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# Nebit – a unit of negative probability

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Quantum Contextuality in Quantum Mechanics and Beyond IV

17 May 2021

# Motivation – why negative probabilities

- Negative probabilities in QT are not new

  **Wigner, Dirac, Feynman, ...**

  **Mückenheim et. al., Phys. Reps. 133, 337 (1986)**

- Consistent mathematical theory capable of 'explaining' paradoxes

  **Khrennikov, Székely, ...**

- Non-signalling + Non-locality $\Rightarrow$ Negative JPD

  **Al-Safi & Short, PRL 111, 170403 (2013)**

  **Abramsky & Brandenburger, in 'Horizons of the Mind' (2014)**

- Quantum computation is working, but how?

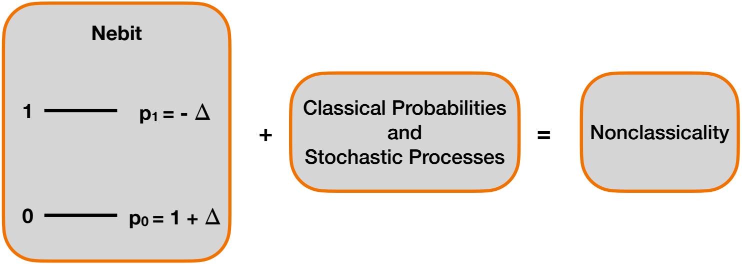
  **Howard et. al., Nature 510 351 (2014)**

  **Veitch et. al., New J. Phys. 14, 113011 (2012)**

  **Raussendorf et. al., PRAA 95, 052334 (2017)**

- Unsolved problem – interpretation (like probability amplitudes)

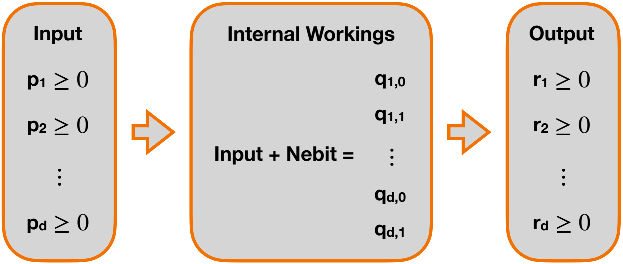
# General idea




- Which nonclassical phenomena can be simulated that way?
- Can we simulate QT? (how large $\Delta$ to do this?)

**DK & PK, Found. Phys. 51, 55 (2021)**

# Rules of the game




Input and output probability distributions must be classical (nonnegative)

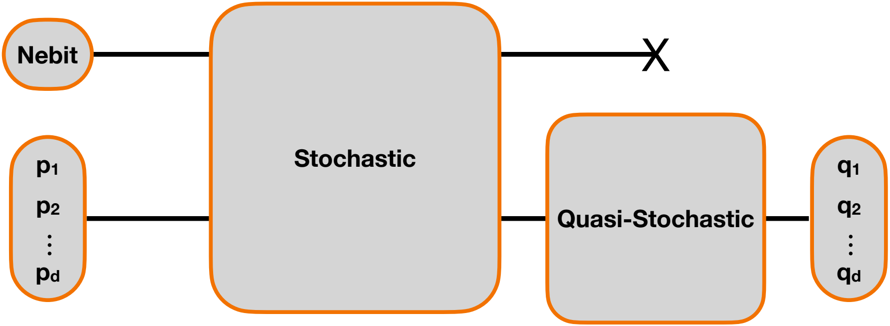
# Stochastic and Quasi-Stochastic transformations

$$\mathrm{p}(t) = \begin{pmatrix} p_1 \\ \vdots \\ p_d \end{pmatrix}, \quad S = \begin{pmatrix} s_{11} & \dots & s_{1d} \\ \vdots & \ddots & \vdots \\ s_{d1} & \dots & s_{dd} \end{pmatrix}, \quad \sum_i s_{ij} = 1,$$

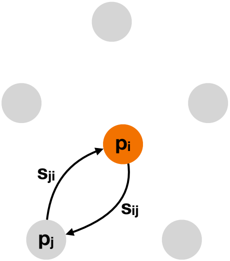
$$\mathrm{p}(t+1) = S\mathrm{p}(t),$$

If some $s_{ij} < 0$, then **quasi-stochastic** transformation

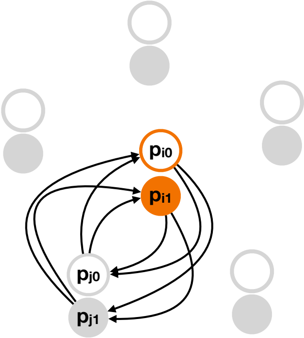
# Stochastic + Nebit = Quasi-Stochastic



# Negative random walk



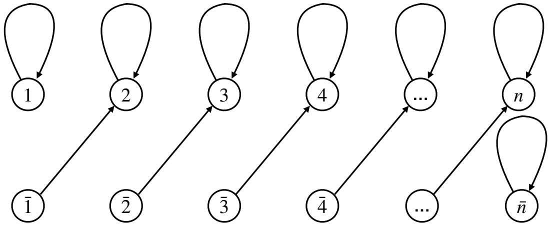

**Random Walk**




**Negative Random Walk**

$$p_i = p_{i0} + p_{i1} \geq 0$$

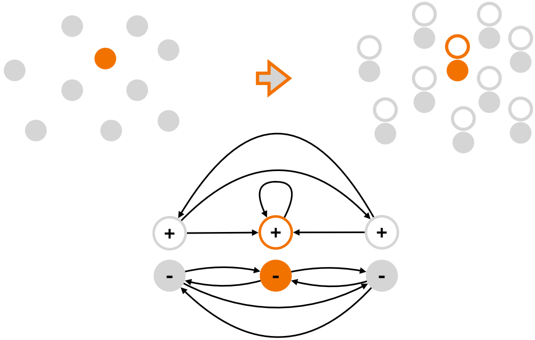
# Negative random walk – example




$$p(0) = (0, \underbrace{-1, \ldots, -1}_{n-1}, \underbrace{1, \ldots, 1}_{n})^T \qquad p_{x=1} = 1+0 = 1, \quad p_{x\neq 1} = 1-1 = 0,$$

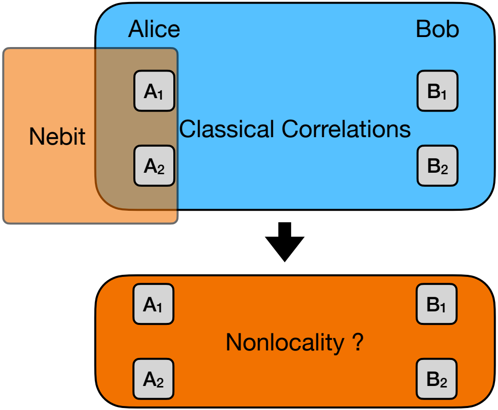
$$p(1) = (\underbrace{0, \ldots, 0}_{n}, \underbrace{0, \ldots, 0}_{n-1}, 1)^T \qquad p_{x=n} = 1+0 = 1, \quad p_{x\neq n} = 0+0 = 0,$$

# Search algorithm




$$p(0) = \begin{pmatrix} \frac{1}{N} \\ \vdots \\ \frac{1}{N} \end{pmatrix} \otimes \begin{pmatrix} 1+\Delta \\ -\Delta \end{pmatrix} \qquad T = O\left(\frac{N}{\Delta+1}\right)$$

# Local activation of nonlocality




**DK, KO & PK, in preparation**

# Local activation of the CHSH nonlocality

- Binary observables: $A_1, A_2, B_1, B_2 = \pm 1$
- JPD: $p(a_1, a_2, b_1, b_2) = \frac{1}{16}\left(1 + \frac{\alpha}{2}(a_1 b_1 + a_1 b_2 + a_2 b_1 - a_2 b_2)\right)$
- $0 \leq \alpha \leq 2$ (C: $\alpha \leq 1$, Q: $\alpha \leq \sqrt{2}$, PRB: $\alpha \leq 2$)
- Alice applies:

$$S(\beta)_{a_1,a_2} = S = \frac{1}{4}\begin{pmatrix} 1+3\beta & 1-\beta & 1-\beta & 1-\beta \\ 1-\beta & 1+3\beta & 1-\beta & 1-\beta \\ 1-\beta & 1-\beta & 1+3\beta & 1-\beta \\ 1-\beta & 1-\beta & 1-\beta & 1+3\beta \end{pmatrix}, \quad \Delta = \frac{3}{4}(\beta - 1)$$

$$S(\beta)_{a_1,a_2} \cdot p(a_1, a_2, b_1, b_2) = \frac{1}{16}\left(1 + \frac{\alpha\beta}{2}(a_1 b_1 + a_1 b_2 + a_2 b_1 - a_2 b_2)\right)$$

# Local activation of the multipartite (GHZ) nonlocality

- Assuming maximal classical correlations ($\alpha = 1$)
- Maximum quantum **2-partite** nonlocality $\beta = \sqrt{2}$
- Maximum quantum **3-partite** nonlocality $\beta = 2$
- Maximum quantum **4-partite** nonlocality $\beta = 4$
- $\vdots$
- Maximum quantum **N-partite** nonlocality $\beta = 2^{\lfloor N/2 \rfloor}$

# Conclusions

- Some nonclassical phenomena (correlations, dynamics, computational speedup) can be simulated using

  **Classical Probabilities and Stochastic Processes + Nebit**

- Open question: can we use this approach to design new nonclassical algorithms and then to translate them into quantum algorithms?