel Kurzynski)

We introduce nebit, a classical bit with a signed probability distribution, and show that classical stochastic dynamics supplemented with nebits can achieve or even exceed the speedup of Grover's quantum search algorithm. The proposed classical dynamics never reveals negativity of nebits and thus we do not need any operational interpretation of negative probabilities. We will use this results to speculate how to simulate quantum mechanical antics, including quantum contextuality.

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Noncontextual Hamiltonians: Quasi-Quantization and Classical Simulation

(joint work with Peter J. Love)

We use contextuality to evaluate the variational quantum eigensolver (VQE), a promising tool for near-term quantum simulation. We present an efficiently computable test to determine whether or not the Hamiltonian in a VQE procedure is noncontextual (in the sense that its Pauli terms admit consistent joint outcome assignments). In the noncontextual case, we show how to construct a quasi-quantized model for the set of Pauli terms, composed of a set of allowed joint outcome assignments, together with probability distributions over them, representing classical uncertainty. We demonstrate how to use the quasi-quantized model to classically simulate a noncontextual VQE procedure. We also use it to show that the NONCONTEXTUAL HAMILTONIAN PROBLEM — the problem of deciding whether the ground state of a noncontextual Pauli Hamiltonian lies below a given energy gap — is NP-complete. These results support the notion of noncontextuality as classicality in VQE.

[1] Kirby, W. M. and Love, P. J., Phys. Rev. Lett. 2019, 123, 200501.

[2] Spekkens, R. W., in Quantum Theory: Informational Foundations and Foils, Chiribella, G., Spekkens, R. W., Eds.; Springer Netherlands: Dordrecht, 2016, pp 83-135.

[3] Kirby, W. M. and Love, P. J., Classical Simulation of Noncontextual Pauli Hamiltonians, arXiv preprint (2020), arxiv:2002.05693 [quant-ph].

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Copula-Based Analysis of Contextuality

Following the notation of Contextuality-by-Default (CbD), we consider a system of random variables indexed by contents (properties) and contexts, with random variables sharing a context being jointly distributed (observed together). In CbD, one considers couplings of all random variables, with a certain condition imposed on the subcouplings corresponding to connections (random variables sharing the same content) to allow analysis of contextuality under inconsistent connectedness (when marginal distributions of random variables in a connection are not the same across contexts) generalizing the requirement of identity subcouplings implied in traditional analysis when the system is consistently connected.

In the present variant of CbD, we propose to analyze the contextuality of a system of real-valued random variables based on the copulas of random variables sharing context. The copula represents the interdependencies of random variables by representing them as variablewise increasing transformations of jointly distributed $[0, 1]$-uniform random variables. These increasing transformations encode how the values of random variables sharing a content relate to each other in different contexts. Therefore, in effect, the original system is transformed in a certain well-defined way into a new system satisfying consistent connectedness (all marginals $[0, 1]$-uniform) and contextuality analysis can then be performed essentially in the traditional way for this new, simpler system.

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Contextuality of quantum dynamics

(joint work with T. Kopyciuk and M. Lewandowski)

I am going to discuss a simple single-partite quantum dynamical model known as a quantum walk (QW). Many of its features are simulable by classical light. Nevertheless, this model can be used to efficiently implement quantum computation (e.g. Grover-like search algorithm). Therefore, there must be some non-classicality in QW and the problem is to find it. It was recently proven that logical pre- and post-selection (PPS) paradoxes are equivalent to proofs of contextuality [1,2]. The goal of this presentation is to show that logical PPS paradoxes can be easily found in QW [3], which would shed some light on the non-classicality of this model.

[1] M. F. Pusey, Phys. Rev. Lett. 113, 200401 (2014)
[2] M. F. Pusey and M. S. Leifer, Proc. QPL, EPTCS vol 195, p 295 (2015)
[3] T. Kopyciuk, M. Lewandowski, and P. K., New J. Phys. 21, 103054 (2019)

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Self-testing mutually unbiased bases in higher dimensions with space-division multiplexing technology

(joint work with: Máté Farkas, Jaime Cariñe, Gustavo Cañas)

In the new approach of device-independent quantum information processing (QIP), the proper implementation of a given task can be self-tested solely from the recorded statistics and without detailed models of the devices used. Thus, promising practical verification schemes for new quantum technologies. Here, we experimentally study whether self-testing certification schemes can be adopted to validate the new platform of space-division multiplexing technologies to QIP. Specifically, we consider the prepare-and-measure protocol of M. Farkas and J. Kaniewski (Phys. Rev. A 99, 032316) for self-testing measurements corresponding to mutually unbiased bases (MUBs) in a dimension $d > 2$. In our scheme, the preparation and measurement procedures are implemented with a multi-arm interferometer based on modern multi-core fibers and related technology. Given the high optical quality achieved, we are able to self-test the implementation of two four-dimensional MUBs. We also quantify two operational quantities of the measurements: (i) the incompatibility robustness, connected to Bell violations, and (ii) the randomness extractable from the outcomes. Since MUBs lie at the core of several quantum information protocols, our results constitute an important step towards practical self-testing of new high-dimensional quantum technologies.

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Variational Quantum Eigensolvers and contextuality

(joint work with Andrew Zhao, Andrew Tranter, William M. Kirby, Shu Fay Ung, Akimasa Miyake)

The variational quantum eigensolver (VQE) is the leading candidate for practical applications of Noisy Intermediate Scale Quantum (NISQ) devices. The method has been widely implemented on small NISQ machines in both superconducting and ion trap implementations. I will review progress to date and discuss two questions: how quantum mechanical are small VQE demonstrations and can VQE be implemented at the scale of devices capable of exhibiting quantum supremacy, around 50 qubits? I will discuss some recent techniques to reduce the number of measurements required, which use the concept of contextuality.

[1] Measurement reduction in variational quantum algorithms, Andrew Zhao, Andrew Tranter, William M. Kirby, Shu Fay Ung, Akimasa Miyake, Peter Love arXiv:1908.08067 [quant-ph]

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Contextual Advantage in a Communication Task

(joint work with Pierre-Emmanuel Emeriau, Mark Howard and Anna Pappa)

I will present a communication task with quantum-over-classical advantage for information-carrying systems of fixed dimesion. Optimal quantum strategies for this task rely on the use of magic states with maximal Wigner negativity, which in turn can be related both to contextuality in the sense of Bell–Kochen–Specker and to sequential contextuality [1].

[1] Mansfield, S., & Kashefi, E. (2018). Quantum advantage from sequential-transformation contextuality. Physical Review Letters, 121(23), 230401.

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Wigner's Friends and Contextuality

(joint work with Marek Żukowski)

Recently we observe growing interest in extended versions of famous Wigner's Friend paradox ([1], [2]), which question the internal consistency of quantum theory as well as objectivity of outcomes of quantum measurements. In this talk I will explain the supposed paradoxes within an operational approach to quantum theory indicating their relations with quantum contextuality.

[1] D. Frauchiger, R. Renner, Nat. Comm. **9**, 3711 (2018).
[2] C. Brukner, Entropy **20**, 350 (2018);

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Contextuality Studies: From Physics and Cognition to Language and AI

Contextuality is a transdisciplinary phenomenon that may be observed across the sciences. Contextuality in quantum physics and cognitive science has been discussed extensively in the QCQMB community, and so in this talk, we focus upon contextuality in natural language semantics, natural language processing, and artificial intelligence, which is in a stark contrast with the more traditional principle of compositionality. We explicate and articulate, inter alia, the relationships between contextuality and compositionality in language and AI. We also attempt to give a classification of contextuality in different sciences.

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What determines orthomodular lattices and orthoalgebras

(joint work with John Harding, Chris Heunen, and Bert Lindenhovius)

Orthomodular lattices, as one of candidate structures of events in a quantum system, are not determined by their spaces of states (probability measures); these can be arbitrary compact convex sets [4]. Orthomodular lattices ar also not characterized by their automorphism groups; these can be arbitrary groups. Even the knowledge of both the state space and automorphism group is not sufficient to determine an orthomodular lattice uniquely [1].

In contrast to these results, every orthomodular lattice can be reconstructed from its lattice of subalgebras, even from the lattice of *Boolean* subalgebras [2]. Recently, this result was generalized to orthoalgebras [3]. For the reconstruction, we need only the structure of the poset of Boolean subalgebras, without any reference to their elements, and Boolean subalgebras isomorphic to 2^n, $n \leq 3$, suffice. As a by-product, we obtained a new representation of orthomodular lattices and orthoalgebras.

[1] Harding, J., Navara, M.: Embeddings into orthomodular lattices with given centers, state spaces and automorphism groups. *Order* **17** (2000), 239–254. DOI 10.1023/A:1026593007940

[2] Harding, J., Navara, M.: Subalgebras of orthomodular lattices. *Order* **28** (2011), 549–563. DOI 10.1007/s11083-010-9191-z

[3] Harding, J., Heunen, Ch., Lindenhovius, B., Navara, M.: Boolean subalgebras of orthoalgebras. *Order* **36** (2019), 563–609. DOI 10.1007/s11083-019-09483-6

[4] Shultz, F. W.: A characterization of state spaces of orthomodular lattices. *J. Comb. Theory* **17** A (1974), 317–328.

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Stable homotopy and quantum contextuality

Linear constraint systems (LCS) provide instances of Kochen–Specker type contextuality proofs generalizing the well-known example of Peres–Mermin square. A LCS is specified by a linear system of equations. Solutions in the unitary group are called quantum realizations. In the contextual case no solution exists in the group of scalar matrices. Homotopy theory has proved to be useful in detecting contextual LCS [1] and extending earlier results such as Arkhipov's graph theoretic characterization of contextuality. In the present work we extend the homotopical approach to classify quantum realizations in terms of homotopy classes of maps. For this we introduce a topological version of quantum realization which uses classifying spaces [2] tailored for contextuality. These classifying spaces can be "stabilized" in a way similar to the stabilization of vector bundles to obtain topological K-theory. This brings in a stable notion of contextuality detected by a generalized cohomology theory known as commutative K-theory. This procedure is in close analogy with the classification of symmetry-protected topological phases via generalized cohomology theories as in the work of Kitaev et al. We apply our machinery to prove various results about LCS.

[1] Homotopical approach to quantum contextuality. Cihan Okay, Robert Raussendorf, Quantum 4, 217 (2020).

[2] Classifying space for quantum contextuality. Cihan Okay, Daniel Sheinbaum, arXiv:1905.07723 (2019).

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Intersubjectivity of Quantum Measurement II

In a previous workshop, QCQMB 2017, it was shown that if two observers simultaneously measure the same observable, represented by the same self-adjoint operator, then they always obtain the same outcome [1]. Thus, the outcome of a measurement of a specific observable at a specific time is uniquely determined independent of observers to measure. This suggests that the measurement ascertains the pre-existing value of the measured observable in contrast to the standard view. Here, we show that if one measures an observable A in an unknown state ψ in a Hilbert space $\mathcal{H}$ by an apparatus prepared in a state ξ in another Hilbert space $\mathcal{K}$ with the meter M by the interaction from time 0 to τ with the time evolution described by a unitary operator U on $\mathcal{H} \otimes \mathcal{K}$, then $A(0) = A \otimes I$ and $M(\tau) = U^{\dagger}(I \otimes M)U$ commute in a common invariant subspace of $\mathcal{H} \otimes \mathcal{K}$ including $\mathcal{H} \otimes [\xi]$ and have the joint probability distribution $P(A(0) = x, M(\tau) = y)$ such that $P(A(0) = x, M(\tau) = y) = 0$ if $x \neq y$ in any state in $\mathcal{H} \otimes [\xi]$ [2]. Therefore, a measurement reproduces the value of A at the time just before the measurement as the value of the meter at the time just after the measurement. We argue about reconciliations of apparent contradictions with the Copenhagen interpretation, the Quantum Baysean interpretation, and the common interpretation of the Kochen-Specker theorem. We also argue that our result enforces Bohr's complementarity view that the measurement arrangement defines the value of the observable to be measured. We conclude that the pre-existing value of the observable to be measured can be uniquely defined contextually to be revealed by the measurement independent of observers, whereas quantum mechanics does not predict the value but only its probability distribution.

[1] M. Ozawa (2017), Intersubjectivity of Quantum Measurement, QCQMB 2017, Abstracts, pp. 27–28, available at http://www.psych.purdue.edu/~ehtibar/workshop/schedule.html.

[2] M. Ozawa (2019), Intersubjectivity of outcomes of quantum measurements, preprint available at https://arxiv.org/abs/1911.10893.

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Information and disturbance

(joint work with Giacomo Mauro D'Ariano and Alessandro Tosini)

The origin of many counterintuitive features of quantum theory is complementarity, by which we mean the existence of measurements that are mutually exclusive: if an observer performs one measurement on a given system, he will not be able to know what would have happened had he chosen to perform a complementary one. This fact, in turn, is strictly linked to the impossibility of extracting relevant information from a given system without irretrievably affecting the outcomes of any subsequent measurement, as quintessentially described in Heisenberg's account of the gamma ray microscope thought experiment. The principle of *no information without disturbance* will be discussed here in the context of general Operational Probabilistic Theories (OPTs) — a class of theories ruling the processes of hypothetical elementary systems, playing the role of foils or candidate alternatives to classical or quantum systems for physics, as well as their probabilities. This approach allows one to understand the features of quantum mechanics in a deeper way, distinguishing what phenomena are genuinely quantum, and what are typical of most theories. We will show necessary and sufficient conditions for *no information without disturbance*, discussing their operational interpretation. We will also illustrate the geometric features of a state space that embody the possibility or impossibility to extract information without disturbance. Particular care is taken in the definition of disturbance, considering not only direct disturbance on the system undergoing the measurement, i.e. on statistics of other measurements, but also on correlations with external systems. The role of composition rules will be highlighted, illustrating the unexpected features of theories without local discriminability in this respect. All the results will be proved without assuming causality, which implies that disturbance can affect both *preceding and subsequent* measurements.

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A structure theorem for transformations in noncontextual models

(joint work with John Selby, David Schmid and Rob Spekkens)

A well-motivated criteria for classicality is the existence of a noncontextual ontological model [1]. Existing work on such models has mostly considered preparations and measurements, with much less attention paid to transformations. I will diagrammatically prove a structure theorem about the representation of transformations in noncontextual ontological models of locally tomographic operational theories (including quantum theory). One striking consequence of this theorem is a severe restriction on the number of ontic states in the model. For example, a noncontextual model of a qubit could have at most four ontic states. Combining this with the fact that any ontological model of a qubit must in fact have an infinite number of ontic states immediately gives a new proof that qubits do not admit a noncontextual model. Time permitting, I will mention an analogous structure theorem regarding quasiprobability representations [2] and reevaluate the representation of transformations in various ontological models and quasiprobability representations in the literature.

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[2] C. Ferrie, Quasi-probability representations of quantum theory with applications to quantum information science. Rep. Prog. Phys. **74**, 116001 (2011).

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Phase space simulation method for quantum computation with magic states on qubits

(joint work with Juani Bermejo-Vega, Emily Tyhurst, Cihan Okay, and Michael Zurel)

We propose a method for classical simulation of finite-dimensional quantum systems, based on sampling from a quasiprobability distribution, i.e., a generalized Wigner function. Our construction applies to all finite dimensions, with the most interesting case being that of qubits. For multiple qubits, we find that quantum computation by Clifford gates and Pauli measurements on magic states can be efficiently classically simulated if the quasiprobability distribution of the magic states is non-negative. This provides the so far missing qubit counterpart of the corresponding result [V. Veitch *et al.*, New J. Phys. **14**, 113011 (2012)] applying only to odd dimension. Our approach is more general than previous ones based on mixtures of stabilizer states. Namely, all mixtures of stabilizer states can be efficiently simulated, but for any number of qubits there also exist efficiently simulable states outside the stabilizer polytope. Further, our simulation method extends to negative quasiprobability distributions, where it provides probability estimation. The simulation cost is then proportional to a robustness measure squared. For all quantum states, this robustness is smaller than or equal to robustness of magic.

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Generalized probabilities from Cauchy's Functional Equation

One interpretation of Gleason's theorem is in terms of solutions of a Cauchy Functional Equation – but not with scalar arguments in its usual and most elementary, classical form. Instead, the arguments of Cauchy Functional Equation are taken to be unit vectors/one-dimensional orthogonal projections, and only (mutually) orthogonal vectors in a Hilbert space are allowed to enter the sum. Indeed, as Victoria J. Wright and Stefan Weigert write [1] *"Gleason-type theorems … can be viewed as results about the solutions of Cauchy's functional equation for vector-valued arguments: additive functions on subsets of a real vector space, subject to some additional constraints, are necessarily linear."* To quote the first two paragraphs of Gleason [2] this is guaranteed by the Pythagorean property if one takes the square of the length of the orthogonal projections of some pure state vector onto elements of some orthonormal basis. In that way, a "linear" function(al) $f(x+y) = f(x)+f(y)$ can be constructed via a state vector ψ, with $f_\psi(x) = |\langle\psi|x\rangle|^2$ for x, y in some orthonormal basis: summation over the absolute squares of the inner products of such (mutually orthogonal) vectors with a state vector guarantees positivity and additivity up to value one.

One straightforward generalization is to consider Cauchy's Functional Equation with *tensor arguments* (multilinear forms), or to consider arguments which are other entities and more general formal objects not necessarily associated with Hilbert spaces. The only remaining "underlying" structure may be intertwined Boolean algebras or (mutual) exclusivity; maybe forming orthomodular lattices.

It might also be quite natural to "invert" the question by "fixing" the functional form of f, say, to some elementary analytic function; and then ask: (i) what are the mathematical entities x, y satisfying Cauchy's Functional Equation $f(x + y) = f(x) + f(y)$? (ii) Which empirical data might require such an analysis?

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Experimental computational advantage from quantum superposition of multiple temporal orders of logic gates

(joint work with Márcio M. Taddei, Jaime Cariñe, Daniel Martínez, Tania García, Nayda Guerrero, Alastair A. Abbott, Mateus Araújo, Cyril Branciard, Esteban S. Gómez, Stephen P. Walborn, Leandro Aolita, Gustavo Lima)

Advanced models for quantum computation where even the circuit connections are subject to the quantum superposition principle have been recently introduced [1,2]. There, a control quantum system can coherently control the order in which a target quantum system undergoes N gate operations. This process is known as the quantum N-switch, which has been identified as a resource for several information-processing tasks[2]. In particular, the quantum N-switch provides a quadratic computational advantage − over all known circuits with fixed gate orders − for phase-estimation problems involving N unknown unitary gates. However, the corresponding algorithm requires the target-system dimension to grow (super-)exponentially with N, making it experimentally demanding. In fact, all implementations of the quantum N-switch reported so far have been restricted to $N = 2$ [3]. Here, we introduce a promise problem for which the quantum N-switch also gives a quadratic speed-up but where the target-system dimension can be as small as 2 regardless of N. We use state-of-the-art multi-core optical fiber technology [4] to experimentally demonstrate the quantum N-switch with N = 4 gates acting on a photonic-polarization qubit. This is the first observation of a quantum superposition of more than 2 temporal orders, and also demonstrates its usefulness for efficient phase-estimation.

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[3] Lorenzo M. Procopio et. al, "Experimental superposition of orders of quantum gates", Nature Comm. 6, 7913 (2015).
[4] J. Cariñe et. al, "Multi-port beamsplitters based on multi-core optical fibers for high-dimensional quantum information", (2020), arXiv:2001.11056 [quant-ph].