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Intersubjectivity of Quantum Measurement II

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Introduction

- **The common interpretation of the Kochen-Specker theorem denies the pre-existing value of an observable. Is the outcome of a measurement the result of the observer's personal gambling?**

- **Suppose that two observers simultaneously measure the same observable. Then it is predicted that their probability distributions are the same. Are the outcomes identical and perfectly correlated, or different and uncorrelated?**

- **Here, we shall show that quantum mechanics answers the question, so that only the first case occurs.**

- **This suggests that the common outcome of the two measurements reveals the value pre-existed before the measurement.**

- **Indeed, we show that the state-dependent approach enables quantum mechanics to answer the question: there is a time-like perfect correlation between the observable just before the measurement and the meter just after the measurement.**

Postulates for Quantum Mechanics

(P1) Quantum systems $\leftrightarrow$ Hilbert spaces; Observables $\leftrightarrow$ self-adjoint operators; States $\leftrightarrow$ unit vectors.

(P2) Born formula for commuting observables: For any commuting observables X, Y,

$$\Pr\{X = x, Y = y \| \Psi\} = \langle \Psi | P^X(x) P^Y(y) | \Psi \rangle. \tag{1}$$

(P3) Schrödinger-Heisenberg time-evolution: For any commuting observables X, Y,

$$\begin{aligned} \Pr\{X(t) = x, Y(t) = y \| \Psi\} &= \langle U(t)\Psi | P^X(x) P^Y(y) | U(t)\Psi \rangle & (2) \\ &= \langle \Psi | P^{X(t)}(x) P^{Y(t)}(y) | \Psi \rangle, & (3) \end{aligned}$$

where $U(t) = e^{-itH/\hbar}$ for the Hamiltonian H of the system, and $X(t) = U(t)^\dagger X U(t)$ and $Y(t) = U(t)^\dagger Y U(t)$.

Observer Independence of the Outcomes of Measurements

- **S: the system to be measured described by a Hilbert space $\mathcal{H}$.**
- **E: the environment described by a Hilbert space $\mathcal{K}$.**
- **A: an observable of S to be measured.**
- **M_1, M_2: two commuting observables in E representing the meters of the two remote observers.**

The time evolution of the total system with the total Hamiltonian H on $\mathcal{H} \otimes \mathcal{K}$ determines the Heisenberg operators $A(0)$, $M_j(t)$, $(j = 1, 2)$ with $0 < t$, where

$$\begin{aligned} A(0) &= A \otimes I, && (4) \\ M_j(t) &= U(t)^\dagger (I \otimes M_j) U(t), && (5) \\ U(t) &= \exp(-itH/\hbar). && (6) \end{aligned}$$

- **Let ψ and ξ be the initial state of the system S and the environment E. The POVMs of the two observers are defined by**

$$\Pi_j(y) \quad = \quad \langle\xi|P^{M_j(t)}(y)|\xi\rangle. \tag{7}$$

- **Observer j obtains the outcome x in the system state ψ with the probability**

$$\Pr\{M_j(t) = x\|\psi\otimes\xi\} \quad = \quad \langle\psi|\Pi_j(x)|\psi\rangle. \tag{8}$$

- **Then our sole assumption is that the two observers reproduce the probability distribution of A correctly; namely, we require**

$$\Pr\{M_j(t) = x\|\psi\otimes\xi\} \quad = \quad \Pr\{A(0) = x\|\psi\otimes\xi\}. \tag{9}$$

- **Equivalently,**

$$\Pi_j(x) = P^A(x). \tag{10}$$

- **Since $M_1(t)$ and $M_2(t)$ commute, their joint probability distribution in the initial state $\psi \otimes \xi$ is given by**

$$\Pr\{M_1(t) = x, M_2(t) = y \| \psi \otimes \xi\} = \langle \psi \otimes \xi | P^{M_1(t)}(x) P^{M_2(t)}(y) | \psi \otimes \xi \rangle \quad (11)$$

for all $x, y \in \mathbb{R}$. Thus, quantum mechanics can answer whether the above JPD is perfectly correlated or uncorrelated.

- **Theorem 1.** ***The outcomes of the measurements of A by the two observers are identical, i.e.,***

$$\Pr\{M_1(t) = x, M_2(t) = y \| \psi \otimes \xi\} = 0 \quad (12)$$

if $x \neq y$.

- **The proof is based on the theory of quantum perfect correlations developed in [M. Ozawa, Quantum perfect correlations, Ann. Phys. (N.Y.) 321, 744-769 (2006)].**

Value Reproducibility of Quantum Measurements

- **Theorem 1 suggests that the measurement reproduces the pre-existing value, i.e.,**

$$\Pr\{A(0) = x, M_j(t) = y \| \psi \otimes \xi\} = 0 \tag{13}$$

if $x \neq y$, where $j = 1, 2$.

- **There is a difficulty in the above formula, since $A(0)$ and $M_j(t)$ are not necessarily commuting.**

- **However, we shall show that $A(0)$ and $M_j(t)$ are essentially commuting, and Eq. (13) actually makes sense to hold.**

State-Dependent Commutativity

- **von Neumann (1932; English ed. p. 230) observed that any observables X and Y are commuting on the subspace $\mathcal{M}(X,Y)$ generated by the common eigenstates of X and Y.**

- **Definition. We say that X and Y are *commuting in a state* Ψ ($X \leftrightarrow_{\Psi} Y$) iff $\Psi \in \mathcal{M}(X,Y)$.**

- **Definition. $\mu(x,y)$ is a *joint probability distribution (JPD)* of X, Y in Ψ iff**

$$\langle \Psi | f(X,Y) | \Psi \rangle = \sum_{x,y} f(x,y) \mu(x,y) \tag{14}$$

for any polynomial $f(X,Y)$.

- **Theorem 2. The following conditions are equivalent.**

  (i) $X \leftrightarrow_{\Psi} Y$**.**

  (ii) $[P^X(x), P^Y(y)]\Psi = 0$ **for any** $x, y \in \mathbb{R}$**.**

  (iii) JPD μ **of** X, Y **in** Ψ **exists.**

  (iv) Ψ **is a superposition of common eigenstates, i.e.,** $\Psi = \sum_{x,y} c_{x,y} |X = x, Y = y\rangle$**.**

  (v) $\sum_{x,y} P^X(x) \wedge P^Y(y)\Psi = \Psi$**.**

  In this case, we obtain

$$\mu(x, y) = |c_{x,y}|^2 = \langle\Psi|P^X(x)P^Y(y)|\Psi\rangle = \langle\Psi|P^X(x) \wedge P^Y(y)|\Psi\rangle. \tag{15}$$

- **State-Dependent Born Formula: If** $X \leftrightarrow_{\Psi} Y$**, we define**

$$\Pr\{X = x, Y = y\|\Psi\} = \langle\Psi|P^X(x)P^Y(y)|\Psi\rangle. \tag{16}$$

Quantum Perfect Correlations

- **Definition (MO 2006): We say that X, Y are *perfectly correlated in a state* Ψ ($X =_\Psi Y$) iff $\quad X \leftrightarrow_\Psi Y$ and $\Pr\{X = x, Y = y \| \Psi\} = 0$ if $x \neq y$.**

- **Theorem 3 (MO 2006). The following conditions are equivalent.**

  (i) $X =_\Psi Y$.

  (ii) Ψ is a superposition of common eigenstates of X and Y with common eigenvalues, i.e., $\Psi = \sum_x c_x \, |X = x, Y = x\rangle$.

  (iii) $\sum_x P^X(x) \wedge P^Y(x)\psi = \psi$.

- **Theorem 4 (MO2006). The relation $=_\Psi$ is an equivalence relation. In particular, it is transitive, i.e., if $X =_\Psi Y$ and $Y =_\Psi Z$, then $X =_\Psi Z$.**

From Value Reproducibility to Observer Independence

- **Theorem 5 (MO2006). If a measurement of A with the meter $M(t)$ has the correct POVM Π, i.e., $\Pi = P^{A}$, then the value reproducibility holds, i.e.,**

$$A(0) =_{\psi\otimes\xi} M(t) \tag{17}$$

for all ψ.

- **Proof of Theorem 1: From Theorem 5, we have**

$$M_1(t) =_{\psi\otimes\xi} A(0) \quad \textbf{and} \quad A(0) =_{\psi\otimes\xi} M_2(t). \tag{18}$$

By transitivity, we have

$$M_1(t) =_{\psi\otimes\xi} M_2(t). \tag{19}$$

Thus, we have

$$\Pr\{M_1(t) = x, M_2(t) = y \| \psi \otimes \xi\} = 0 \tag{20}$$

if $x \neq y$ for all ψ.

Conclusion

- **The Kochen-Specker theorem denies the "non-contextual" pre-existing value. Nevertheless, the outcomes of simultaneous measurements by remote observers of an observable is always unique and observer independent, shown as the space-like perfect correlation between the two outcomes.**

- **This suggests the existence of the "pre-existing value" as a common cause for the coincidence of outcomes of remote observers.**

- **The state-dependent approach to quantum mechanics can indeed prove that the measurement reveals the "contextual" pre-existing value, as the time-like perfect correlation between the observable just before the measurement and the meter just after the measurement.**

- **Here, the "context" is brought by the measuring interaction that makes the time-like perfect correlation between the observable and the meter. The value of an observable is thus defined by the measuring arrangement as Bohr claimed long ago.**