Griffith UNIVERSITY
Centre for Quantum Dynamics

CENTRE FOR QUANTUM COMPUTATION & COMMUNICATION TECHNOLOGY
AUSTRALIAN RESEARCH COUNCIL CENTRE OF EXCELLENCE

On the relationship between No-fine-tuning and Bell-KS inequalities

Eric Cavalcanti (joint work with Jason Pearl)

QCQMB’21 workshop
18 May 2021

Australian Government
Australian Research Council

FQXi
FOUNDATIONAL QUESTIONS INSTITUTE

J.C Pearl and EGC, arXiv:1909.05434:

Classical causal models cannot faithfully explain Bell nonlocality or Kochen-Specker contextuality in arbitrary scenarios

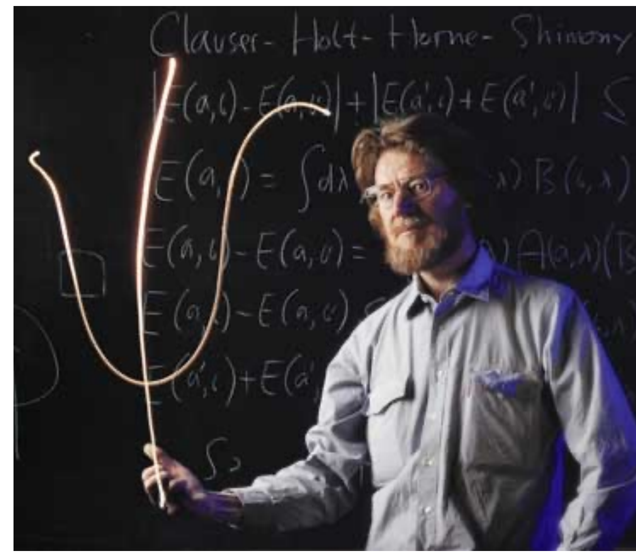
EGC, Phys. Rev. X 8, 021018 (2018):

Classical causal models for Bell and Kochen-Specker inequality violations require fine-tuning

“For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation [of quantum theory] and fundamental relativity…

It may be that a real synthesis of quantum and relativity theories requires not just technical developments but radical conceptual renewal.”

J.S. Bell (1986)



Measurement scenario

Set of measurements: $\mathcal{M} = \{m_1, \ldots, m_k\}$

Set of measurement outcomes: $\mathcal{O}_m = \mathcal{O} \;\; \forall m$

Measurement contexts: $c \in \mathcal{C}$, $c \subseteq \mathcal{M}$

i.e. m_1, m_2 compatible iff $\{m_1, m_2\} \in \mathcal{C}$

Bell scenario: $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2 \cup \ldots \cup \mathcal{M}_n$

$\mathcal{M}_i \cap \mathcal{M}_j = \{\,\} \;\; \forall i \neq j$

$x_1 \in \mathcal{M}_1, \; x_2 \in \mathcal{M}_2, \ldots, x_n \in \mathcal{M}_n$

Kochen-Specker scenario: any scenario that is not a Bell scenario

Experimental test

Measurement settings: $\boldsymbol{X} = \{X_1, X_2, ..., X_n\}$

e.g. $\{X_1 = m_1, ..., X_n = m_n\} \in \mathcal{C}$

Measurement outcomes: $\boldsymbol{A} = \{A_1, A_2, ..., A_n\}$

Measurement-outcome pair: (X_i, A_i) for all $i \in \mathcal{I} = \{1, 2, ..., n\}$

Examples

Bell-CHSH scenario

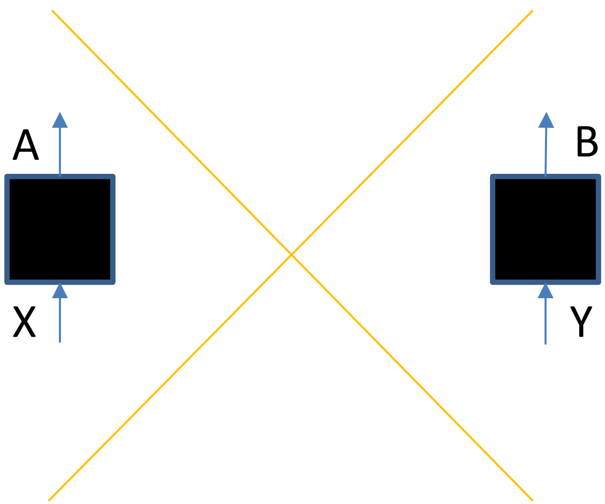
$X \in \mathcal{M}_1 = \{m_1, m_2\}$

$Y \in \mathcal{M}_2 = \{m_3, m_4\}$

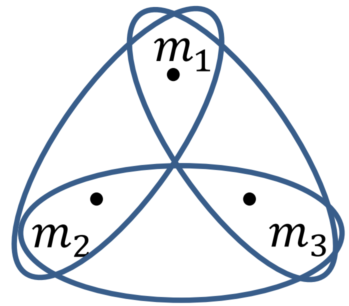
$\mathrm{A} \in \mathcal{O}_1 = \mathcal{O} = \{-1,1\}$

$\mathrm{B} \in \mathcal{O}_2 = \mathcal{O} = \{-1,1\}$

$\mathcal{C} = \{m_1, m_2\} \times \{m_3, m_4\}$



Specker's triangle scenario

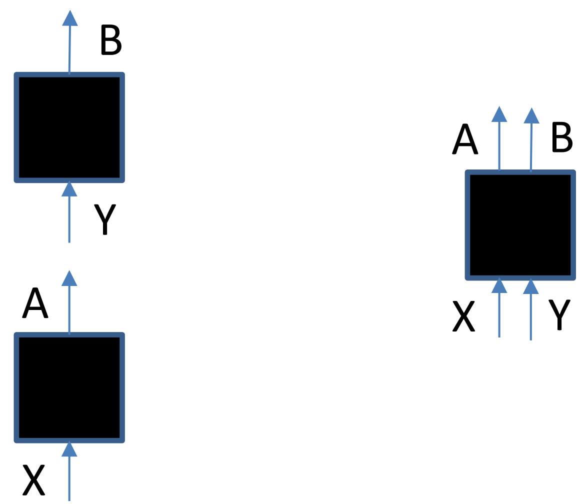


$\mathcal{M} = \{m_1, m_2, m_3\}$

$\mathcal{O}_1 = \mathcal{O}_2 = \mathcal{O}_3 = \mathcal{O} = \{0,1\}$

$\mathcal{C} = \{\{m_1, m_2\}, \{m_2, m_3\}, \{m_1, m_3\}\}$

$\{X, Y\} \in \mathcal{C}$



Kochen-Specker contextuality *vs* Bell nonlocality

Both are violations of *factorisability*:

$$\mathcal{P}(\boldsymbol{A}|\boldsymbol{X}) = \sum_{\boldsymbol{\Lambda}} P(\boldsymbol{\Lambda}) \prod_i P(A_i|\boldsymbol{\Lambda} X_i)$$

s.t. $P(A_i|\boldsymbol{\Lambda} X_i = m) = P(A_j|\boldsymbol{\Lambda} X_j = m)$ (whenever those are defined)

Fine-Abramsky-Brandenburger Theorem:
A phenomenon has a factorisable model IFF it has a KS-noncontextual model.

KSNC		(1964) Bell-locality
Measurement Noncontextuality	⇒	Locality (Parameter Independence)
Outcome Determinism		Outcome Determinism
Freedom of Choice		Freedom of Choice

Spekkens (2005):

- Measurement noncontextuality (as well as analogous notions for preparations and transformations) is motivated by

Leibniz's Principle of the Identity of Indiscernibles* (Einstein's methodological principle):

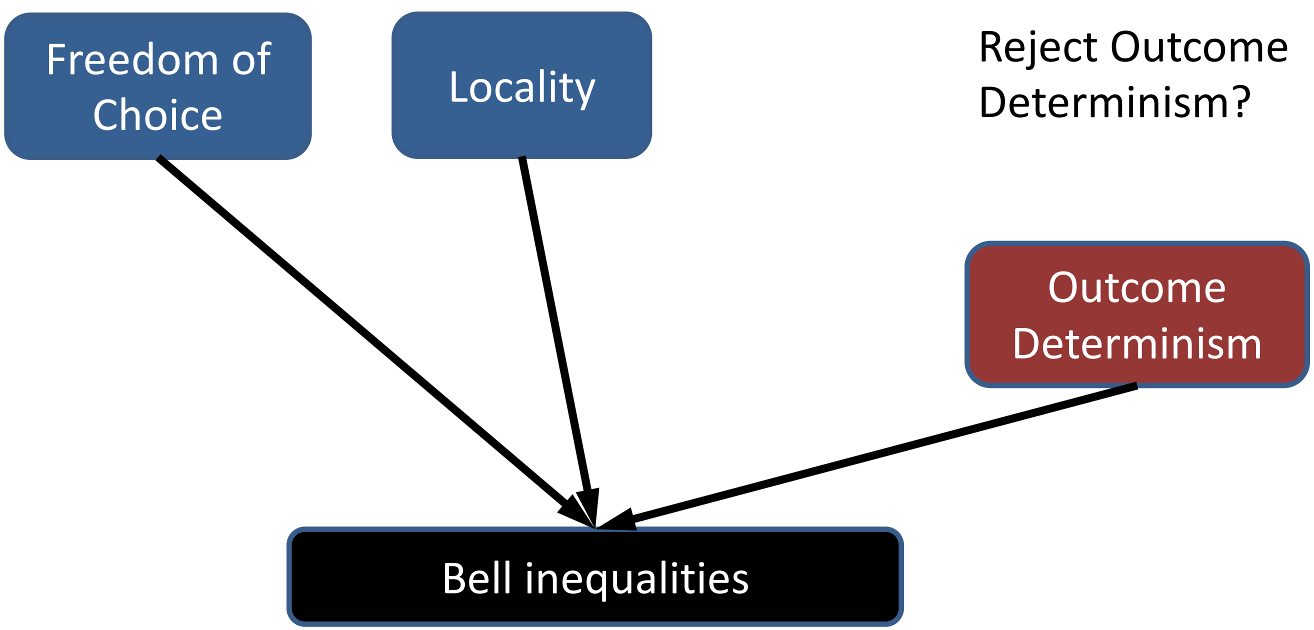
Empirically indistinguishable scenarios should be represented by ontologically identical models.

- However, Outcome Determinism is not justified for unsharp measurements

*Spekkens, arXiv:1909.04628 (2019)

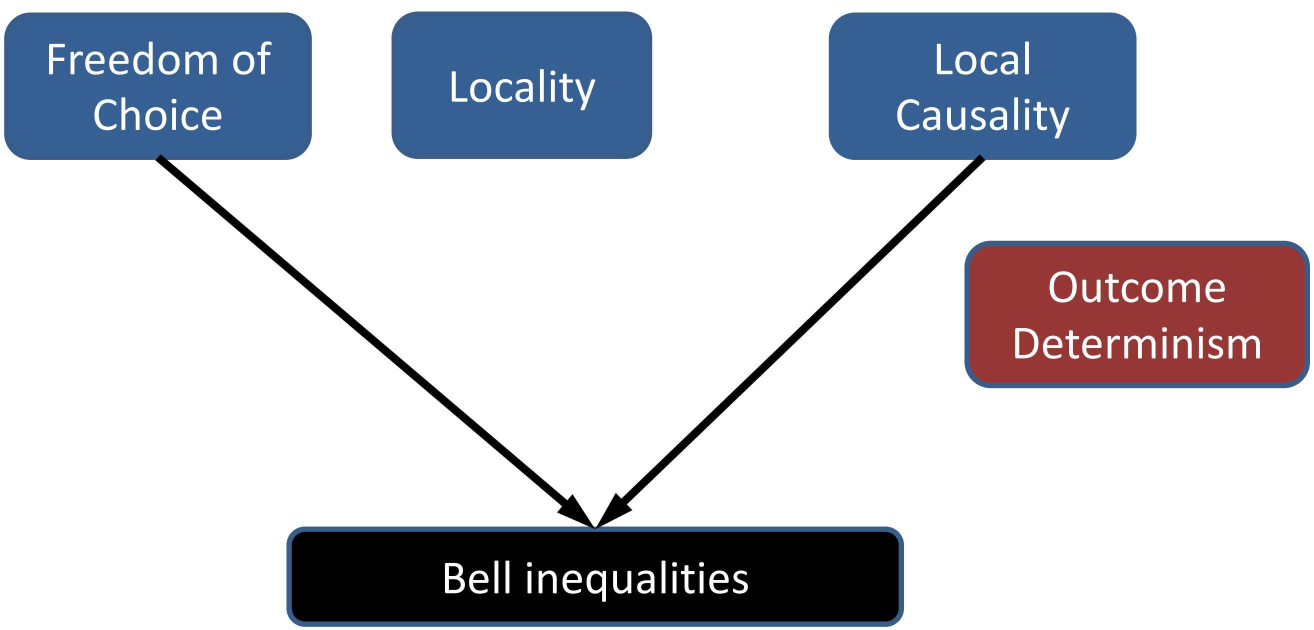
H.M. Wiseman & EGC (2017) "Causarum Investigatio and the Two Bell's Theorems of John Bell". In R. Bertlmann & A. Zeilinger (Eds.), *Quantum [Un]Speakables II,* arXiv:1503.06413

Bell's 1964 theorem: Quantum phenomena violate the conjunction of Freedom of Choice, Locality, and Outcome Determinism.

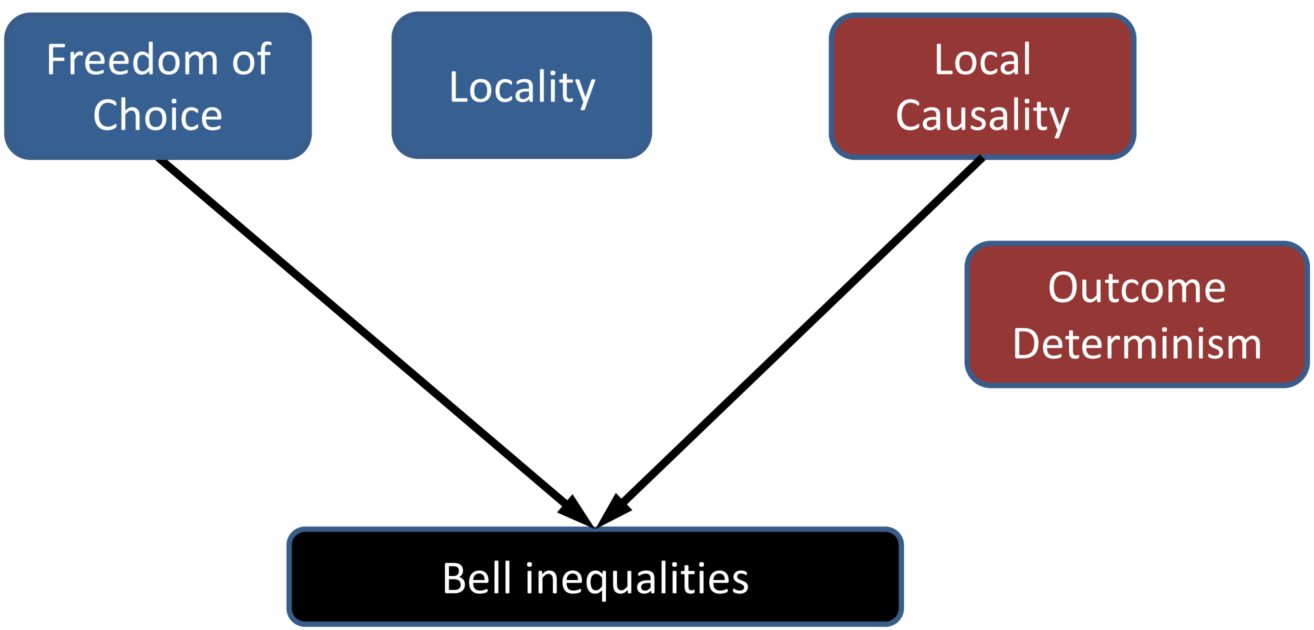


Bell's 1976 theorem: Quantum phenomena violate the conjunction of No Superdeterminism and Local Causality.

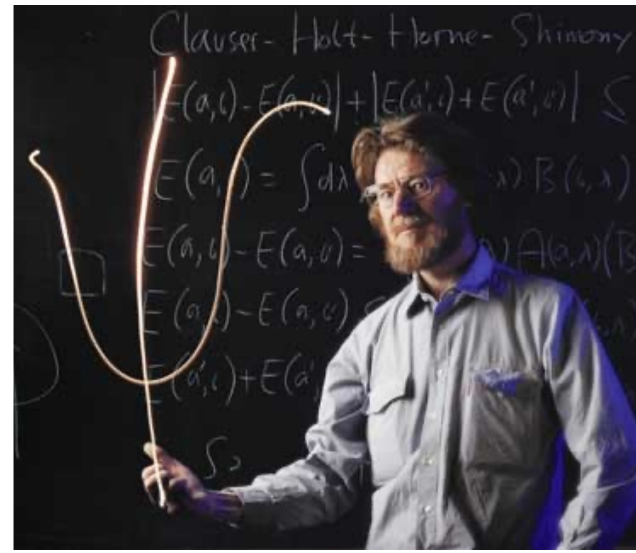
Local Causality: If two space-like separated sets of events A and B are correlated, then there is a set of events C in the intersection of their past light cones such that conditioning on C eliminates the correlation.



Reject Local Causality?



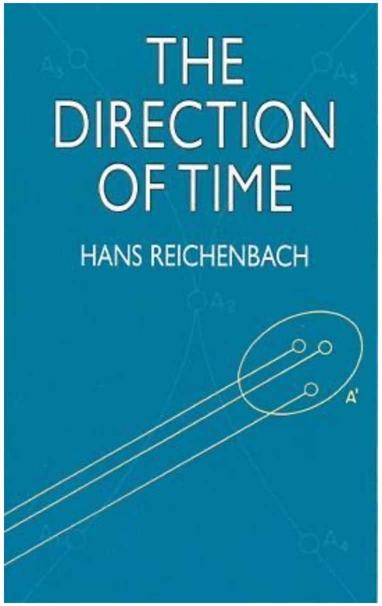
"Do we then have to fall back on "no signalling faster than light" as the expression of the fundamental causal structure of contemporary theoretical physics? That is hard for me to accept. For one thing we have **lost the idea that correlations can be explained, or at least this idea awaits reformulation**".



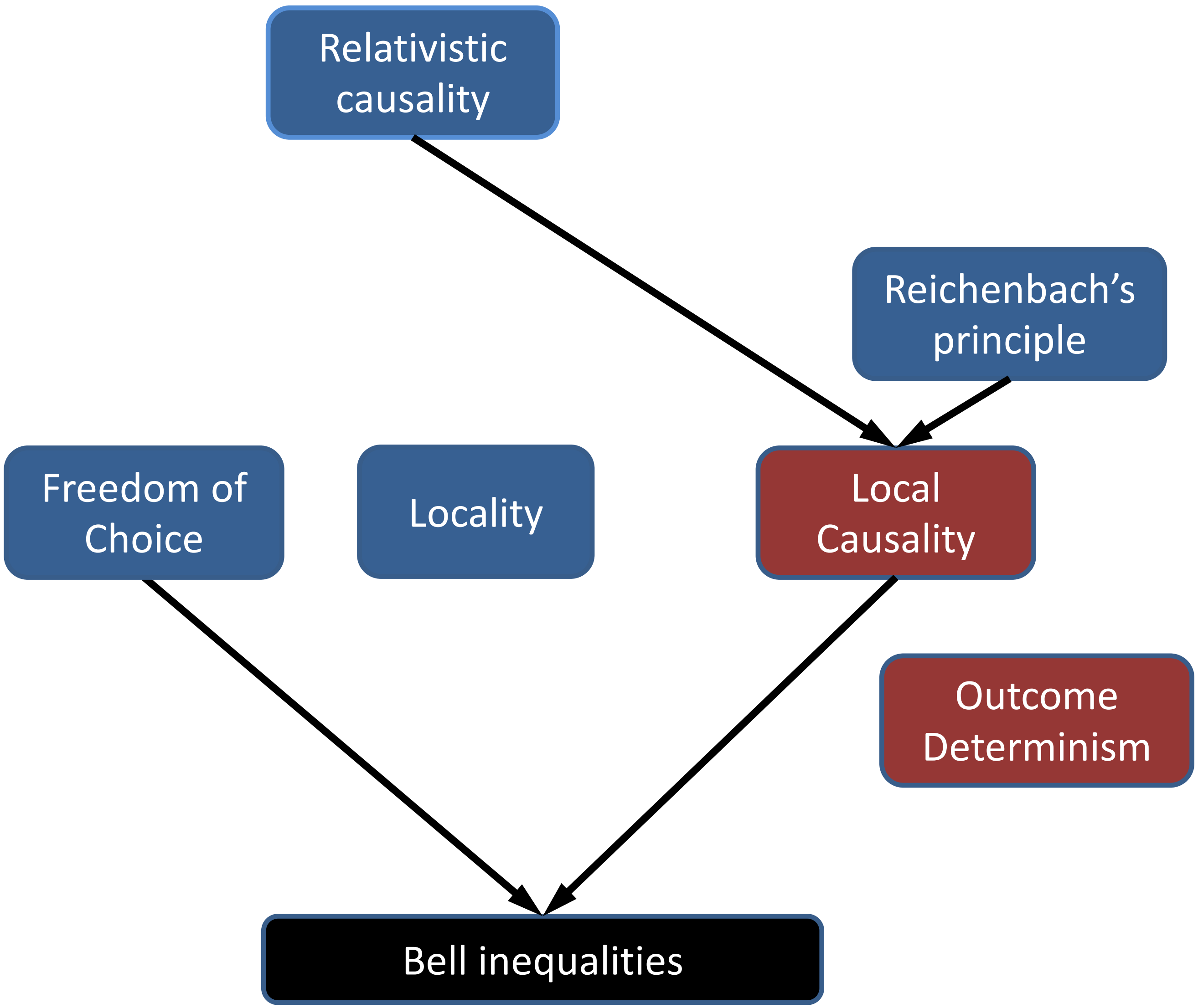
– J.S. Bell, "La Nouvelle Cuisine" (1990)

Causal explanation

Reichenbach's Principle of Common Cause (1956): If two sets of events *A* and *B* are correlated, and no event in either is a cause of any event in the other, then they have a set of common causes Λ, such that conditioning on Λ eliminates the correlation.



Relativistic Causality: The causal past of an event is its past light-cone.



Reject Relativistic Causality?

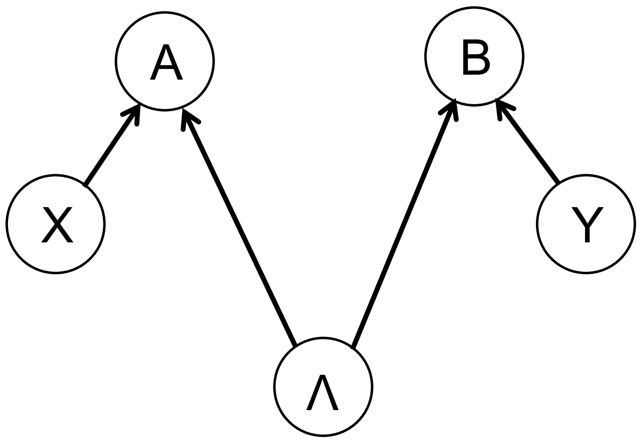
- E.g. Bohmian mechanics
  - violates, at a fundamentally hidden level, apparent operational symmetries.

- I.e., it violates

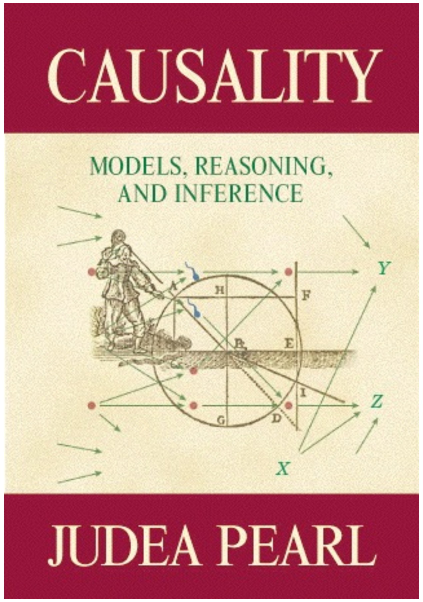
Leibniz's Principle of the Identity of Indiscernibles (Einstein's methodological principle):

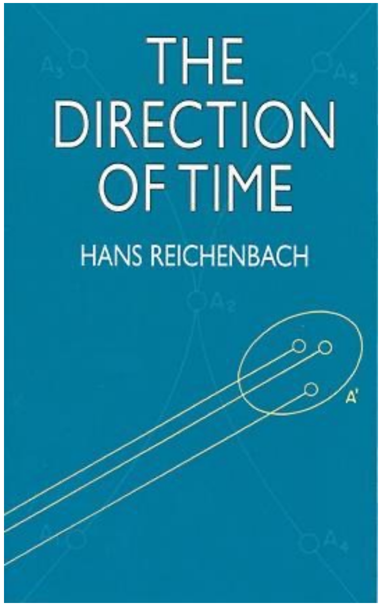
Empirically indistinguishable scenarios should be represented by ontologically identical models.

Classical causal models



Directed Acyclic Graph (DAG)
for a Bell scenario





Causal Markov Condition:

A variable is independent of its non-effects given its direct causes

Causal Markov Condition → Reichenbach's Principle

Reject Reichenbach's Principle?

What about causal explanation?

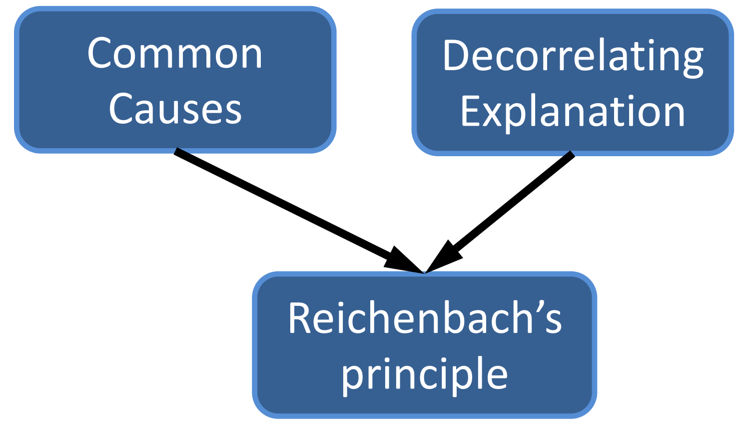
IOP Publishing Journal of Physics A: Mathematical and Theoretical

J. Phys. A: Math. Theor. **47** (2014) 424018 (13pp) doi:10.1088/1751-8113/47/42/424018

On modifications of Reichenbach's principle of common cause in light of Bell's theorem

Eric G Cavalcanti[1,2] and Raymond Lal[1,2]

[1] School of Physics, University of Sydney, NSW 2016, Australia
[2] Department of Computer Science, University of Oxford, Oxford OX1 3QD, UK



Principle of Common Cause: If two sets of events A and B are correlated, and no event in either is a cause of any event in the other, then they have a set of common causes C that explains the correlation.

Principle Of Decorrelating Explanation ("Factorisation of Probabilities"): A set of causes C, common to two sets of events A and B, explains a correlation between them only if conditioning on C eliminates the correlation.

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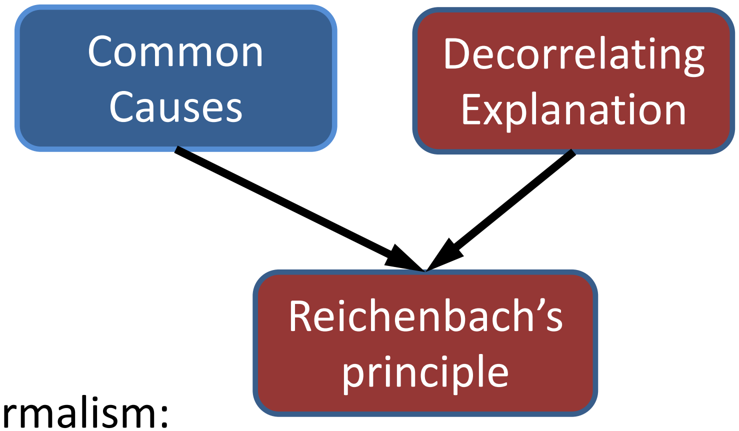

Using Leifer-Spekkens quantum conditional states formalism:
~~Decorrelating explanation~~ → *Factorisation of quantum channels*

Principle of Common Cause: If two sets of events A and B are correlated, and no event in either is a cause of any event in the other, then they have a set of common causes C that explains the correlation.

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Quantum causal models

Leifer and Spekkens, Phys. Rev. A 88, 052130 (2013), arXiv:1107.5849
Wood and Spekkens, New Journal of Physics, 17 (2015), arXiv:1208.4119
Cavalcanti and Lal, J. Phys. A 47, 424018 (2014), arXiv:1311.6852
Fritz, T. Comm. Math. Phys., 341, 391–434 (2016), arXiv:1404.4812
Henson, Lal and Pusey (HLP), New J. Phys. 16, 113043 (2014), arXiv:1405.2572
Pienaar and Brukner (PB), New J. Phys. 17, 073020 (2015), arXiv:1406.0430
Chaves, Majenz and Gross, Nat. Commun. 6, 5766 (2015), arXiv:1407.3800
Costa and Shrapnel, New J. Phys. 18 063032 (2016), arXiv:1512.07106
Allen, Barrett, Horsman, Lee and Spekkens, Phys. Rev. X 7, 031021 (2017), arXiv:1609.09487
Barrett, Lorenz and Oreshkov, arXiv:1906.10726
Giarmatzi and Costa. npj Quantum Information 4, 17 (2018)

- Allow for causal discovery (e.g. Giarmatzi and Costa)
- Compatible with Relativistic Causality
- Reduces to classical causal models as a special case

- Provides a *faithful causal explanation* of Bell correlations

→ Resolution of the ”easy problem”* of Bell

*E. G. Cavalcanti, “Bell’s theorem and the measurement problem: reducing two mysteries to one?”, J. Phys. Conf. Ser. 701, 12002 (2016). arXiv:1602.07404

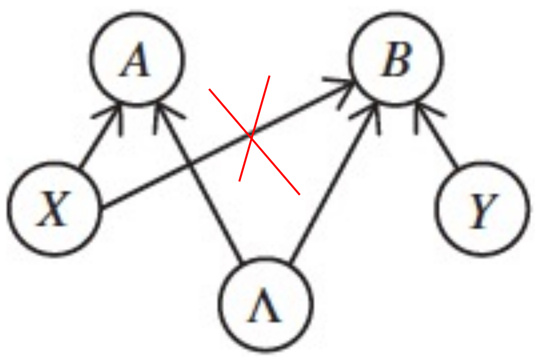
What about contextuality?

- In general contextuality scenarios, causal structure isn’t constrained by relativistic causality.

- Different approach: show that *all possible causal structures* that can explain quantum contextuality must violate a fundamental causal principle: *no-fine-tuning*.

Leibniz’s Principle + Causal Models → *Causal No-fine-tuning*

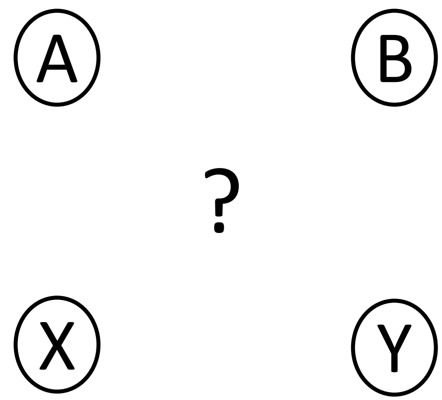
- **Causal No-fine-tuning (Causal Faithfulness):** Every conditional independence between variables should arise as a consequence of the causal graph and not due to special choices of model parameters.



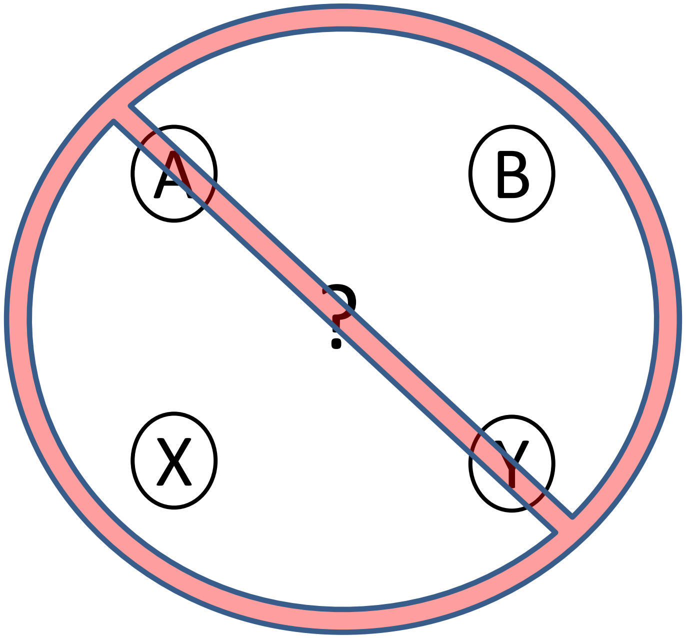
E.g.: if B is independent of X given Y then B must be *d-separated* from X given Y

$(B \perp X|Y) \Rightarrow (B \perp X|Y)_d$

Finely-tuned Bells



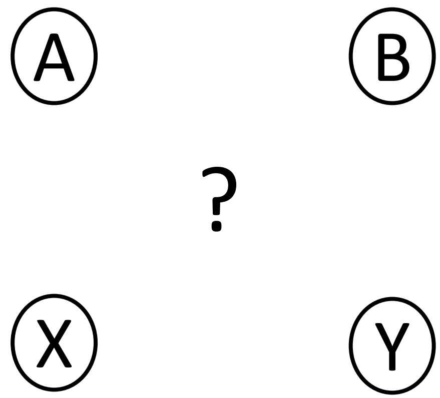
Finely-tuned Bells



Wood and Spekkens, NJP **17**, 33002 (2015):

No classical causal model can explain all instances of bipartite Bell nonlocality without causal fine-tuning.

Finely tuned Bells

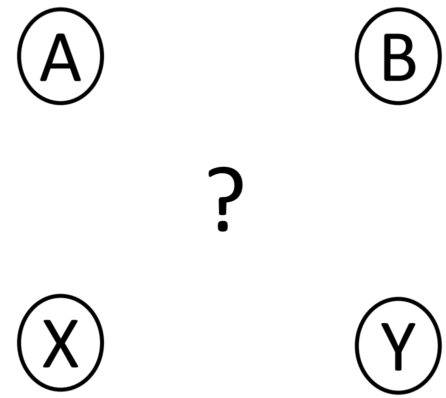


- Choice independence (CI): $(X \perp Y)$
- Local setting dependence (LSD): $\neg(A \perp X), \neg(B \perp Y)$,
- No-signalling (NS): $(B \perp X|Y), (A \perp Y|X)$

- Causal No-fine-tuning (CNFT)

- CI ∧ LSD ∧ NS ∧ CNFT ⊢ BI
  Wood and Spekkens, NJP **17**, 33002 (2015)

Finely tuned Bells



- ~~Choice independence (CI):~~ $\cancel{(X \perp Y)}$
- ~~Local setting dependence (LSD):~~ $\cancel{\neg(A \perp X), \neg(B \perp Y),}$
- No-signalling (NS): $(B \perp X|Y), (A \perp Y|X)$

- Causal No-fine-tuning (CNFT)

- NS ∧ CNFT ⊢ BI
  EGC, PRX **8**, 021018 (2018)

Finely tuned Bells

Theorem 1: *No classical causal model can reproduce violations of bipartite Bell inequalities without fine-tuning.*

→ *Allows generalisation to KS-noncontextuality*

$\{X, Y\} \in \mathcal{C}$

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- ~~Local setting dependence (LSD):~~ $\cancel{\neg(A \perp X), \neg(B \perp Y),}$
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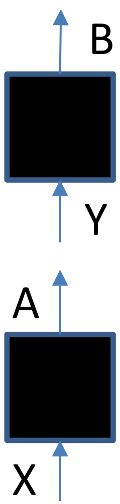
EGC, PRX **8**, 021018 (2018)

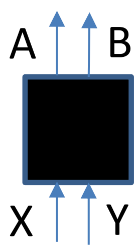
Finely tuned Bells

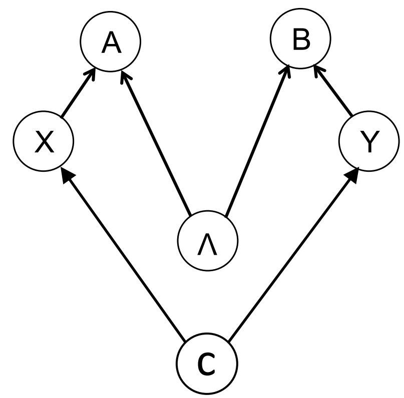
Result 1: *No classical causal model can reproduce violations of bipartite Bell inequalities without fine-tuning.*

→ *Allows generalisation to KS-noncontextuality*

$\{X, Y\} \in \mathcal{C}$







Causal Markov Condition → Factorisability (for Bell scenarios)

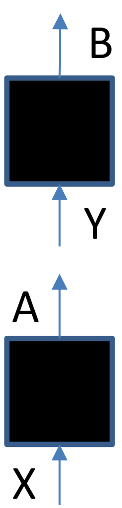
$$\mathcal{P}(\boldsymbol{A}|\boldsymbol{X}) = \sum_{\boldsymbol{\Lambda}} P(\boldsymbol{\Lambda}) \prod_i P(A_i|\boldsymbol{\Lambda} X_i)$$

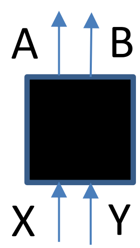
Finely tuned Bells

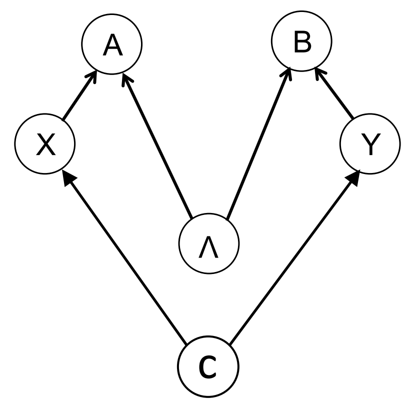
Result 1: *No classical causal model can reproduce violations of bipartite Bell inequalities without fine-tuning.*

→ *Allows generalisation to KS-noncontextuality*

$\{X, Y\} \in \mathcal{C}$







Causal Markov Condition→ Factorisability
(but not for KS scenarios!)

$$\mathcal{P}(\boldsymbol{A}|\boldsymbol{X}) = \sum_{\boldsymbol{\Lambda}} P(\boldsymbol{\Lambda}) \prod_i P(A_i|\boldsymbol{\Lambda} X_i)$$

$$\boxed{P(A_i|\boldsymbol{\Lambda} X_i = m) = P(A_j|\boldsymbol{\Lambda} X_j = m)}$$

Stronger No-fine-tuning

A causal model satisfies **No-fine-tuning** or is **Faithful** relative to a phenomenon $\mathcal{P}$ IFF it satisfies:

Causal No-fine-tuning: Every conditional independence between variables in $\mathcal{P}$ should arise as a consequence of the causal graph and not due to special choices of model parameters.

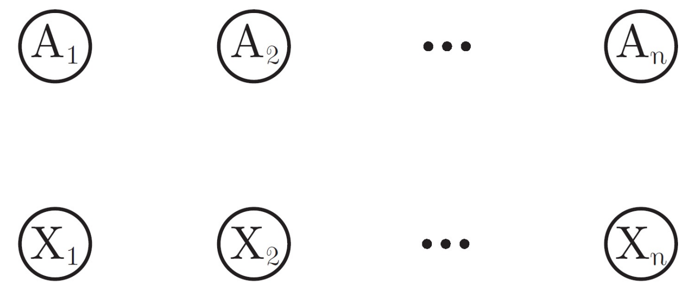
AND

Operational No-fine-tuning: Operational symmetries of $\mathcal{P}$ are reflected by the model, rather than holding only for special values of the model parameters.

$$\mathcal{P}(A_i|X_i = m) = \mathcal{P}(A_j|X_j = m) \longrightarrow P(A_i|\mathbf{\Lambda} X_i = m) = P(A_j|\mathbf{\Lambda} X_j = m)$$

- Our notion of ONFT is analogous to, but weaker than that recently introduced by Catani and Leifer, arXiv:2003.10050.
- It also follows from, but is weaker than, Spekkens' notion of MNC.
- ONFT is implied by Leibniz's principle

No Disturbance



A phenomenon satisfies **no-disturbance** iff

(i) $\mathcal{P}(\boldsymbol{A}_\gamma|\boldsymbol{X}) = \mathcal{P}(\boldsymbol{A}_\gamma|\boldsymbol{X}_\gamma)$ $\forall$ $\{A_i, X_i\}$, $\gamma \subseteq \mathcal{I}$ & $i \in \gamma$.

I.e. any subset of outcomes depends only on the associated subset of settings.

(ii) $\mathcal{P}(A_i|X_i = m) = \mathcal{P}(A_j|X_j = m) \forall i, j$

Marginals for the same measurement are independent of the variable index.

Theorem 2: *Every phenomenon satisfying no-disturbance in an arbitrary contextuality scenario that has a faithful causal model is factorisable.*

J.C Pearl and EGC, arXiv:1909.05434 (2019)

No-fine-tuning and no-disturbance imply factorisability in arbitrary Bell-KS scenarios.

Wrapping up

- **Non-contextuality and Bell-locality** both arise from the requirement of **no-fine-tuning** on classical causal models;
- Unifies KS-NC and Bell-nonlocality as violations of classical causality in full generality;
- **No assumption of determinism** is needed;
- **Theory-independent** derivation;

Wrapping up

- Open problem: quantum causal models give a faithful explanation of Bell correlations but not (yet) of contextuality.

→ Faithful quantum causal explanation of KS inequality violations?

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Thank you