Contextuality and Dichotomizations of Random Variables

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Outline

1. Contextuality-by-Default
   - Systems of Random Variables
   - Couplings and Contextuality
   - Dichotomizations and Split Representations
2. Theory for Choosing Dichotomizations
   - Allowable Coarse-grainings
   - Examples

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Systems of Random Variables

- System of random variables

$$\mathcal{R} = \left\{ R_q^c : q \prec c \right\} \tag{1}$$

- Double-indexed with
  - properties (*contents*) q being measured
  - conditions (*contexts*) c under which measurements made
- *Format relation* $q \prec c$ indicates which contents are measured in which contexts

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Systems of Random Variables

- Technically random variables R_q^c are measurable functions

$$R_q^c : \Omega_c \to E_q$$

  - all R_q^c with given c form a *bunch* (same *sample space* Ω_c so *jointly distributed*)
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Systems of Random Variables

- System $\mathcal{R} = \left\{ R_q^c : q \prec c \right\}$ conveniently illustrated as grid

$$\begin{array}{|c|c|c|c|c} \hline R_1^1 & R_2^1 & & & c=1 \\ \hline R_1^2 & & R_3^2 & R_4^2 & c=2 \\ \hline & R_2^3 & & R_4^3 & c=3 \\ \hline q=1 & q=2 & q=3 & q=4 & \end{array} \tag{2}$$

  - rows are bunches (jointly distributed)
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  - empty cells indicate pairs (q, c) missing from the format relation $q \prec c$

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Outline

Couplings

Definition

A *coupling* of a set $\{X_i : i \in I\}$ of random variables is a *jointly distributed* set $\{Y_i : i \in I\}$ of correspondingly indexed random variables where each Y_i has the same distribution as X_i.

Definition

A coupling $\{Y_1, Y_2\}$ of $\{X_1, X_2\}$ is *maximal* if the probability $\Pr[Y_1 = Y_2]$ is maximal among all couplings of $\{X_1, X_2\}$.

Definition

A *multimaximal* coupling $\{Y_i : i \in I\}$ of $\{X_i : i \in I\}$ is a coupling such that $\{Y_i, Y_j\}$ is a maximal coupling of $\{X_i, X_j\}$ for all $i, j \in I$.

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\end{array} \quad (3)$$

  - bunches (rows) jointly distributed
  - connections (columns) stochastically unrelated
- Coupling $S = \{S_q^c : q \prec c\}$ of system $\mathcal{R}$:

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Contextuality (traditional understanding)

Definition

A system $\mathcal{R} = \left\{ R_q^c : q \prec c \right\}$ of random variables

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is *noncontextual* if it has a coupling $S = \{S_q^c : q \prec c\}$

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\hline
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\end{array} \tag{6}$$

whose connections (columns) are identity couplings (i.e., all random variables in a column are identical, or equal with probability 1).

Contextuality (generalized, Contextuality-by-Default)

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whose connections (columns) are multimaximal couplings (i.e., all pairs of random variables in a column are maximal couplings).

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Dichotomizations

- Distribution of a random variable

$$R_q^c : \Omega_c \to E_q$$

is characterized by the probabilities $\Pr[R_q^c \in A]$ for all measurable subsets $A \subset E_q$

  - E_q can be arbitrarly complex (numbers, functions, sets, etc.), if endowed with suitable collection of measurable subsets
  - Any R_q^c is fully determined by a sufficiently rich collection of *dichotomizations*

$$R_{q,A}^c = \left[R_q^c \in A\right] := \begin{cases} 1, & R_q^c \in A \\ 0, & \text{otherwise} \end{cases}. \tag{9}$$

- In CbD, non-binary systems replaced by *split representation*, an equivalent binary system replacing each content q by several (q, A) such that dichotomizations $R_{q,A}^c$ uniquely determine R_q^c.

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Example of a Split Representation

Let the random variables of each connection of the system

$$\begin{array}{|c|c|c|c}
\hline
R_1^1 & R_2^1 & & c=1 \\
\hline
R_1^2 & & R_3^2 & c=2 \\
\hline
 & R_2^3 & R_3^3 & c=3 \\
\hline
q=1 & q=2 & q=3 &
\end{array} \tag{10}$$

have value-spaces $E_1 = \{1,2,3,4\}$, $E_2 = \{a,b,c\}$, and $E_3 = \{0,1\}$.

We can represent this original system by the binary system

$$\begin{array}{|c|c|c||c|c||c|c}
\hline
[R_1^1 \in \{1\}] & [R_1^1 \in \{1,2\}] & [R_1^1 \in \{1,2,3\}] & [R_2^1 \in \{a\}] & [R_2^1 \in \{b\}] & & c=1 \\
\hline
[R_1^2 \in \{1\}] & [R_1^2 \in \{1,2\}] & [R_1^2 \in \{1,2,3\}] & & & R_3^2 & c=2 \\
\hline
 & & & [R_2^3 \in \{a\}] & [R_2^3 \in \{b\}] & R_3^3 & c=3 \\
\hline
q=(1,\{1\}) & q=(1,\{1,2\}) & q=(1,\{1,2,3\}) & q=(2,\{a\}) & q=(2,\{b\}) & q=3 &
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Extended Understanding of Contextuality in CbD

- The extended understanding of contextuality in CbD is based on two modifications of traditional understanding:
  1. replacement of identity couplings of connections with multimaximal
  2. replacement of systems of random variables with their split representations.
- How are these choices justified?

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- How are these choices justified?

Justification of the choices

- Why multimaximal coupling?
  - The only other reasonable alternative to multimaximal coupling is maximal coupling (i.e., coupling $(Y_1, \ldots, Y_n)$ with maximal $\Pr[Y_1 = Y_2 = \cdots = Y_n]$)
  - however, it does not satisfy
    - *noncontextual nestedness*: any subsystem of a noncontextual system (obtained by removing random variables) is noncontextual
  - counterexample based on the fact if any two random variables X_i, X_j of $(X_1, \ldots, X_n)$ have non-overlapping supports, then any coupling $(Y_1, \ldots, Y_n)$ is maximal

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    - *noncontextual nestedness*: any subsystem of a noncontextual system (obtained by removing random variables) is noncontextual
  - counterexample based on the fact if any two random variables X_i, X_j of $(X_1, \ldots, X_n)$ have non-overlapping supports, then any coupling $(Y_1, \ldots, Y_n)$ is maximal

Justification of the choices

- Why dichotomizations?
  - A non-binary system does not generally satisfy the principle of
    - *coarse-graining*: a noncontextual system remains noncontextual following coarse-graining (certain values lumped together) of its random variables
  - The principle is trivial for binary systems
  - Can be recovered for non-binary systems using (appropriately chosen) dichotomizations:
    - dichotomizations of coarse-grained random variable correspond to dichotomizations of the original random variable
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Choice of Split Representation

- How to choose the split representation?
  - contextuality status may depend on the choice
  - for categorical random variables, use all possible dichotomizations as there is no structure to choose otherwise
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  - for real-valued, continuous random variables, using all possible dichotomizations makes all inconsistently connected systems contextual (and therby reduce to the traditional understanding of contextuality) so a smaller set would seem more reasonable
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Outline

1. Contextuality-by-Default
   - Systems of Random Variables
   - Couplings and Contextuality
   - Dichotomizations and Split Representations

2. Theory for Choosing Dichotomizations
   - Allowable Coarse-grainings
   - Examples

Allowable Coarse-grainings

- Assume value spaces E_q are endowed with a certain additional structure that defines *linked* sets (a certain form of pre-topological connectedness)
  - a more general structure than topology, needed to treat both continuous and discerete spaces
  - main idea: only allow coarse-grainings (or dichotomizations) that do not create new discontinuities in the value space
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Definition

A *coarse-graining* is a measurable surjection $f : E \to E'$;
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A coarse-graining $f : E \to E'$ is *allowable* if and only if

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Examples of Coarse-grainings

- Consider coarse-grainings $f : \mathbb{R} \to \{0, 1, 2\}$:[1]

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$\mapsto 0$ $\mapsto 1$ $\mapsto 0$ $\mapsto 2$

not allowable as the gray area is not linked even though it is the preimage of the linked set $\{0\}$

(b) $-\infty$ ∞

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not allowable as the linked gray–yellow area is coarse-grained into the non-linked set $\{0, 2\}$

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this is allowable (linked sets preserved in both directions)

[1] We assume here that linked sets of $\mathbb{R}$ are its connected subsets and linked sets of $\{0, 1, 2\}$ are are all subsets except $\{0, 2\}$.

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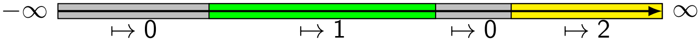

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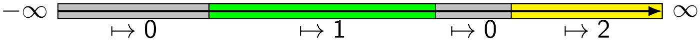

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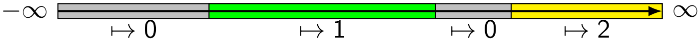
(a) 

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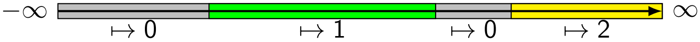
(a) 

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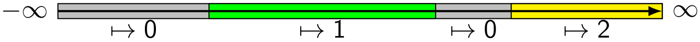
(a) 

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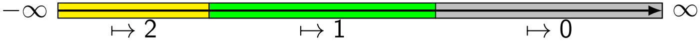
(c) 

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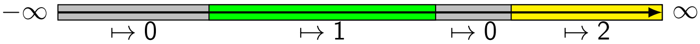
(a) 

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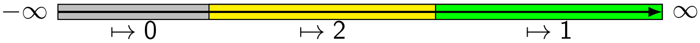
(b) 

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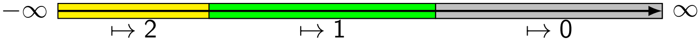
(c) 

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The coarse-graining principle is satisfied due to

Theorem

Allowable coarse-grainings are closed under compositions: if $f : E \to E'$ and $g : E' \to E''$ are allowable coarse-grainings, then $g \circ f : E \to E''$ is an allowable coarse-graining.

- Allowable dichotomization $d : E' \to \{0, 1\}$ of an allowably coarse-grained space $E' = f(E)$ yields an allowable dichotomization $d \circ f : E \to \{0, 1\}$ of the original space E.
- Thus, a dichotomization of a coarse-grained system is merely a subsystem of the dichotomization of the original system, because of which if the latter is noncontextual, then so is the former (due to noncontextual nestedness) .

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Structured Discrete Random Variables

- In a psychophysical experiment[2], subject identified position of visual object as *center*, *left*, *right*, *up*, or *down*: $\boxed{\begin{array}{ccc} & * & \\ * & * & * \\ & * & \end{array}}$
  - Result 5-valued random variable with certain spatial structure.
  - Model spatial structure with value space $E =$

    - linked sets are connected subgraphs (pre-topology generated from edges of the graph)
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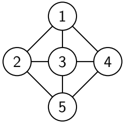
  - Model spatial structure with value space $E =$ 

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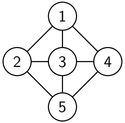
  - Model spatial structure with value space $E =$ 

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Real-valued Random Variables

- For real-valued random variables (with the natural pre-topological structure based on open intervals)
  - allowable dichotomizations are cuts $[R_q^c \leq x]$ (or $[R_q^c < x]$) for all cut-points $x \in \mathbb{R}$ (only one of the two kinds of cuts are needed, the other is redundant)

Theorem

The split representation of a single original column $\mathcal{R} = \{R^c : c = 1, \ldots, n\}$ based on cuts is always noncontextual.

Proof.

The required coupling is $\{S_x^c : c = 1, \ldots, n\}$, where $S_x^c = [F_c^{-1}(U) \leq x]$, U is a $[0, 1]$-uniform random variable, and F_c^{-1} is the quantile function of R^c. □

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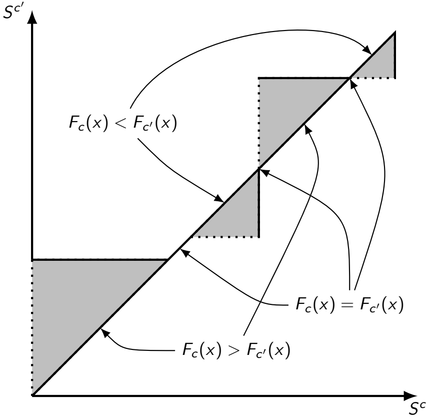
The split representation of a column $\mathcal{R}$ is noncontextual iff there exists a coupling S of $\mathcal{R}$ where for all c, c', the pair $(S^c, S^{c'})$ is supported in the set illustrated below:



Conclusion

- CbD requires every random variable in a system to be represented by equivalent set of dichotomous random variables
- We have presented general principles that justify the use of dichotomizations and determine their choice, based on respecting the "topological" structure of the value-spaces
- Enables treatment of systems of real-valued random variables
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