The 4th Workshop:
Quantum Contextuality in Quantum Mechanics and Beyond

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Abstracts (alphabetic order)

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Partial boolean algebras and the logical exclusivity principle

(joint work with Rui Soares Barbosa)

Kochen and Specker's seminal work on contextuality used the formalism of partial boolean algebras. Unlike quantum logic in the sense of Birkhoff – von Neumann, partial boolean algebras only admit physically meaningful operations. We describe a refinement of current approaches to contextuality, in particular the sheaf-theoretic and graph-theoretic approaches, to incorporate partial boolean algebras. We discuss some striking and little-known results of Conway and Kochen (*not* the so-called "Free Will Theorem"!) in relation to this. We introduce a new axiom for partial boolean algebras, the Logical Exclusivity Axiom, and show that all probability models based on partial boolean algebras satisfying this axiom obey Specker's Exclusivity Principle.

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Using a resource theoretic perspective to witness and engineer quantum generalized contextuality for prepare-and-measure scenarios

We use the resource theory framework of generalized contextuality as a tool for analyzing the structure of prepare-and-measure scenarios. We argue that this framework is capable of simplifying proofs for witnessing quantum contextuality and straightens known arguments regarding the robustness of experimental implementations thereof. As a case study, we witness quantum contextuality associated with any nontrivial noncontextuality inequality for a class of scenarios by noticing a connection between the resource theory and measurement simulability. We also expose a rule for composing behaviours that allows one to build complex scenarios from simpler ones, which provides an explanation of the non-contextual polytope structure for complex scenarios and facilitates finding possible quantum violations. In particular, we show that the state-dependent cloning scenario inherits contextuality from a simpler scenario, and we engineer a complex prepare-and-measure scenario that has quantum contextual violations for any noncontextuality inequality. Witnessing contextuality from the abstract formalism of resource theories represents a potential new framework for demonstrating quantum-over-classical advantages for practical tasks.

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Emergence of noncontextuality and Classical limits

(joint work with Marcelo Terra Cunha, Rafael Wagner, Cristhiano Duarte, and Bárbara Amaral)

Quantum contextuality is one of the fundamental features responsible for distinguishing the quantum and the classical descriptions of nature. On the other hand, the so-called 'Classical limits' research area attempts to understand how our classical everyday experience may arise from the quantum realm. For instance, under this kind of program, we can understand loss of coherence (so-called Decoherence) and emergence of 'objectivity' (quantum Darwinism). However, one important question remains: if contextuality is such an important nonclassical feature, how classical limits explain its disappearance to allow for a reconciliation?

In this talk, we discuss classical limits of quantum theory in the perspective of (non)contextuality. In particular, we analyze how noncontextuality can emerge in a suitable setting resembling the collisional approach to open systems [1], discussing its relation to other kinds of emergence of classicality in a similar setting. Moreover, we also show that if a Darwinism process is successful enough, noncontextuality will emerge: independent observers will have a noncontextual explanation to what they collectively see [2]. In other words, if interaction of a system and portions of its environment leads to objectivity of observables and objectivity of outcomes, as defined by Brandão, Piani and Horodecki [3], noncontextuality emerges.

[1] R.D.Baldijão, M. Terra Cunha, Classical limits and contextuality in a scenario with multiple observers, *Phys. Rev. A*,**102**, 052226

[2] R.D.Baldijão, R. Wagner, C. Duarte, B. Amaral, M. Terra Cunha, Emergence of noncontextuality under quantum Darwinism, *in prep.; will be available on arXiv by the time of the QCQMB 2021 Workshop. The current version of the draft can be sent upon request.*

[3] F.L.S.Brandão, M. Piani., P. Horodecki, Generic emergence of classical features under quantum Darwinism, *nature physics*,**6**, Article number: 7908

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From Vorob'ev's theorem to monogamy of non-locality and local macroscopic averages

Vorob'ev's theorem characterises those configurations of contexts (i.e. measurement scenarios) which are inherently classical in the sense that the no-signalling or no-disturbance condition is enough to ensure locality or non-contextuality. Even if at first glance these scenarios may look a bit boring from the point of view of contextuality, we will discuss how Vorob'ev's result can be applied to provide an elegant structural explanation for two related phenomena: (1) monogamy of non-locality, which establishes a trade-off between strength of non-locality shared between a party and multiple others, and (2) locality of average macroscopic behaviour, regardless of the non-classicality present in the microscopic state of a system. Since Vorob'ev's theorem depends solely on the compatibility structure of measurements, these results hold not just for quantum theory, but for any empirical behaviours satisfying only the no-signalling condition.

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Local certification of programmable quantum devices of arbitrary high dimensionality

(joint work with Maharshi Ray, Antonios Varvitsiotis, Adán Cabello, and Leong-Chuan Kwek)

The onset of the era of fully-programmable error-corrected quantum computers will be marked by major breakthroughs in all areas of science and engineering. These devices promise to have significant technological and societal impact, notable examples being the analysis of big data through better machine learning algorithms and the design of new materials. Nevertheless, the capacity of quantum computers to faithfully implement quantum algorithms relies crucially on their ability to prepare specific high-dimensional and high-purity quantum states, together with suitable quantum measurements. Thus, the unambiguous certification of these requirements without assumptions on the inner workings of the quantum computer is critical to the development of trusted quantum processors. One of the most important approaches for benchmarking quantum devices is through the mechanism of self-testing that requires a pair of entangled non-communicating quantum devices. Nevertheless, although computation typically happens in a localized fashion, no local self-testing scheme is known to benchmark high dimensional states and measurements. Here, we show that the quantum self-testing paradigm can be employed to an individual quantum computer that is modelled as a programmable black box by introducing a noise-tolerant certification scheme. We substantiate the applicability of our scheme by providing a family of outcome statistics whose observation certifies that the computer is producing specific high-dimensional quantum states and implementing specific measurements.

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Measuring locality vs free choice

(joint work with E. M. Pothos, Y. M. Yearsley, C. Gallus, and E. Borsuk)

Bell inequalities rest on three fundamental assumptions: *realism*, *locality*, and *free choice*, which lead to nontrivial constraints on correlations in very simple experiments. If we retain realism, then violation of the inequalities implies that at least one of the remaining two assumptions must fail, which can have profound consequences for the causal explanation of the experiment. We investigate the extent to which a given assumption needs to be relaxed for the other to hold at all costs, based on the observation that a violation need not occur on every experimental trial, even when describing correlations violating Bell inequalities. How often this needs to be the case determines the degree of, respectively, *locality* or *free choice* in the observed experimental behavior. Despite their disparate character, we show that both assumptions are *equally costly.* Namely, the resources required to explain the experimental statistics (measured by the frequency of causal interventions of either sort) are exactly the same. Furthermore, we compute such defined measures of locality and free choice for any *nonsignaling* statistics in a Bell experiment with binary settings, showing that it is directly related to the amount of violation of the so-called Clauser-Horne-Shimony-Holt inequalities. This result is theory independent as it refers directly to the experimental statistics. Additionally, we show how the local fraction results for quantum-mechanical frameworks with infinite number of settings translate into analogous statements for the measure of free choice we introduce. Thus, concerning statistics, causal explanations resorting to either locality or free choice violations are *fully* interchangeable.

[1] Blasiak, P., Pothos, E. M., Yearsley, J. M., Gallus, C. and E. Borsuk *Violations of locality and free choice are equivalent resources in Bell experiments.* PNAS **118** e2020569118 (2021)

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Causal reappraisal of the quantum three box paradox

(joint work with P. Błasiak)

Quantum three box paradox is a prototypical example of some bizarre predictions for intermediate measurements made on pre- and post-selected systems. Although in principle those effects can be explained by measurement disturbance, it is not clear what mechanisms are required to fully account for the observed correlations. In this talk, three box paradox is scrutinised from a causal point of view. We consider an array of potential causal structures behind the experiment, eliminating those without enough explanatory power. This gives us a way of differentiating between the various mechanisms in which measurement disturbance can propagate in the system. Specifically, we distinguish whether it is just the *measurement outcome* or the *full measurement context* that is required for the causal explanation of the observed statistics. We show that the latter is indispensable, but only when the full statistics is taken into account (with any of the three boxes being checked in the experiment). We also discuss a distinction between the *pure* and *realist causal setting*, where the latter assumes that measurements reveal preexisting values. While in the pure case the original version of the paradox can be explained without resorting to any measurement disturbance, this fails when the full statistics is taken into account. The realist case is more stringent in this respect. These various results illustrate the richness of the paradox which is better appreciated from the causal perspective.

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Quantum Contextuality: from logical contradictions to experimental tests

Contextuality refers to the impossibility of reproducing the outcome statistics of freely and independently chosen measurements, under the assumption that ideal measurements reveal pre-existing values that are independent of the context each measurement belongs to.

Originated in the work of Kochen and Specker in the 1960s, this notion has been further developed in the subsequent fifty years by several authors in different directions. In particular, several approaches has been proposed for the problem of performing experimental tests of contextuality.

In this contribution, we provide an overview of some of the main approaches to quantum contextuality and its experimental tests; we address the different definitions of noncontextual hidden variable theories and noncontextuality inequalities, the physical assumptions on measurement operations involved in experimental tests of contextuality, and so on.

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From nonlocality to contextuality and back

We introduce a map that captures all the connections between Bell nonlocality and contextuality in quantum theory. Due to Neumark's dilation theorem, every matrix of quantum nonlocal correlations can be mapped to an identical matrix of contextual correlations. A more difficult problem is identifying connections from contextuality into nonlocality. We show that there are "one-to-one" and partial connections for some contextuality scenarios, but not for all of them. However, we also present a method that transforms any matrix of contextual correlations into a matrix of Bell nonlocal correlations.

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On the relationship between no-fine-tuning and Bell-KS inequalities

(joint work with Jason C. Pearl)

In this talk I will discuss conceptual aspects of the relationship between the assumption of no-fine-tuning applied to classical causal models and Bell-Kochen-Specker inequalities, following the results of [1,2,3].

[1] C. J. Wood and R. W. Spekkens, New Journal of Physics 17, 033002 (2015).
[2] E. G. Cavalcanti, Physical Review X 8, 021018 (2018).
[3] J. C. Pearl and E. G. Cavalcanti, arXiv:1909.05434 (2019).

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A hierarchical measure of contextuality

(joint work with Ehtibar N. Dzhafarov)

I will present a hierarchical measure of (non)contextuality that we introduced in [1]. Many systems in which contextuality is studied (e.g., those represented by cyclic systems) have in common that their (non)contextuality is determined by particular configurations of pairwise correlations. This characterization leads to the incorrect intuitive idea that all contextuality appears at the level of pairwise correlations, perhaps even in cyclic subsystems. The hierarchical measure presented helps to determine whether contextuality arises at that level or rather at a higher one (triples, quadruples, etc.).

I will apply this measure to several systems. As objects of analysis we use impossible figures of the kind created by the Penroses and Escher. We make no assumptions as to how an impossible figure is perceived, taking it instead as a fixed physical object allowing one of several deterministic descriptions. Systems of epistemic random variables are obtained by probabilistically mixing these deterministic systems. This probabilistic mixture reflects our uncertainty or lack of knowledge rather than random variability in the frequentist sense.

[1] Cervantes, V. H. & Dzhafarov, E. N. Contextuality Analysis of Impossible Figures. Entropy 22, 981 (2020).

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The Quantum Sets of some Coloured Graphs in between CHSH Bell Inequality and CHSH NC Inequality

(joint work with Anna Lina Vandré)

The graph approach to contextuality [1] is a major contribution to the understanding of Kochen-Specker contextuality [2]. The first worked example is Clauser-Horne-Shimony-Holt scenario and inequality [3]. As any Bell inequality, CHSH is also a Non-Contextuality inequality. When written as a sum of eight probabilities, its quantum bound is the same in both cases: with or without Bell restriction. What about behaviours? Is it true that any behaviour obtained by the Born rule under the restriction of CHSH NC-inequality can also be obtained in the CHSH Bell scenario? The Coloured Graph Approach [4], originally proposed for Nonlocality, is much more general: it applies to Contextuality with Parts, which includes the so called Generalised Bell Scenarios [5], when each part is also allowed to make locally compatible measurements. With this apparatus, we answer negatively the question posed above, by exhibiting families of coloured graphs with different nested quantum sets, the outer most being CHSH NC-inequality one, the inner most the CHSH Bell-inequality one [6,7].

[1] Adán Cabello, Simone Severini, and Andreas Winter, *Phys. Rev. Lett.* **112**, 040401 (2014).
[2] Simon Kochen and Ernest Specker, *J. Math. Mech.* **17**, 59 (1967).
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[4] Rafael Rabelo *et. al*, *J. Phys. A: Math. Theor.* **47** 424021 (2014).
[5] Tassius Temistocles, Rafael Rabelo, and Marcelo Terra Cunha, *Phys. Rev. A* **99** 042120 (2019).
[6] Anna Lina Vandré and Marcelo Terra Cunha, in preparation.
[7] Anna Lina Vandré, Masters Thesis, University of Innsbruck.

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All About Cyclic Systems

(joint work with Janne V. Kujala and Víctor H. Cervantes)

Cyclic systems of dichotomous random variables have played a prominent role in contextuality research, describing such experimental paradigms as the KCBS, EPR/Bell, and Leggett-Garg ones. We present a theoretical analysis of the degree of contextuality in cyclic systems (if they are contextual) and the degree of noncontextuality in them (if they are not). The Contextuality-by-Default (CbD) theory allows us to drop the commonly made assumption that systems of random variables are consistently connected (i.e., we allow for all possible forms of "disturbance" or "signaling" in them). By contrast, all previously proposed measures of contextuality are confined to consistently connected systems, and most of them cannot be extended to measures of noncontextuality. Our measures of contextuality and noncontextuality are defined by the L_1-distance between a point representing a cyclic system and the surface of the polytope representing all possible noncontextual cyclic systems with the same single-variable marginals. We completely characterize this polytope, and establish that, in relation to the maximally tight Bell-type CbD inequality for cyclic systems, the measure of contextuality is proportional to the absolute value of the difference between its two sides. For noncontextual cyclic systems, the measure of noncontextuality is shown to be proportional to the smaller of the same difference and the L_1-distance to the surface of the hyperbox circumscribing the noncontextuality polytope. These simple relations, however, do not generally hold beyond the class of cyclic systems, and noncontextuality of a system does not follow from noncontextuality of its cyclic subsystems. We also compute the volumes of the noncontextuality polytope and the circumscribing hyperbox to answer the following question: if one chooses a cyclic system "at random" (i.e., uniformly within the hyperbox), what are the odds that it will be (non)contextual? We find that the odds of contextuality rapidly tend to zero as the size of the system increases.

[1] Dzhafarov, E.N., Kujala, J.V., & Cervantes, V.H. Contextuality and noncontextuality measures and generalized Bell inequalities for cyclic systems. Physical Review A 101:042119. + Erratum Note in Physical Review A 101:06990 (arXiv:1907.03328)

[2] Dzhafarov, E.N., Kujala, J.V., & Cervantes, V.H. (2021). Epistemic odds of contextuality in cyclic systems. To be published in European Physics Journal - Special Topics. (arXiv:2002.07755.)

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Bounds on overlaps give us coherence, contextuality and non-locality inequalities

(joint work with Daniel J. Brod and Rui Soares Barbosa)

A SWAP test is a quantum circuit that measures the overlaps $r_{\rho\sigma} = Tr(\rho\sigma)$ between two states ρ, σ. If we consider a set of n quantum states, different bounds on two-state overlaps result when we consider either i) diagonal, coherence-free states, or ii) general quantum states. The difference between i) and ii) allowed us to propose novel basis-independent coherence witnesses [1]. I will show that the inequalities for overlaps of coherence-free states correspond to noncontextuality and locality inequalities, which suggests a unified framework for resource theories of coherence, contextuality and nonlocality.

[1] E. F. Galvão and D. J. Brod. Quantum and classical bounds for two-state overlaps. Phys. Rev. A 101, 062110 (2020). https://arxiv.org/abs/1902.11039

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Proof of the Peres conjecture for contextuality

(joint work with Zhen-Peng Xu and Jing-Ling Chen)

A central result in the foundations of quantum mechanics is the Kochen-Specker theorem. In short, it states that quantum mechanics cannot be reconciled with classical models that are noncontextual for compatible observables. The first explicit derivation by Kochen and Specker was rather complex, but considerable simplifications have been achieved thereafter. We propose a systematic approach to find Hardy-type and Greenberger-Horne-Zeilinger-type (GHZ-type) proofs of the Kochen-Specker theorem, these are characterized by the fact that the predictions of classical models are opposite to the predictions of quantum mechanics. With this approach, we find the provably minimal GHZ-type proof. Based on our results, we show that the Kochen-Specker set with 18 vectors from Cabello et al. [1] is the minimal set for any dimension, verifying a long-standing conjecture by Peres. Our results allow to identify minimal contextuality scenarios and to study their usefulness for information processing.

[1] A. Cabello, J. Estebaranz, and G. García-Alcaine, Phys. Lett. A **212**, 183 (1996).

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On global states of collections of random variables

One of the characteristic traits of quantum theory is that the description of a quantum system involves a collection of incompatible measurement contexts. Each context can be seen as a classical random variable, defined by a complete set of commuting observables. But it turns out that contexts are intertwined: quantum probabilistic models can be described as very specific pastings of Boolean algebras, which are globally non-Boolean. States are represented by density operators that define global states, and give place to classical probabilities when restricted to the maximal Boolean subalgebras associated to measurement contexts. The characterization of the peculiar pasting occurring in the quantum domain has been a topic of much research, and is related to the understanding of quantum contextuality. In this talk we discuss different techniques for combining collections of (possibly non-compatible) random variables in such a way that one obtains –as in the quantum case– a global state that yields classical probabilities when restricted to the local Boolean subalgebras. After commenting different approaches related to the possibility of using negative probabilities, we address the well known problem of pasting families of Boolean algebras. We discuss some of our findings with regard to the problem of defining global objects representing states of contextual probabilistic theories.

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General Causal-Model Characterization of Contextuality

This work extends my previous results [1] characterizing contextuality in terms of probabilistic causal models, paralleling [2] but applicable to cases of inconsistent connectedness (i.e., violation of no-disturbance). There, I proved the equivalence of three definitions for contextuality of any measurement system M: (1) there exists no causal model for M that simultaneously minimizes all direct influences of context on measurement outcomes, (2) any causal model for M contains hidden influences (influences that go in opposite directions for different latent states, or equivalently signaling that carries no information), and (3) contextuality as defined in the Contextuality-by-Default (CbD) theory [3]. These previous results were limited to a class of causal models having a particular canonical structure. Here I generalize the results to arbitrary causal models.

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Neither contextuality nor non-locality admits catalysts

In [1] we show that the resource theory of contextuality does not admit catalysts. As a corollary, we observe that the same holds for non-locality. This adds a further example to the list of "anomalies of entanglement", showing that non-locality and entanglement behave differently as resources. We also show that catalysis remains impossible even if instead of classical randomness we allow some more powerful behaviors to be used freely in the free transformations of the resource theory.

[1] Karvonen, M. Neither contextuality nor non-locality admits catalysts. `https://arxiv.org/abs/2102.07637`

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Formalization of Bohr's contextuality within theory of open quantum systems

In quantum physics, the notion of contextuality has a variety of interpretations which are typically associated with the names of their inventors, say Bohr, Bell, Kochen and Specker, and recently Dzhafarov. In fact, Bohr was the first who pointed to contextuality of quantum measurements as a part of formulation of his principle of complementarity. (Instead of "contextuality", he considered dependence on "experimental conditions.") Unfortunately, the contextuality counterpart of the complementarity principle was overshadowed by the issue of incompatibility of observables. And the interest for contextuality of quantum measurements rose again only in connection with the Bell inequality. The original Bohr's contextuality, as contextuality of each quantum measurement, was practically forgotten. It was highlighted in the works of the author of this paper, with applications both to physics and cognition. In this note, the theory of open quantum systems is applied to formalization of Bohr's contextuality within the the scheme of indirect measurements. This scheme is widely used in quantum information theory and it leads to the theory of quantum instruments (Davis-Lewis-Ozawa). In this scheme, Bohr's viewpoint on contextuality of quantum measurements is represented in the formal mathematical framework.

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Exploiting Contextuality in Variational Quantum Eigensolvers

(joint work with Andrew Tranter and Peter Love)

We describe how contextuality may be used to advantage in variational quantum eigensolvers. Contextuality is a characteristic feature of quantum mechanics, and identifying contextuality in quantum algorithms provides a means for distinguishing them from their classical counterparts. We first describe how contextuality may be identified in variational quantum eigensolvers (VQEs), which are a leading algorithm for noisy intermediate-scale quantum computers [1]. We then show how to construct a classical phase-space model for any noncontextual Hamiltonian, which provides a classical simulation algorithm for noncontextual VQE and allows us to prove that the noncontextual Hamiltonian problem is only NP-complete, rather than QMA-complete [2]. Finally, we describe an approximation method called contextual subspace VQE that permits us to partition a general Hamiltonian into a noncontextual part and a contextual part, and estimate its ground state energy using a technique that combines classical simulation of the noncontextual part with quantum simulation of the contextual part. By using more quantum resources (in qubits and simulated terms of the Hamiltonian), we can increase the accuracy of the approximation. We tested contextual subspace VQE on electronic structure Hamiltonians, and found that to reach chemical accuracy in most cases it requires fewer qubits and simulated terms than standard VQE [3].

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[2] W. M. Kirby and P. J. Love, Phys. Rev. A 102, 032418 (2020).
[3] W. M. Kirby, A. Tranter, and P. J. Love, arXiv preprint (2020), arXiv:2011.10027 [quant-ph].

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Quantity-with-Uncertainty Defines the Physics of Quantum Mechanics and Space Time

(joint work with John Skilling)

Science relies on numeric quantification, which can be traced back to Euclid, Galileo, and Newton. Here we develop the formalism of numeric quantification in the face of intrinsic uncertainty, since at the smallest scales it is not possible to be able to perform measurements with arbitrary precision. As a result, a faithful numeric description must depend on a *pair of numbers* representing a fusion of quantity and uncertainty, which is potentially more intimate than the familiar quantity $\pm$ error bar. We show that the basic symmetries of combination and partition impose a specific calculus on number pairs, in keeping with but more subtle than standard scalar arithmetic. We derive complex arithmetic operating on pairs which we recognize as quantum amplitudes, observable through modulus-squared probabilities. Not only do we construct the Feynman picture of quantum mechanics, but we find that these same symmetries also lead to the Pauli matrices which generate spin, energy and momentum, and beyond that to $3 + 1$-dimensional relativistic spacetime. Not surprisingly, we find that uncertainty is related to the quantum phase, which is in turn, related to time. The result is that the physics of quantity-with-uncertainty in conjunction with the basic symmetries of combination and partition define much of the mathematical formalism of physics.

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Contextuality and dichotomizations of random variables

(joint work with Ehtibar Dzhafarov)

The Contextuality-by-Default approach to determining and measuring the (non)contextuality of a system of random variables requires that every random variable in the system be represented by an equivalent set of dichotomous random variables. We present general principles that justify the use of dichotomizations and determine their choice. We primarily focus on two types of random variables most often encountered in practice: categorical and real-valued ones (including continuous random variables, underrepresented in the contextuality literature). A categorical variable (one with a finite number of unordered values) is represented by all of its possible dichotomizations. If the values of a random variable are real numbers, then they are dichotomized by intervals above and below a variable cut point.

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Contextuality-by-Default description of Bell Tests

In a system of n dichotomous jointly distributed random variables $\{X_1, .., X_n\}$ taking ± 1 values some marginal expectations obey several non-contextuality inequalities (NCI). Of particular interest are those satisfied by pairwise cyclic expectations $\{\langle X_1 X_2\rangle, \langle X_2 X_3\rangle, \ldots, \langle X_n X_1\rangle\}$ such as Boole, Bell-LHCH and KCBS inequalities. If only pairs of random variables may be measured jointly NCI may be violated by the data due statistical fluctuations in finite samples, even if a studied statistical population may be described by a joint probability distribution of all variables. Quantum mechanics and behavioral studies taught us that there exist contextual random variables 'measuring' the same content (an answer to the same Yes or No question), which may vary, if they are 'measured' jointly with other random variables. Therefore, according to Contextuality-by-Default (CbD) in a real experiment we do not have in a cyclic system, obeying NCI, but a system $\{\langle X_1' X_2\rangle, \langle X_2' X_3\rangle, \langle X_n' X_1\rangle\}$ in which random variables X_i and X_i' are statistically unrelated. In order to test the degree of contextuality of such system one derives, in CbD, modified NCI. It is often believed that, because of Einsteinian no-signaling, we don't have this problem in Bell Tests because Alice's and Bob's measurement may not depend on distant settings. In fact, if Alice and Bob use two different PBS settings and two detectors for each setting, then in synchronized time windows of the width W, they may describe their raw data using 4 random variables A_i and B_j taking the values ± 1 or 0. The no-signaling is confirmed and $\langle A_i\rangle$ and $\langle B_j\rangle$ do not depend on what was 'measured' on other side. However, to perform a Bell Test we have to define a W-dependent coupling between Alice's and Bob's non vanishing outcomes and new coupled post-selected samples, for different settings are described now by 4 pairs of 8 context dependent random variables $\{(A_{11}, B_{11}), (A_{12}, B_{21}), (A_{21}, B_{12}), (A_{22}, B_{22})\}$. The data show that the resulting random variables are not consistently connected $\langle A_{11}\rangle \neq \langle A_{12}\rangle, \langle B_{21}\rangle \neq \langle B_{22}\rangle$, thus to study the contextuality of this system, more in detail, one should use CbD approach instead of CHSH inequalities. It is obvious that the violation of these inequalities has nothing to do with nonlocal physical influences between distant experimental settings, similarly as it is a case in KCBS experiments and in cognitive psychology. This is why one should not talk about signaling and nonlocality, if a significant violation of NCI is observed. Since the correlations observed in Bell Tess may be explained in a locally causal way such terminology is not only imprecise but misleading.

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[2] J. V. Kujala, E. N. Dzhafarov, and J.-Å. Larsson, Necessary and sufficient conditions for extended noncontextuality in a broad class of quantum mechanical systems, Phys. Rev. Lett. 115, 150401 (2015).

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Nebit - a unit of negative probability

(joint work with Dagomir Kaszlikowski)

We introduce nebit, a classical bit with a signed probability distribution. We study its properties and basic transformations that can be applied to it. Then, we introduce a simple dynamical model - a classical random walk supplemented with nebits. We show that such a model exhibits some counterintuitive non-classical properties and that it can achieve or even exceed the speedup of Grover's quantum search algorithm. The proposed classical dynamics never reveals negativity of nebits and thus we do not need any operational interpretation of negative probabilities.

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Loophole-free Contextuality Inequalities

(joint work with Damian Markham)

As it is usually presented, the physical consequence of contextuality is that it forces one to give up on either determinism or parameter-independence. The latter ensures that measurement responses are independent of context, i.e. any other compatible measurements that are performed jointly. These are assumptions that apply to hidden variable models, which are supposed to encompass any reasonable physical description that could underlie the empirical data. In realistic, noisy experiments however, the validity of those assumptions can be called into question. This introduces loopholes to the physical conclusions one can draw from instance of contextuality. A number of previous works have addressed one or other of these loopholes. We build on this to provide a unified approach to closing both loopholes by introducing appropriate correction terms to the contextual fraction measure of contextuality.

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Contextuality in Artificial Intelligence and Cognitive Science

Contextuality is a transdisciplinary phenomenon that may be observed across the sciences. Contextuality in quantum physics and cognitive science has been discussed extensively in the QCQMB community, and so in this talk, we focus upon contextuality in natural language semantics, natural language processing, and artificial intelligence, which is in a stark contrast with the more traditional principle of compositionality. We explicate and articulate, inter alia, the relationships between contextuality and compositionality in natural language and artificial intelligence. We elucidate, moreover, what contextual language models in natural language processing, especially state-of-the-art Transformer models such as BERT and GPT-3, tell us about the relationships between compositionality and contextuality. We also discuss, in passing, how (contextual) linguistic cognition connects with (compositional) mathematical cognition through state-of-the-art models in natural language processing.

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A generalized cohomology theory for quantum contextuality

Linear constraint systems (LCS) provide instances of Kochen-Specker type contextuality proofs generalizing the well-known example of Peres-Mermin square. A LCS is specified by a linear system of equations. Homotopy theory has proved to be useful in detecting contextual LCSs [1] and extending earlier results such as Arkhipov's graph-theoretic characterization of contextuality. In the present work we extend the homotopical approach to a generalized cohomology theory that can be used to classify quantum solutions of a LCS. For this we introduce a topological version of quantum solutions which relies on classifying spaces [2] tailored for contextuality. These classifying spaces can be "stabilized" in a way similar to the stabilization of vector bundles to obtain topological K-theory. This brings in a stable notion of contextuality detected by a generalized cohomology theory known as commutative K-theory. This procedure is in close analogy with the classification of symmetry-protected topological phases via generalized cohomology theories as in the work of Kitaev et al. We apply our machinery to prove various results about LCSs. This talk is based on the arxiv preprint [3].

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[3] Commutative d-torsion K-theory and its applications. Cihan Okay, arXiv:2006.07542 (2020).

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Intersubjectivity of Quantum Measurement II

In a previous workshop, QCQMB 2017, it was shown that if two observers simultaneously measure the same observable, represented by the same self-adjoint operator, then they always obtain the same outcome [1]. Thus, the outcome of a measurement of a specific observable at a specific time is uniquely determined independent of observers to measure. This suggests that the measurement ascertains the pre-existing value of the measured observable in contrast to the standard view. Here, we show that if one measures an observable A in an unknown state ψ in a Hilbert space $\mathcal{H}$ by an apparatus prepared in a state ξ in another Hilbert space $\mathcal{K}$ with the meter M by the interaction from time 0 to τ with the time evolution described by a unitary operator U on $\mathcal{H} \otimes \mathcal{K}$, then $A(0) = A \otimes I$ and $M(\tau) = U^{\dagger}(I \otimes M)U$ commute in a common invariant subspace of $\mathcal{H} \otimes \mathcal{K}$ including $\mathcal{H} \otimes [\xi]$ and have the joint probability distribution $P(A(0) = x, M(\tau) = y)$ such that $P(A(0) = x, M(\tau) = y) = 0$ if $x \neq y$ in any state in $\mathcal{H} \otimes [\xi]$ [2]. Therefore, a measurement reproduces the value of A at the time just before the measurement as the value of the meter at the time just after the measurement. We argue about reconciliations of apparent contradictions with the Copenhagen interpretation, the Quantum Baysean interpretation, and the common interpretation of the Kochen-Specker theorem. We also argue that our result enforces Bohr's complementarity view that the measurement arrangement defines the value of the observable to be measured. We conclude that the pre-existing value of the observable to be measured can be uniquely defined contextually to be revealed by the measurement independent of observers, whereas quantum mechanics does not predict the value but only its probability distribution.

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Classical causal models cannot faithfully explain Bell nonlocality or Kochen-Specker contextuality in arbitrary scenarios

(joint work with Eric G. Cavalcanti)

In this work [1], we present a proof that Bell-Kochen-Specker inequality violations in phenomena satisfying no-disturbance (a generalisation of the no-signalling condition) cannot in general be explained by a classical causal model without fine-tuning. This result generalises the previous results of [2] and [3], by considering Bell scenarios with an arbitrary number of parties, or Kochen-Specker-contextuality scenarios with an arbitrary number of measurements per context. This unifies Bell nonlocality and Kochen-Specker contextuality as violations of a fundamental principle of classical causality.

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[2] C. J. Wood and R. W. Spekkens, New Journal of Physics 17, 033002 (2015).
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Information and disturbance

(joint work with Giacomo Mauro D'Ariano and Alessandro Tosini)

The origin of many counterintuitive features of quantum theory is complementarity, by which we mean the existence of measurements that are mutually exclusive: if an observer performs one measurement on a given system, he will not be able to know what would have happened had he chosen to perform a complementary one. This fact, in turn, is strictly linked to the impossibility of extracting relevant information from a given system without irretrievably affecting the outcomes of any subsequent measurement, as quintessentially described in Heisenberg?s account of the gamma ray microscope thought experiment. The principle of No-information without disturbance will be discussed here in the context of general Operational Probabilistic Theories (OPTs)?a class of theories ruling the processes of hypothetical elementary systems, playing the role of foils or candidate alternatives to classical or quantum systems for physics, as well as their probabilities. This approach allows one to understand the features of quantum mechanics in a deeper way, distinguishing what phenomena are genuinely quantum, and what are typical of most theories. We will show necessary and sufficient conditions for no information without disturbance, dis- cussing their operational interpretation. We will also illustrate the geometric features of a state space that embody the possibility or impossibility to extract information without disturbance. Particular care is taken in the definition of disturbance, considering not only direct disturbance on the system undergoing the measurement, i.e. on statistics of other measurements, but also on correlations with external systems. The role of composition rules will be highlighted, illustrating the unexpected features of theories without local discriminability in this respect. All the results will be proved without assuming causality, which implies that disturbance can affect both preceding and subsequent measurements.

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A structure theorem for transformations in noncontextual models

(joint work with John Selby, David Schmid, and Rob Spekkens)

A well-motivated criteria for classicality is the existence of a noncontextual ontological model [1]. Existing work on such models has mostly considered preparations and measurements, with much less attention paid to transformations. I will diagrammatically prove a structure theorem [2] about the representation of transformations in noncontextual ontological models of locally tomographic operational theories (including quantum theory). One striking consequence of this theorem is a severe restriction on the number of ontic states in the model. For example, a noncontextual model of a qubit could have at most four ontic states. Combining this with the fact that any ontological model of a qubit must in fact have an infinite number of ontic states immediately gives a new proof that qubits do not admit a noncontextual model. Time permitting, I will mention an analogous structure theorem regarding quasiprobability representations [3] and a recent application [4] of our structure theorem to the stabilizer formalism.

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A hidden variable model for universal quantum computation with magic states on qubits

(joint work with Michael Zurel and Cihan Okay)

We show that every quantum computation can be described by a probabilistic update of a probability distribution on a finite phase space. Negativity in a quasiprobability function is not required in states or operations. Our result is consistent with Gleason's Theorem and the Pusey-Barrett- Rudolph theorem.

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Varieties of contextuality – emphasizing (non)embeddability

The notion of "contextuality" in terms of nonclassical (paradoxical) properties of probability distributions is refined by concentrating on the nonclassical logico-algebraic structures of the observables supporting such probabilities. These structures often, but not always, induce and are reflected by nonclassical properties of the probability distributions. Two types of contextuality are introduced, originating either in nonclassical probabilities on classical logics, or in genuine nonclassical logics of observables which cannot be faithfully (homomorphically) embedded into (extended) Boolean algebras. It is suggested to substitute the term "contextual" by "indeterminate" or "value indefinite" in the spirit of partial functions of theoretical computer sciences.

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Generalised Kochen-Specker Theorem in Three Dimensions

(joint work with Mirko Navara)

The Bell–Kochen–Specker theorem is an important no-go theorem in quantum mechanics, which proves the incompatibility of quantum physics with local hidden-variable theories. The Kochen–Specker proof is based on a construction of a set of 117 three-dimensional vectors admitting no $\{0, 1\}$-coloring.

There are numerous generalizations of the Kochen–Specker theorem, e.g., using rational vectors, or replacing a $\{0, 1\}$-coloring with a $\mathbb{Z}_2$-coloring. It was shown in [1], that there is no $\mathbb{Z}_2$-coloring of vectors in $\mathbb{R}^4$ and consequently no $\mathbb{Z}_2$-coloring in dimensions greater than or equal to 5, see [2]. We show that there is no $\mathbb{Z}_2$-coloring even for $\mathbb{R}^3$, which was an open question, formulated, e.g., in [3,4]. The existence of a $\mathbb{Z}_2$-coloring in $\mathbb{R}^2$ is trivial, hence we answer the only remaining case.

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In search of true contextuality in natural language

(joint work with Mehrnoosh Sadrzadeh, Samson Abramsky, and Víctor H. Cervantes)

Motivated by ideas of Firth and Harris, computational linguists argue that if two words often occur in the same context, they have similar meanings. Despite the successes of this idea in disambiguation tasks in NLP [1], the systematic existence of ambiguity in natural language and its shades and nuances are notchallenges faced head on. Contextuality and its degrees are well studied topics in Quantum Mechanics. Here, the mathematical framework of Contextuality-by-Default (CbD) has become a useful tool when dealing with systems that are signalling and will also be convenient for natural language. Our line of research comes closest to the concept combination examples of [2]. However, as shown in [3], neither of the 23 examples of [2] are truly contextual. In our work, we did find combinations that are truly contextual for the first time. These are from a dataset of rank-2 cyclic (verb, noun) phrases constructed from [4,5] and the probabilities are tabulated from occurrences in corpora (BNC and uKWaC). More general types of systems were considered too, initially using the sheaf-theoretic approach to contextuality, but we showed that none of them are truly contextual. For our rank-2 cyclic dataset, we made use of the degrees of signalling and direct influence from M-Contextuality to find quantitative empirical evidence that the context affects ambiguous senses and meanings of nouns and verbs differently. So far contextuality has not been found in more general systems, although nothing seems to preclude the existence of contextual examples in certain types of models.

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