Varieties of contextuality – emphasizing (non)embeddability

http://tph.tuwien.ac.at/~svozil/publ/
2021-QCQMB2021-pres.pdf

Karl Svozil

ITP TU Wien, Vienna Austria
svozil@tuwien.ac.at

Online presentation, Wednesday, May 19, 2021

Three main varieties of (quantum) contextuality: #1 & #2

1. Bohr & Heisenberg forms based on complementarity and measurement uncertainty/nesting; cf. Khrennikov, Jaeger, ...;

Three main varieties of (quantum) contextuality: #1 & #2

1. Bohr & Heisenberg forms based on complementarity and measurement uncertainty/nesting; cf. Khrennikov, Jaeger, ...;

2. nonclassical probability distributions such as of Born-Gleason-Lovász-type, and forms derived from generalizations of Cauchy-type functional equations yielding violations of classical predictions; these forms are both quasi–"empirical" through counterfactuals & stochastic (Specker 1960):

Three main varieties of (quantum) contextuality: #1 & #2

1. Bohr & Heisenberg forms based on complementarity and measurement uncertainty/nesting; cf. Khrennikov, Jaeger, ...;

2. nonclassical probability distributions such as of Born-Gleason-Lovász-type, and forms derived from generalizations of Cauchy-type functional equations yielding violations of classical predictions; these forms are both quasi–"empirical" through counterfactuals & stochastic (Specker 1960):

    2.1. Boole-Bell type inequalities discussed by Bell, Froissart, Pitowsky, Tsirelson, CHSH, Suppes-Zanotti, Cabello, ... ;

Three main varieties of (quantum) contextuality: #1 & #2

1. Bohr & Heisenberg forms based on complementarity and measurement uncertainty/nesting; cf. Khrennikov, Jaeger, ...;

2. nonclassical probability distributions such as of Born-Gleason-Lovász-type, and forms derived from generalizations of Cauchy-type functional equations yielding violations of classical predictions; these forms are both quasi–"empirical" through counterfactuals & stochastic (Specker 1960):

   2.1. Boole-Bell type inequalities discussed by Bell, Froissart, Pitowsky, Tsirelson, CHSH, Suppes-Zanotti, Cabello, ... ;

   2.2. based on gadget graphs with input/output terminals—aka pre-/postselection of pure quantum states: (Kochen-)Specker bug (1965, aka Hardy-type, cf Stigler's law of eponymy), Belinfante, Stairs, Cabello, ...;

Three main varieties of (quantum) contextuality: #3

3. nonempirical / logical / algebraic / theoretical / counterfactual structure of observables with nonclassical interpretations:

Three main varieties of (quantum) contextuality: #3

3. nonempirical / logical / algebraic / theoretical / counterfactual structure of observables with nonclassical interpretations:

   3.1. **nonfaithful embedding into Boolean algebras associated with inseparability, nonunital value assignments, and other nonclassical properties;**

Three main varieties of (quantum) contextuality: #3

3. nonempirical / logical / algebraic / theoretical / counterfactual structure of observables with nonclassical interpretations:

   3.1. **nonfaithful embedding into Boolean algebras associated with inseparability, nonunital value assignments, and other nonclassical properties;**

   3.2. nonexistence of any classical interpretation aka two-valued (even partial) states: Gleason, Specker, Zierler-Schlessinger, Kamber, Kochen-Specker, Pitowsky, Hrushovski-Pitowsky, Cabello, Abbot-Calude-Svozil ...;

A brief note on nonclassical probability distributions

Tactics   what "we do" tactically:

A brief note on nonclassical probability distributions

Tactics   what "we do" tactically:

   BOO   take some suitable bag / collection of (maybe quantum or partition logic) observables which are in different (intertwined) contexts;

A brief note on nonclassical probability distributions

Tactics what "we do" tactically:

     BOO take some suitable bag / collection of (maybe quantum or partition logic) observables which are in different (intertwined) contexts;

     CL see how a classical interpretation (aka two-valued states) performs on them—classical predictions;

A brief note on nonclassical probability distributions

Tactics   what "we do" tactically:

BOO   take some suitable bag / collection of (maybe quantum or partition logic) observables which are in different (intertwined) contexts;

CL   see how a classical interpretation (aka two-valued states) performs on them—classical predictions;

QU   see how a quantum interpretation (eg, vertex labeling by vectors) performs on them—quantum predictions;

A brief note on nonclassical probability distributions

Tactics   what "we do" tactically:

        BOO   take some suitable bag / collection of (maybe quantum or partition logic) observables which are in different (intertwined) contexts;

        CL   see how a classical interpretation (aka two-valued states) performs on them—classical predictions;

        QU   see how a quantum interpretation (eg, vertex labeling by vectors) performs on them—quantum predictions;

CL/QU@BOO   hopefully establish a discrepancy between classical & quantum predictions —bingo!

A brief note on nonclassical probability distributions

Tactics what "we do" tactically:

BOO take some suitable bag / collection of (maybe quantum or partition logic) observables which are in different (intertwined) contexts;

CL see how a classical interpretation (aka two-valued states) performs on them—classical predictions;

QU see how a quantum interpretation (eg, vertex labeling by vectors) performs on them—quantum predictions;

CL/QU@BOO hopefully establish a discrepancy between classical & quantum predictions —bingo!

Note There are three important issues to consider:

A brief note on nonclassical probability distributions

Tactics what "we do" tactically:

    BOO take some suitable bag / collection of (maybe quantum or partition logic) observables which are in different (intertwined) contexts;

    CL see how a classical interpretation (aka two-valued states) performs on them—classical predictions;

    QU see how a quantum interpretation (eg, vertex labeling by vectors) performs on them—quantum predictions;

CL/QU@BOO hopefully establish a discrepancy between classical & quantum predictions —bingo!

Note There are three important issues to consider:

    Fact in general the logic / algebra does not uniquely determine the probability distribution aka the predictions;

A brief note on nonclassical probability distributions

Tactics what "we do" tactically:

  BOO take some suitable bag / collection of (maybe quantum or partition logic) observables which are in different (intertwined) contexts;

  CL see how a classical interpretation (aka two-valued states) performs on them—classical predictions;

  QU see how a quantum interpretation (eg, vertex labeling by vectors) performs on them—quantum predictions;

CL/QU@BOO hopefully establish a discrepancy between classical & quantum predictions —bingo!

Note There are three important issues to consider:

  Fact in general the logic / algebra does not uniquely determine the probability distribution aka the predictions;

  Question "given some logic or some observables, what possible probability distributions are allowed relative to which axioms of probability?"

A brief note on nonclassical probability distributions

Tactics   what "we do" tactically:

     BOO   take some suitable bag / collection of (maybe quantum or partition logic) observables which are in different (intertwined) contexts;

     CL   see how a classical interpretation (aka two-valued states) performs on them—classical predictions;

     QU   see how a quantum interpretation (eg, vertex labeling by vectors) performs on them—quantum predictions;

CL/QU@BOO   hopefully establish a discrepancy between classical & quantum predictions —bingo!

Note   There are three important issues to consider:

     Fact   in general the logic / algebra does not uniquely determine the probability distribution aka the predictions;

     Question   "given some logic or some observables, what possible probability distributions are allowed relative to which axioms of probability?"

     Choice   of the distribution depends on the physical / psychological *etc* realization of the *BOO*.

Anecdotal example: probabilities on a cyclic logic whose respective hypergraph is a pentagon aka pentagram aka house
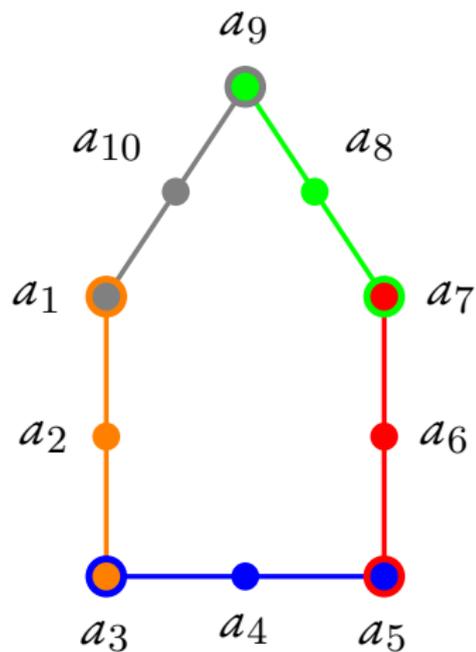
1) classical probability distributions in terms of convex combinations of the 11 two-valued states thereon;

2) quantum probability distributions according to Born, Gleason, and Lovász;

3) exotic probability according to Gerelle & Greechie & Miller (1974) and Wright (1978)

4) — ... ?

So far we only spoke about comparing
different probability distributions on fixed collections of
(interwined)observables ...

❧ *interlude* ☙

... now we shall be talking about
"weird" nonclassical collections of (interwined)observables ...

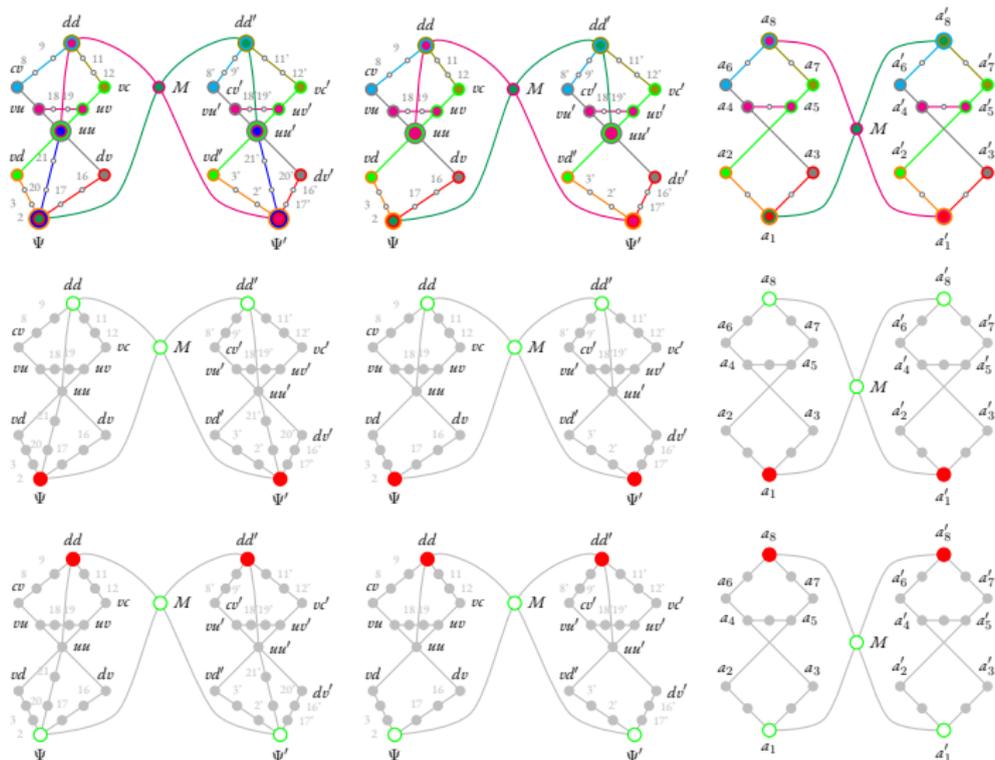# Inseparability 101: Kochen & Specker's demarcation criterion
1967, Theorem 0 of DOI: 10.1512/iumj.1968.17.17004

Theorem 0. Let $\mathfrak{R}$ be a partial Boolean algebra. A necessary and sufficient condition that $\mathfrak{R}$ is imbeddable in a Boolean algebra B is that for every pair of distinct elements a, b in $\mathfrak{R}$ there is a homomorphism $h: \mathfrak{R} \to Z_2$ such that $h(a) \neq h(b)$.
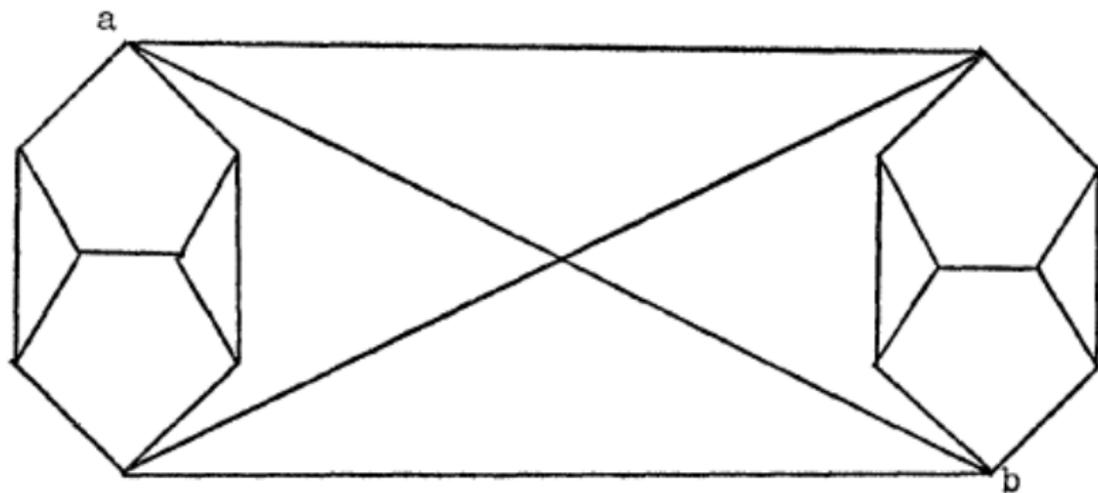
Graph of Γ_3

Hypergraphs with nonseparable set of two-valued states
third column is Kochen & Specker (1967, Γ_3)

Hypergraph with nonunital set of 6 value assignments

Fig. 2 'Almost' Greechie diagram of a suborthoposet of $L(\mathbf{R}^3)$ without a unital set of two-valued states [e.g., $12\bar{1} = \mathrm{Sp}(1, 2, -1)$].

Hypergraph with exotic contextuality derived from coloring

Hypergraph of biconnected intertwined contexts representing complete graphs with a separating set of 6 two-valued states which is non-partitionable: G_{32}, cf. Figure 6, p. 121 Greechie (1971) DOI: 10.1016/0097-3165(71)90015-X
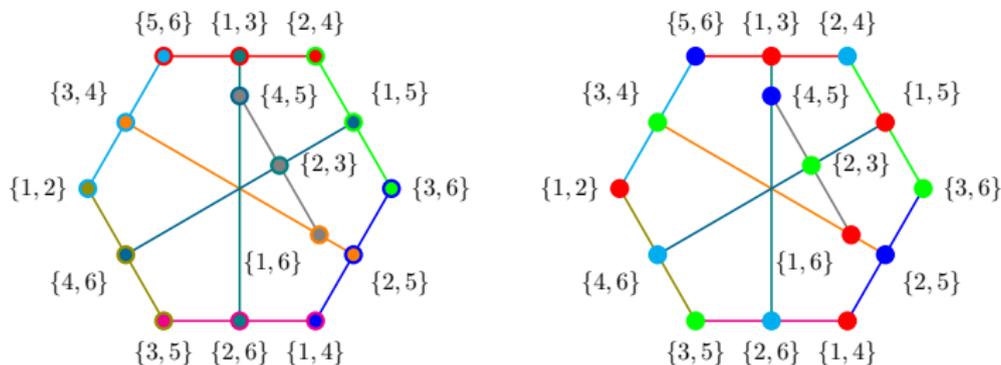
Mohammad H. Shekarriz & KS, vertex labeling by partitions of $\{1, 2, 3, 4, 5, 6\}$ with no faithful orthogonal representation arXiv:2105.08520.

Thank you for your attention!