PAOLO PERINOTTI

INFORMATION AND DISTURBANCE

In operational probabilistic theories

QUit quantum information theory group

UNIVERSITÀ DI PAVIA

OUTLINE

- **Information and disturbance from Heisenberg to quantum information**

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- **Widening the playground: Operational Probabilistic Theories**

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- **Disturbance and correlations**

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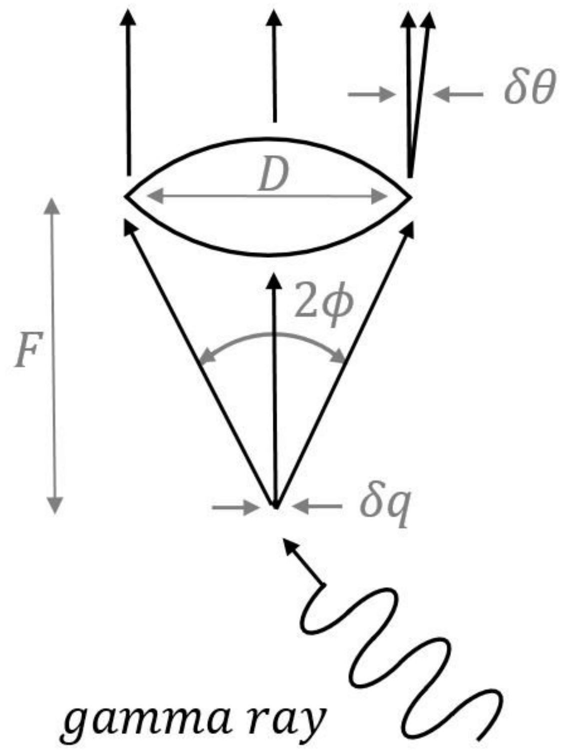
- Necessary and sufficient conditions for NIWD
- Conclusion

HEISENBERG'S GAMMA-RAY EXPERIMENT

Thought experiment used to justify intuitively the uncertainty principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

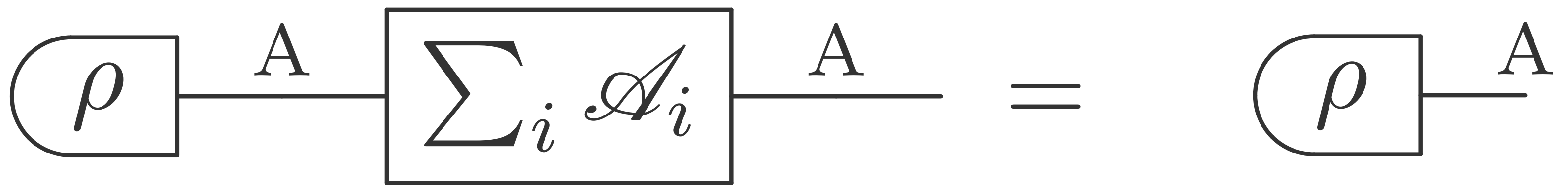


Statistical meaning: there is no quantum state such that predictions of x and p have $\Delta x \Delta p < \frac{\hbar}{2}$

The thought experiment actually introduces a different but related problem: can we measure a system without disturbing its state?

DISTURBANCE

- In quantum information theory: definition by negation
  - Non-disturbing measurement: state after the measurement equal to the one before
  - Quantum instrument $\{\mathscr{A}_i\}$ such that $\sum_i \mathscr{A}_i = \mathscr{I}$



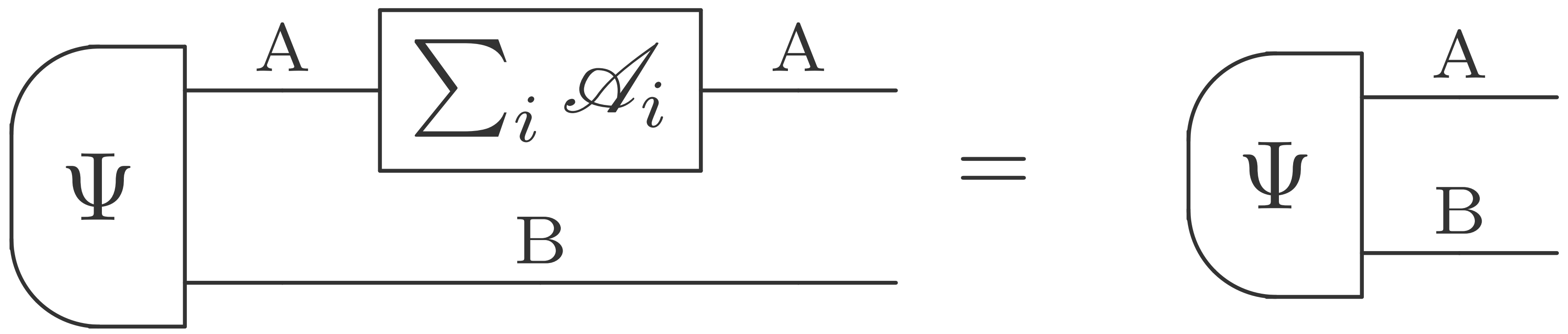
  - This is possible only if $\mathscr{A}_i = p_i \mathscr{I}$

- No disturbance implies no information:

  “No information without disturbance”

EQUIVALENT DEFINITION OF DISTURBANCE

- Equivalent notion of (no-)disturbance:



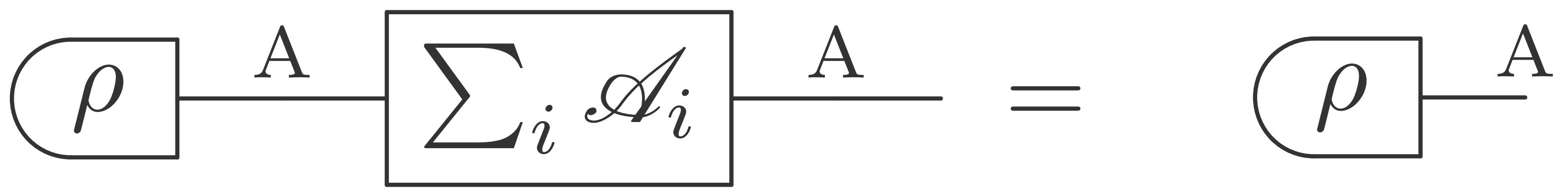
- Quantum information is quantum entanglement:

  "…we conclude that the deepest answer to the question is that quantum information lies in the entanglement between systems. Quantum communication, in this view, is fundamentally about the transfer of that entanglement from one system to another…"
  B. Schumacher and M. Westmoreland, "Quantum Processes, Systems & Information", Cambridge University Press (2010)

DISTURBANCE OF CORRELATIONS

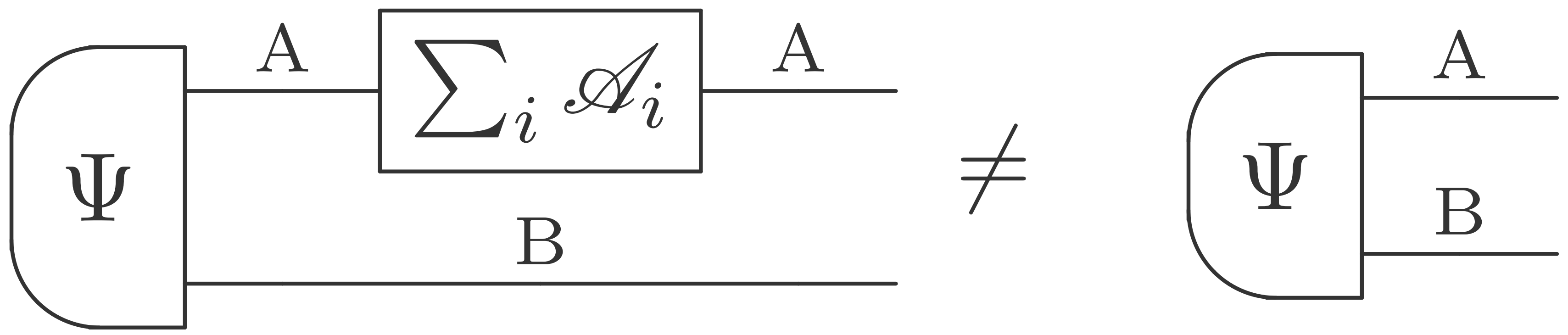
- (No-)Disturbance on correlations: becomes the definition for general theories
- There are indeed situations where

$\forall \rho$



but

$\exists \Psi$



OPERATIONAL PROBABILISTIC THEORIES (IN A NUTSHELL)

System: A

G. Chiribella, G. M. D'Ariano, and PP, Phys. Rev. A 81, 062348 (2010),
G. M. D'Ariano, G. Chiribella, and PP, Quantum Theory from first principles, CUP (2017)

OPERATIONAL PROBABILISTIC THEORIES (IN A NUTSHELL)

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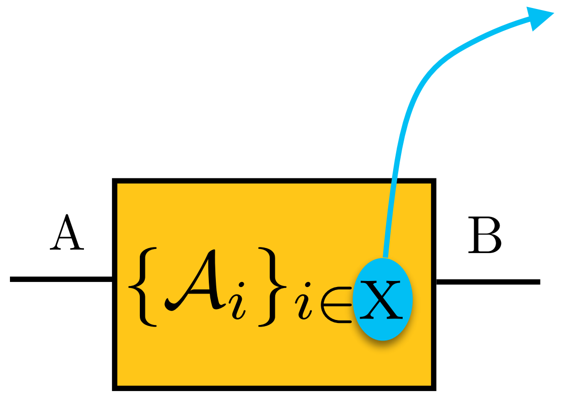
finite set of outcomes

Test:



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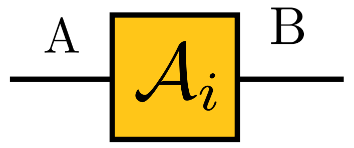

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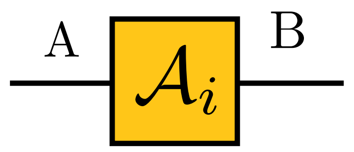
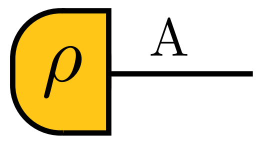
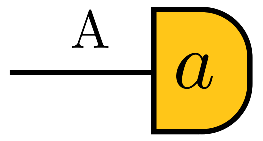
collection of **events:** 

Preparation test: collection of **states** 

Observation test: collection of **effects** 

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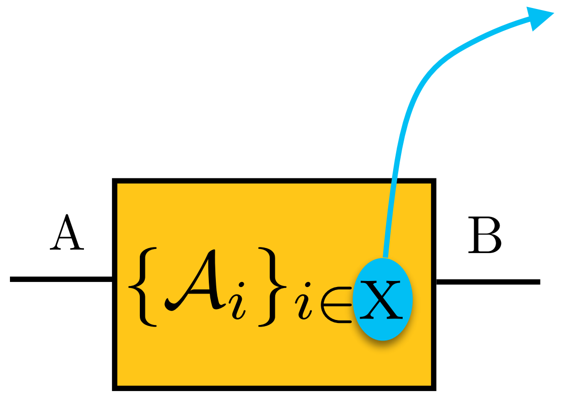

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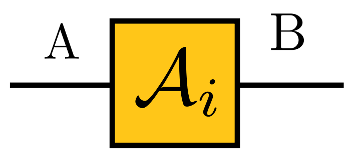

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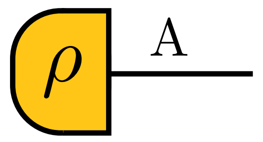

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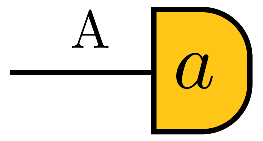

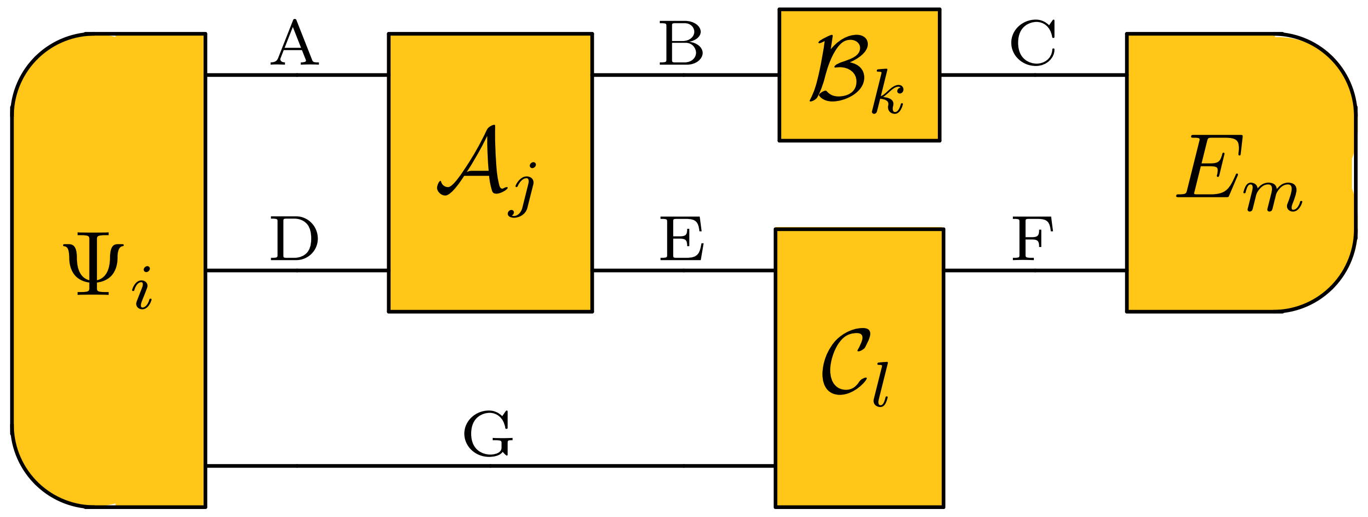

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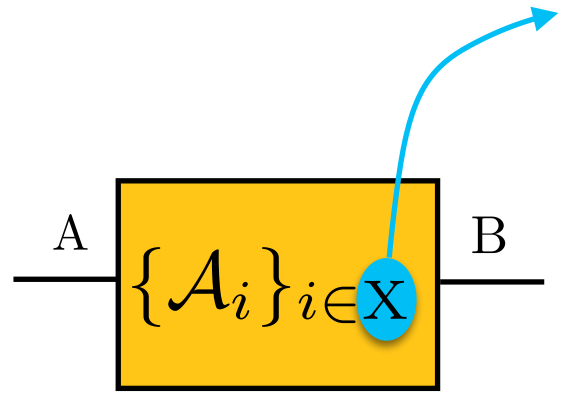

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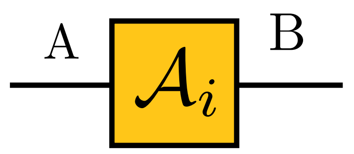

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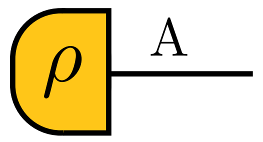

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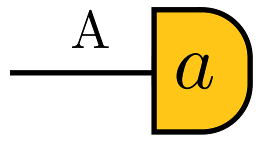
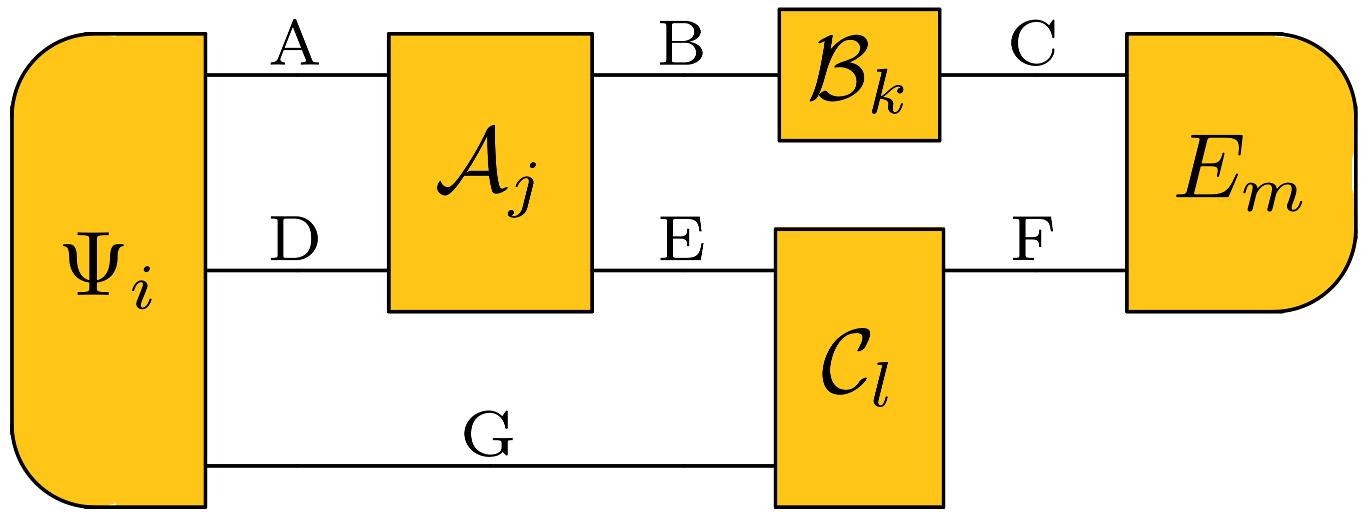
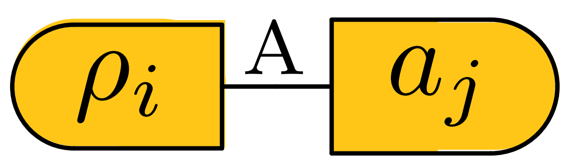




Probabilistic structure:



$:= \Pr[i, j]$

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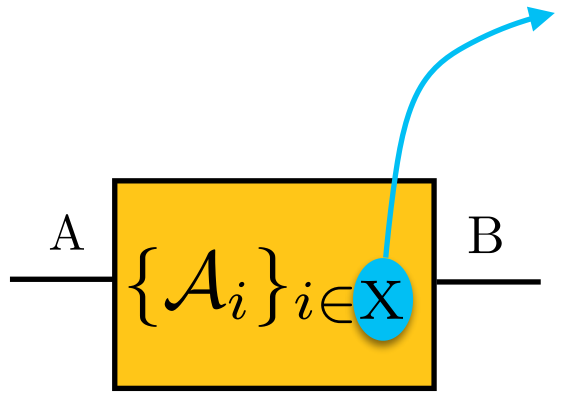

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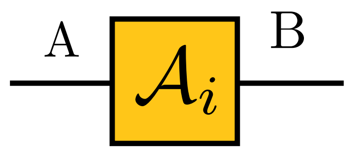

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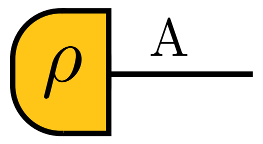

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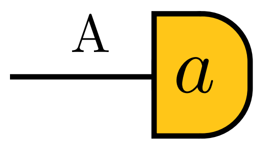

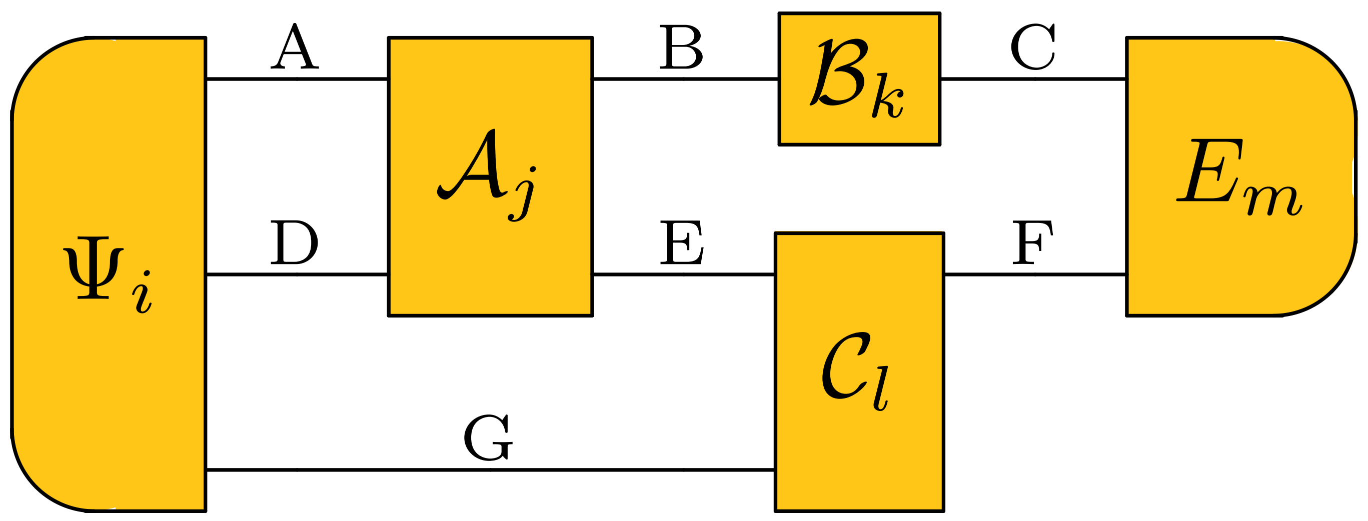

$= \Pr[i, j, k, l, m]$

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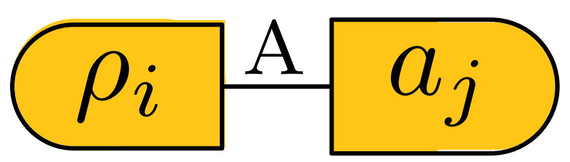

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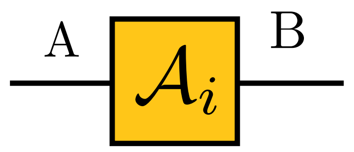
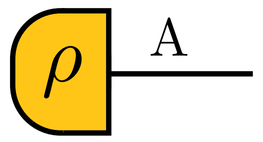
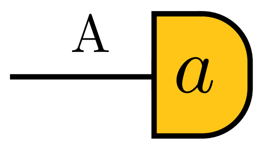
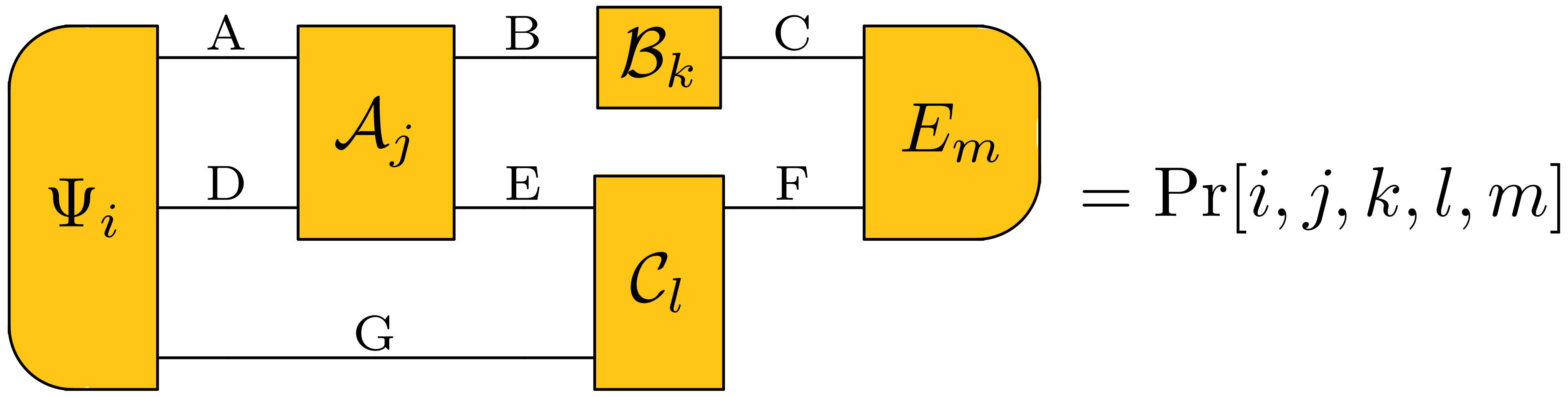
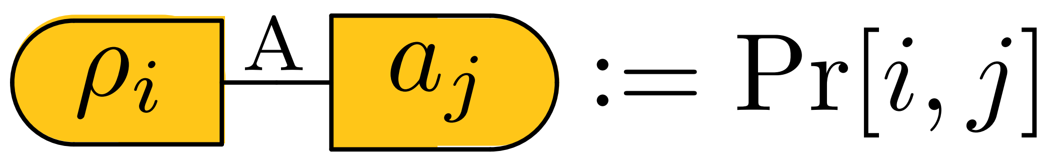
collection of **events:** 

Preparation test: collection of **states** 

Observation test: collection of **effects** 



Probabilistic structure: 

Coarse graining: $\forall j \in Z\ Y_j \subseteq X, \quad j_1 \neq j_2 \Rightarrow Y_{j_1} \cap Y_{j_2} = \emptyset, \quad \bigcup_j Y_j = X \quad \Rightarrow \quad \exists\{\mathscr{A}_{Y_j}\}_{j\in Z}$

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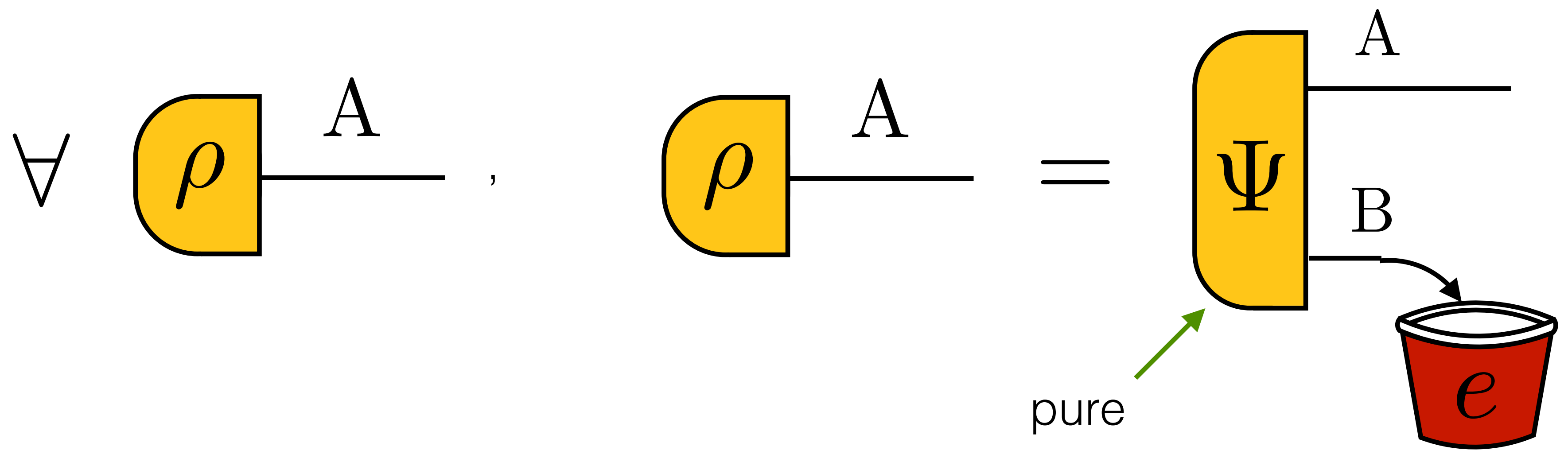
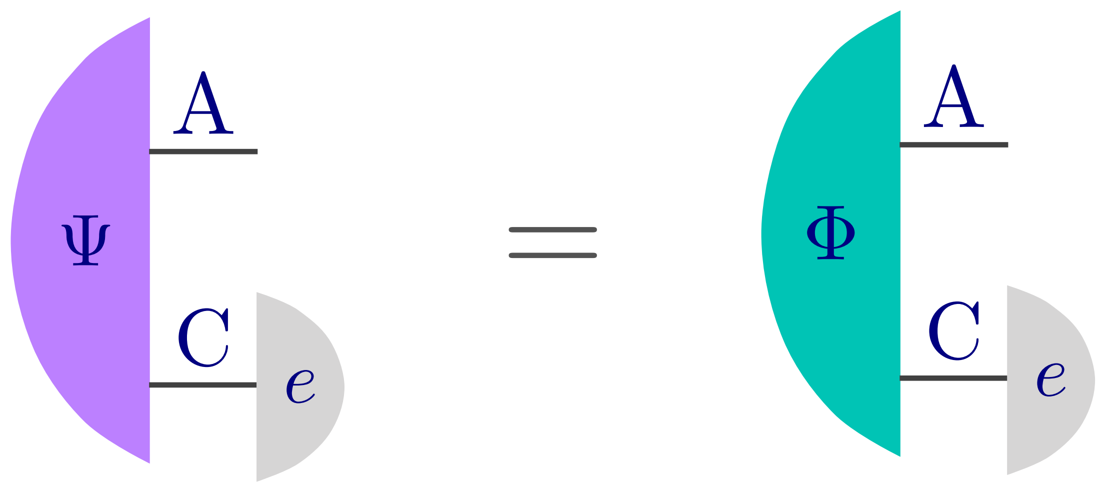
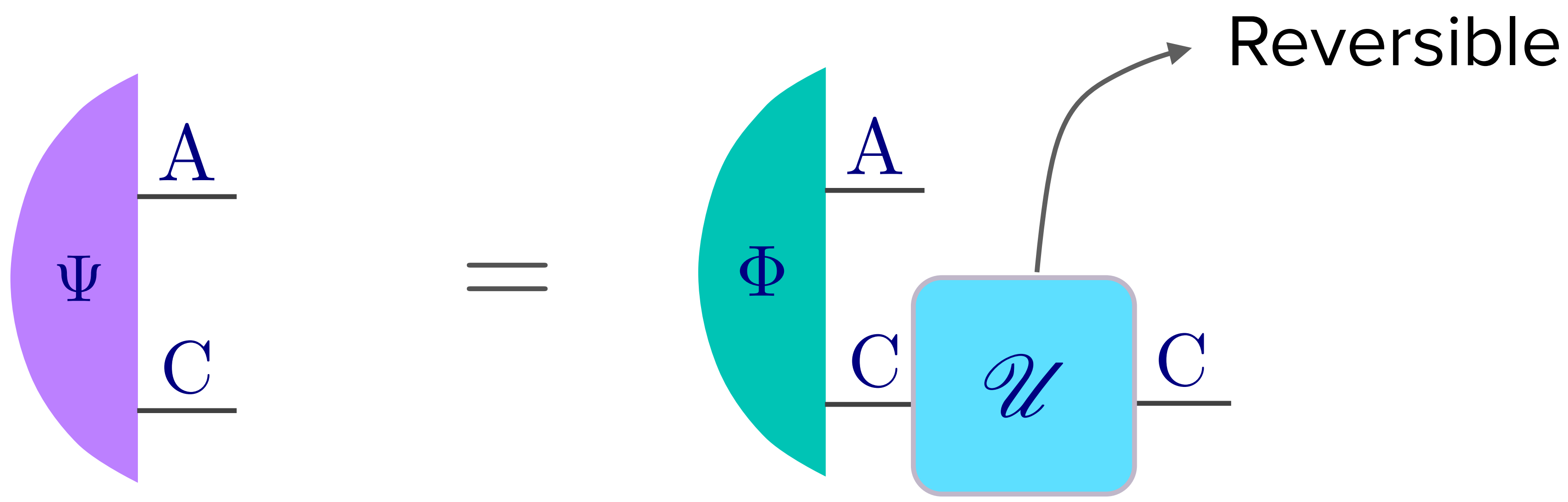
ATOMIC EVENTS AND PURIFICATION

- Atomic event: an event that can be refined only trivially $\mathscr{A} = \sum_j \mathscr{A}_j \quad \Rightarrow \quad \mathscr{A}_j = p_j \mathscr{A}$
- Atomic states in a causal theory are called pure
- Existence of purification:



- Uniqueness of purification





EXAMPLE: FERMIONIC THEORY

- Systems: collections of Local Fermionic Modes (identified by the integer number N)
- To a system of N modes one associates the Hilbert space of N qubits
- Choose the computational basis $|0\rangle, |1\rangle$ and form the basis of tensor products
- Pure states correspond to rays

Fock space: $\mathcal{F} = \mathcal{F}_e \oplus \mathcal{F}_o$

even sector

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Parity superselection

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Any state is of the form:

$$\rho = \left(\begin{array}{c|c} p\rho_e & 0 \\ \hline 0 & (1-p)\rho_o \end{array}\right)$$

STATE SPACE

- States are represented by density matrices
- Parity superselection determines a block structure
- The state space of a composite system is larger than the tensor product of state spaces of component systems
- No local discriminability

$$\mathrm{St}(\mathrm{A}) = \begin{pmatrix} \mathrm{E} & \\ & \mathrm{O} \end{pmatrix}$$

$$\mathrm{St}(\mathrm{B}) = \begin{pmatrix} \mathrm{E} & \\ & \mathrm{O} \end{pmatrix}$$

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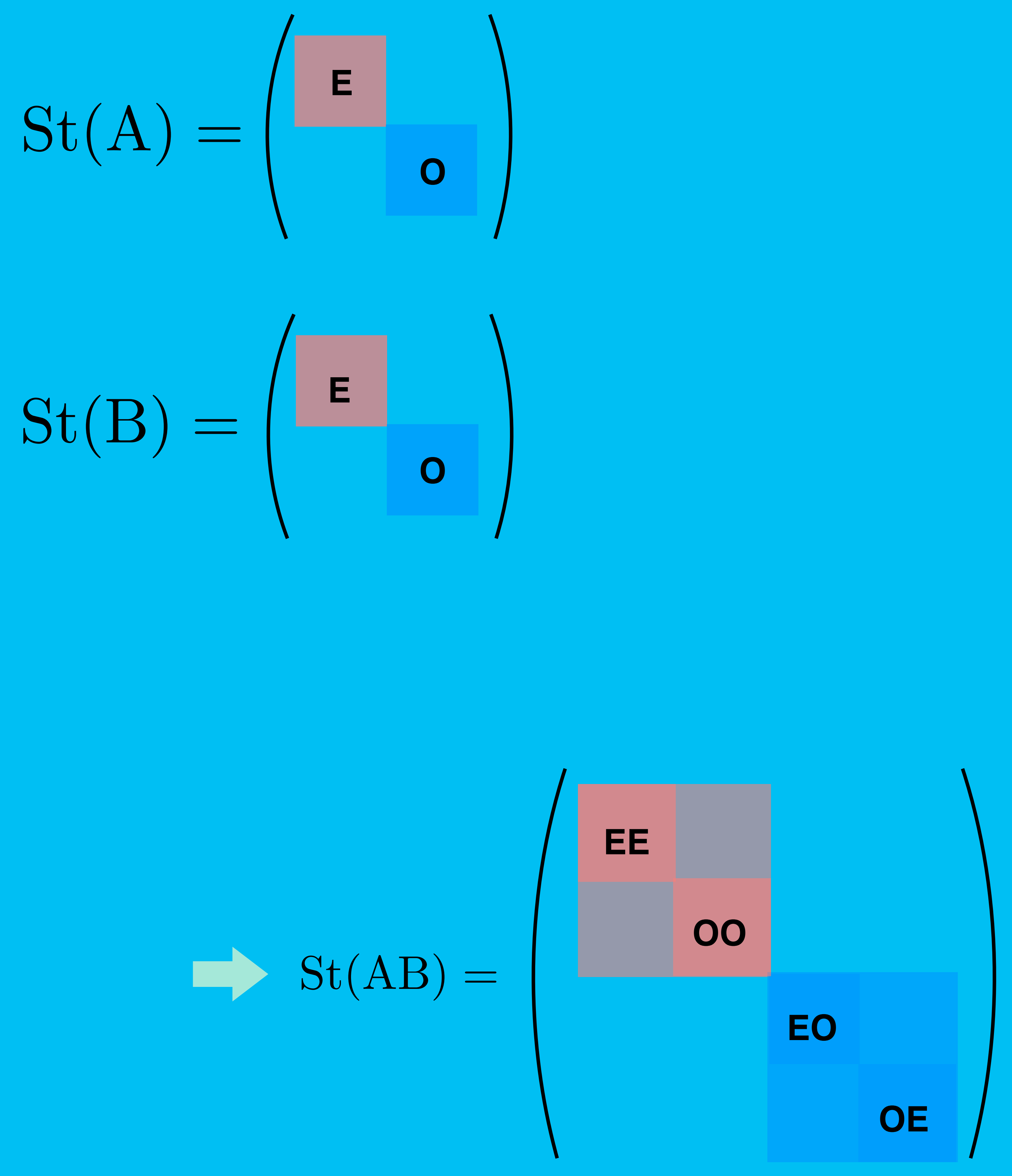
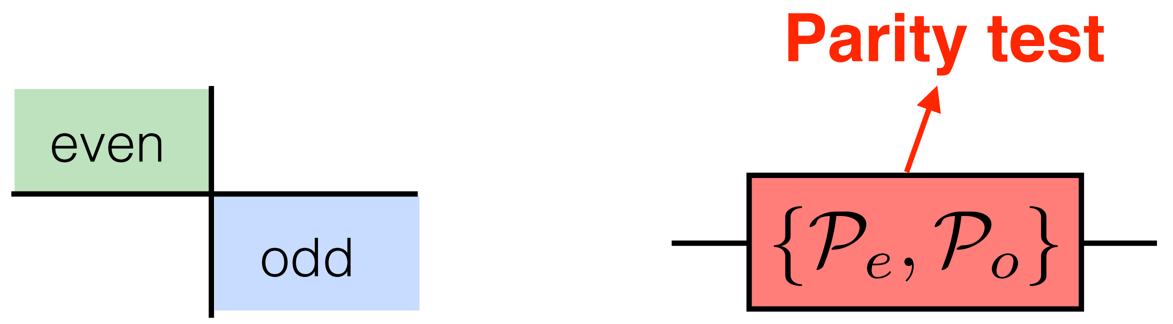
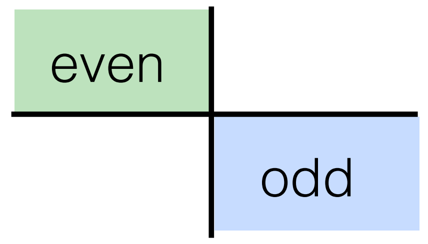
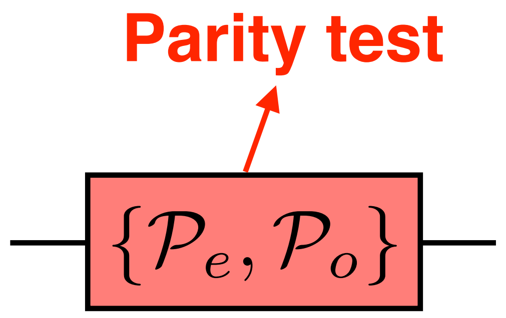


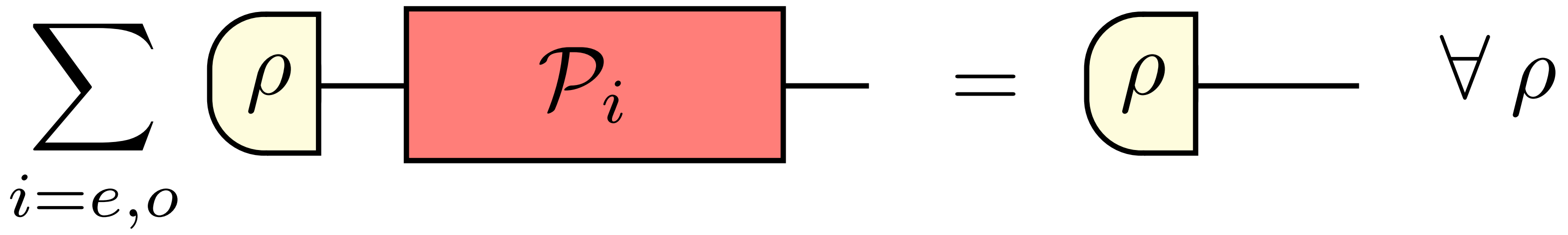


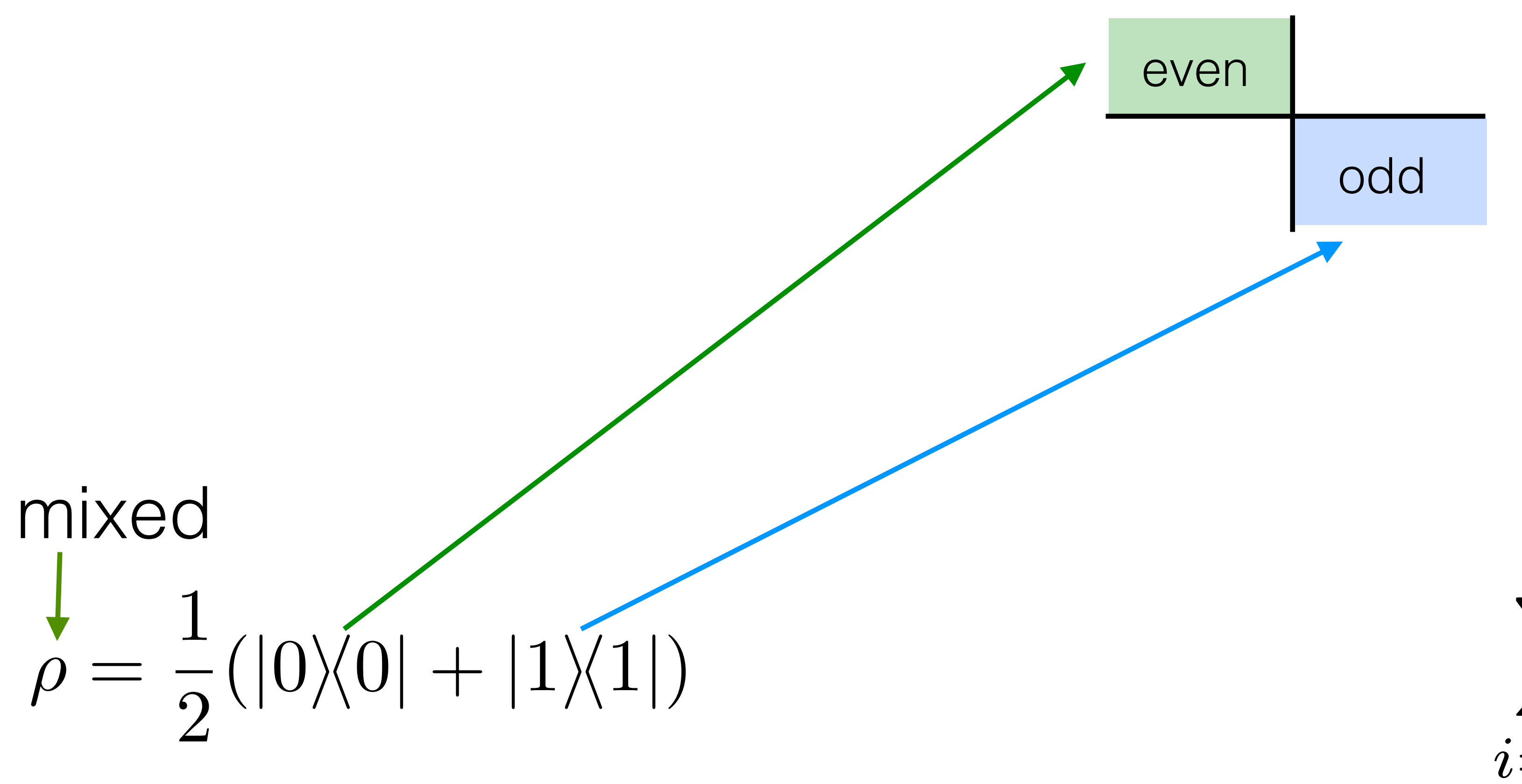
even
odd
Parity test
$\{\mathcal{P}_e, \mathcal{P}_o\}$

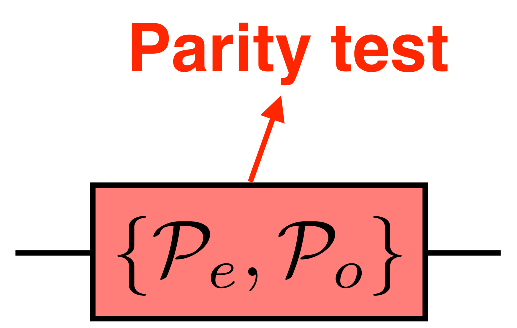




Parity test does not disturb locally







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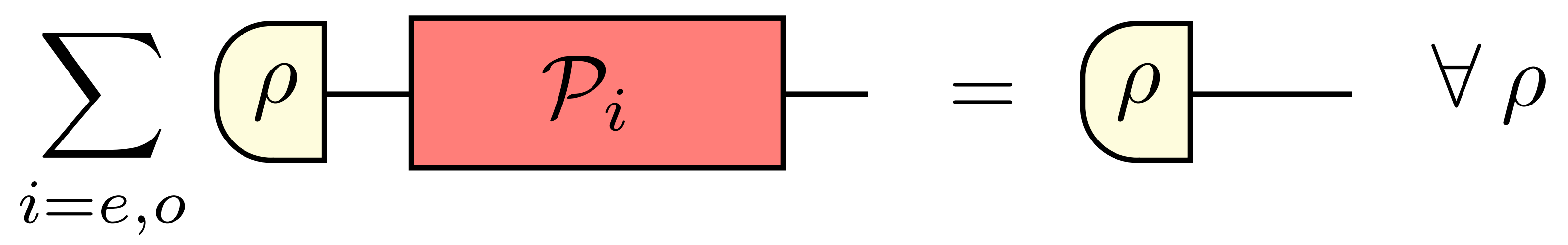
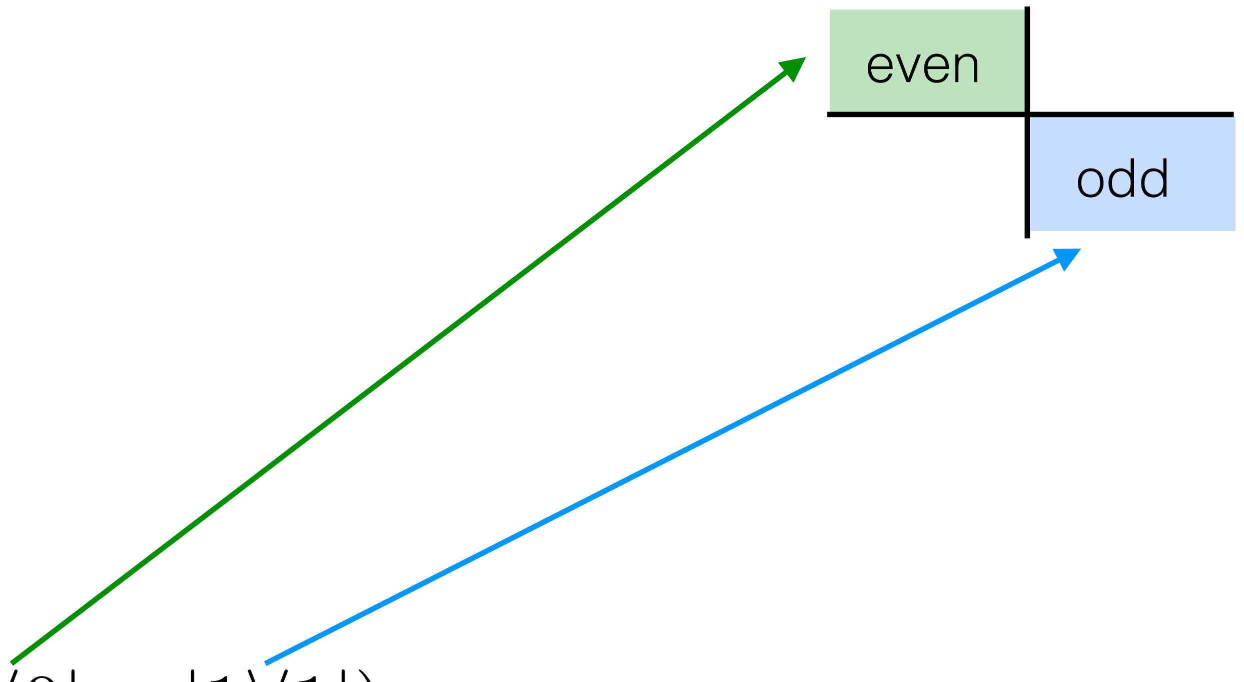
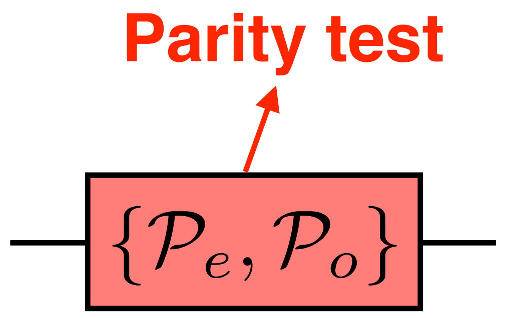


mixed

$$\rho = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$





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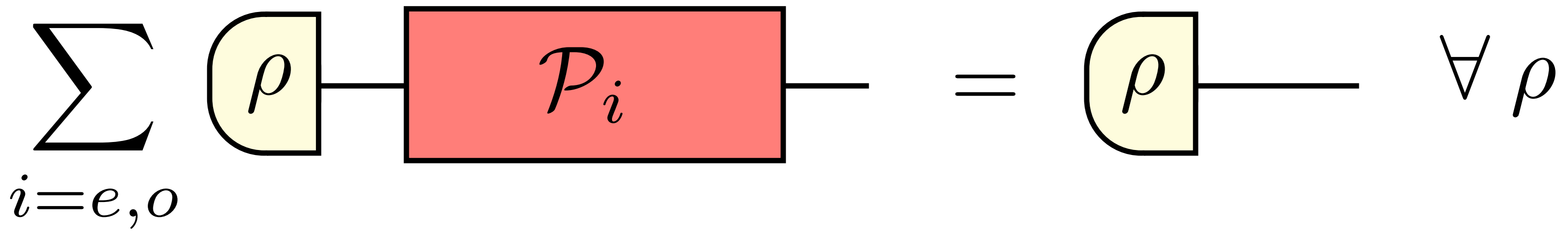
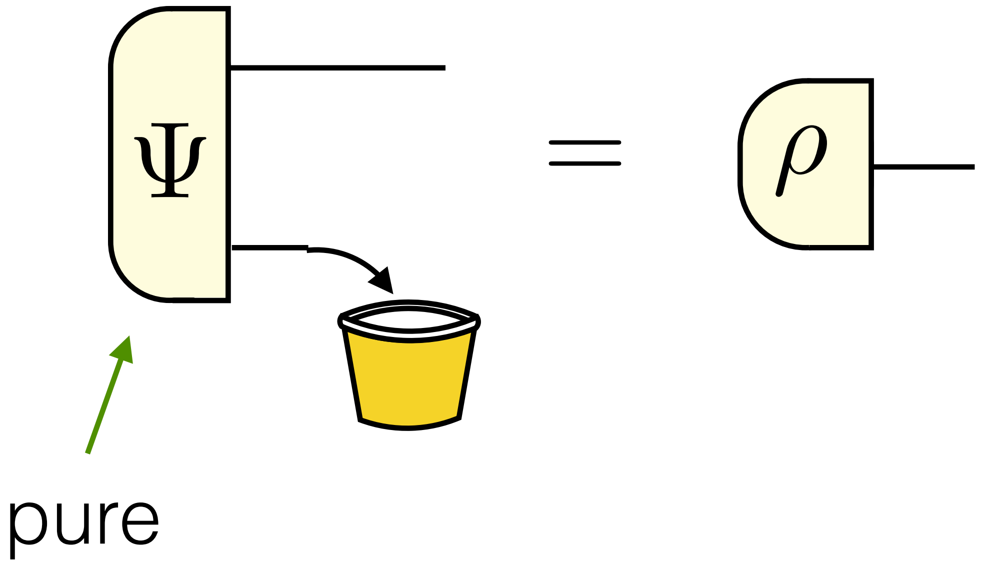
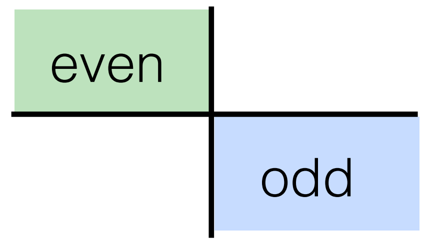


Purification of ρ

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

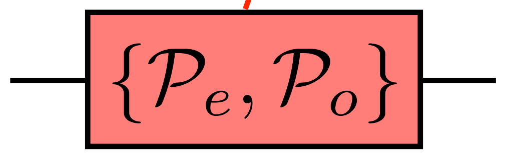




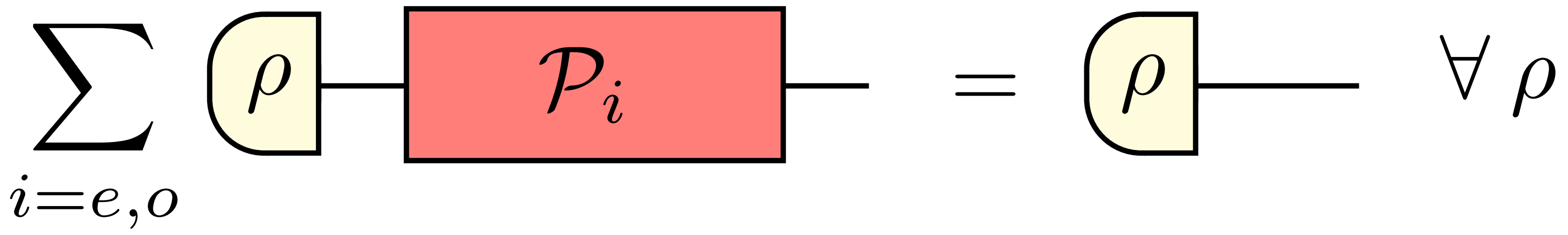
mixed

$$\rho = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

Parity test



Parity test does not disturb locally



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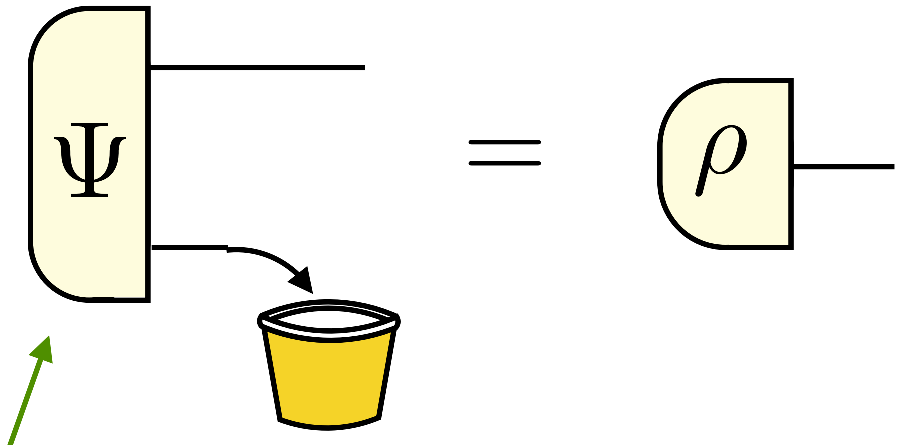
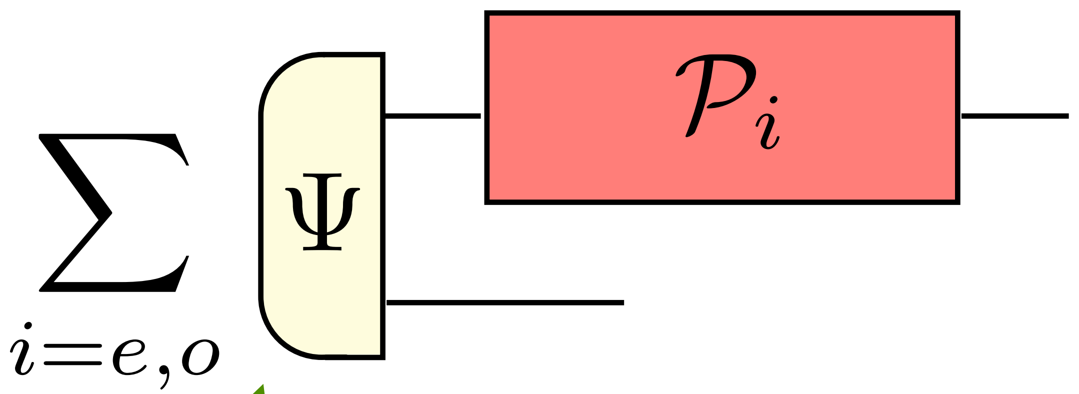


pure

Parity test disturbs purifications



$$= \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

pure

mixed

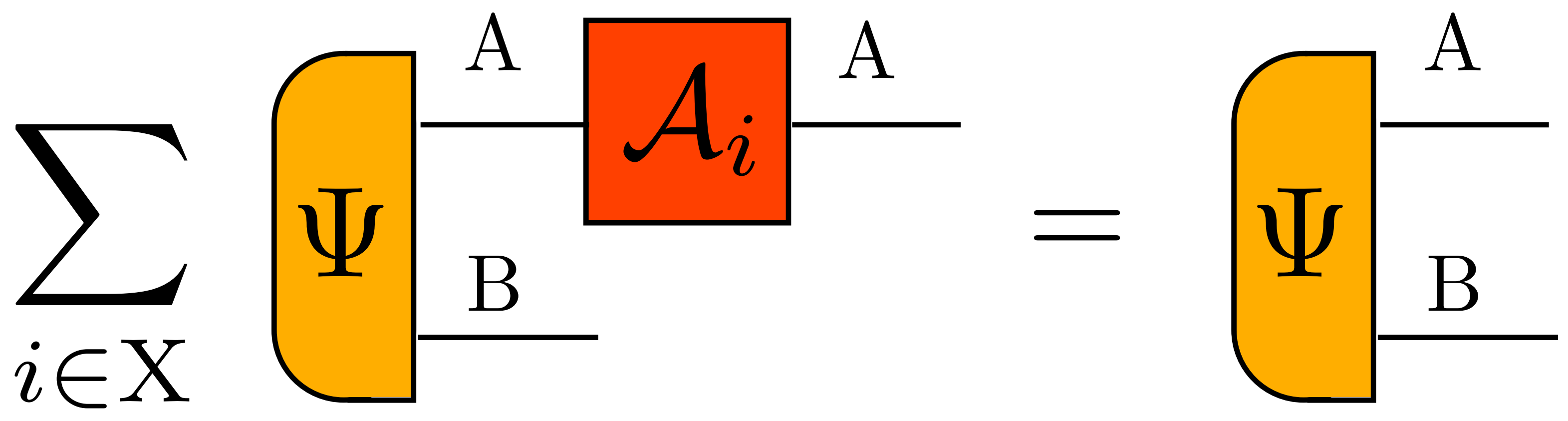
NON-DISTURBING TESTS

- The usual definition is inadequate in the absence of local discriminability

NON-DISTURBING TESTS

- The usual definition is inadequate in the absence of local discriminability
- Definition (non-disturbing test): $\{\mathcal{A}_i\}_{i\in \mathrm{X}}$ is non-disturbing if

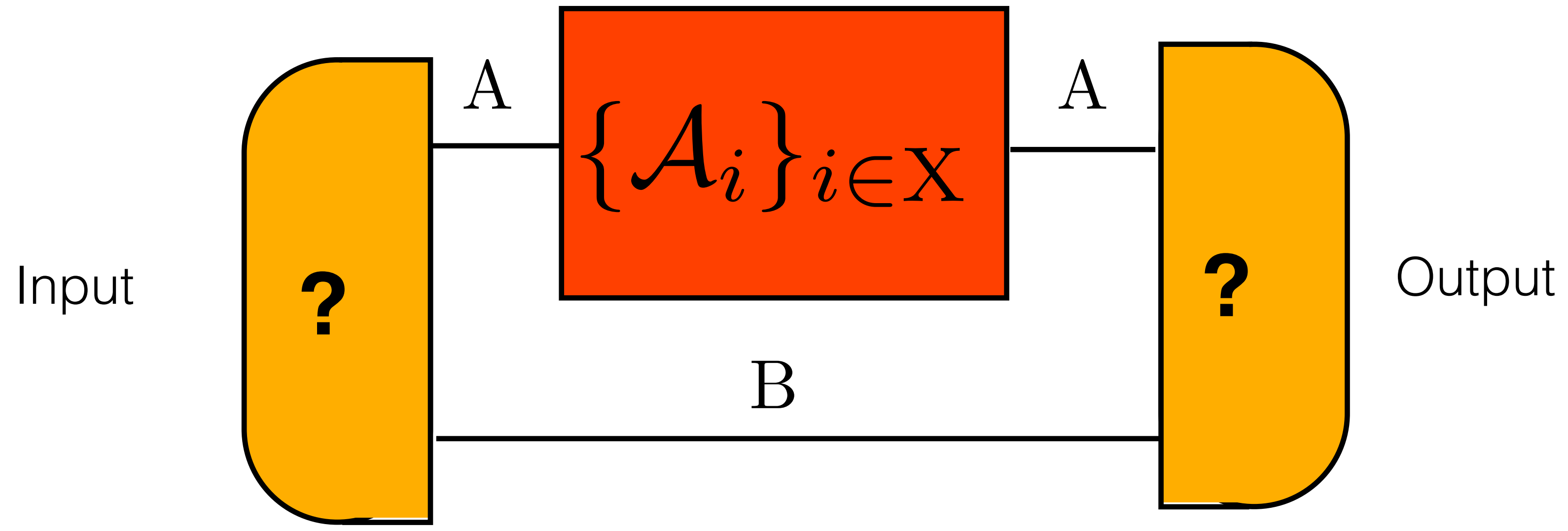
$\forall \mathrm{B}$



$\forall\, \Psi \in \mathrm{St}(\mathrm{AB})$

INFORMATION FROM A TEST

- Consider a test of a theory



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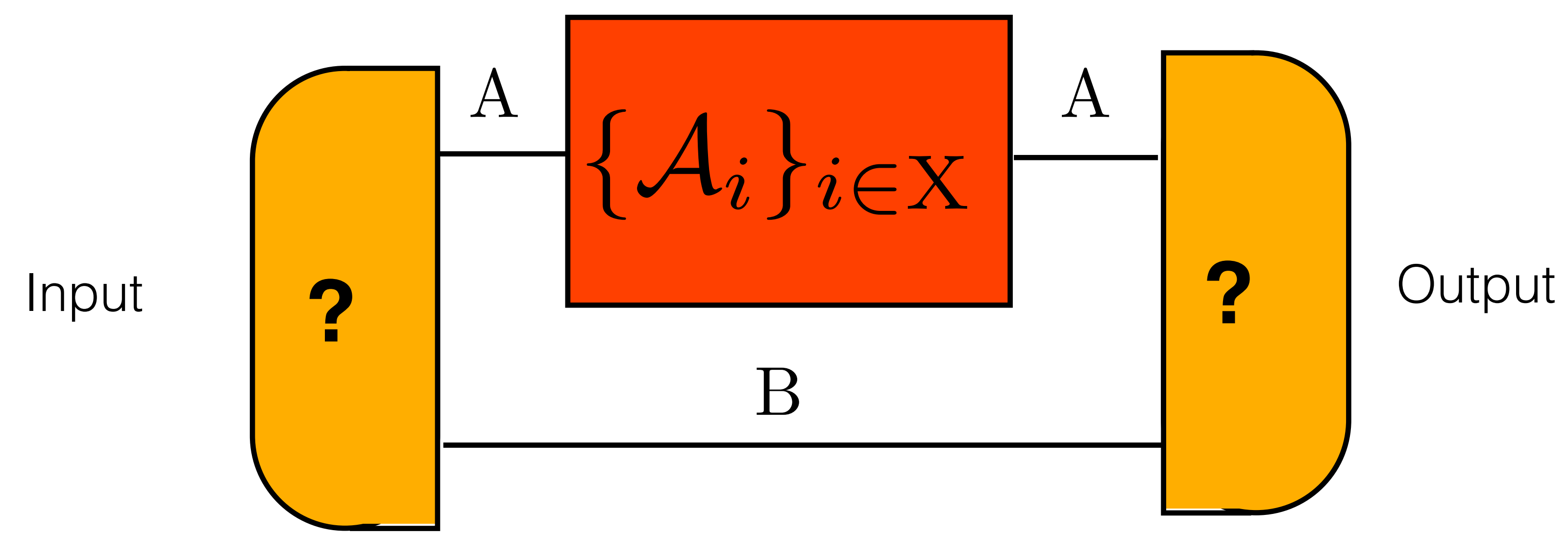

- When does the test provide info on the input?

INFORMATION FROM A TEST

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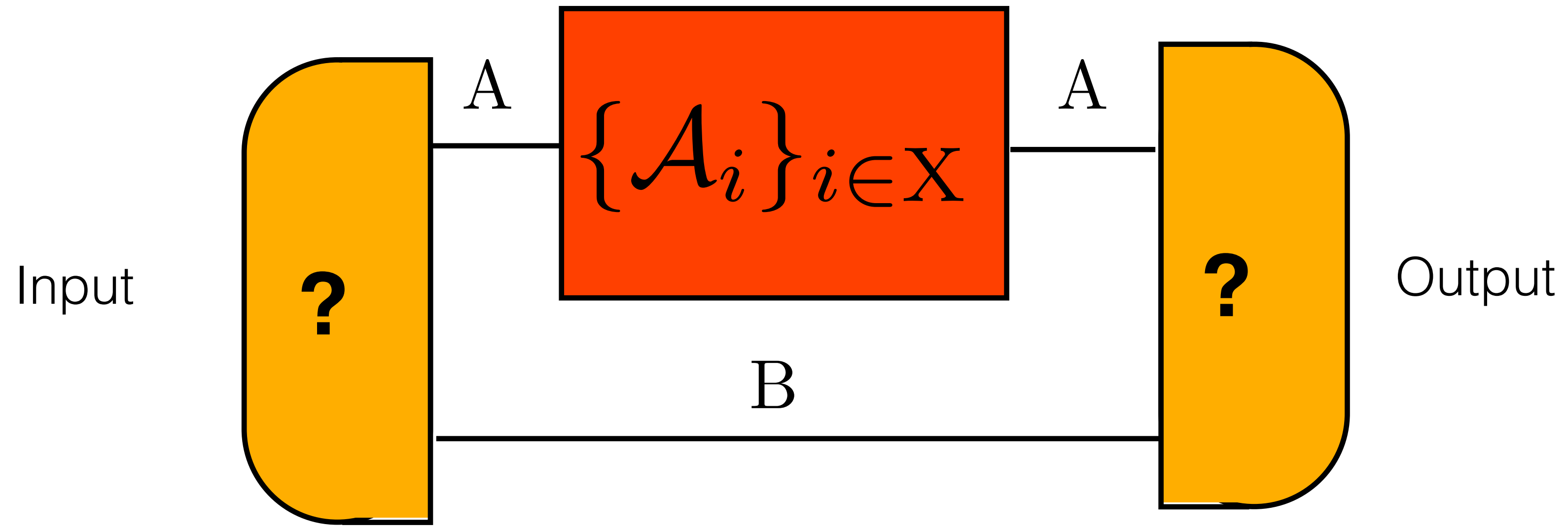

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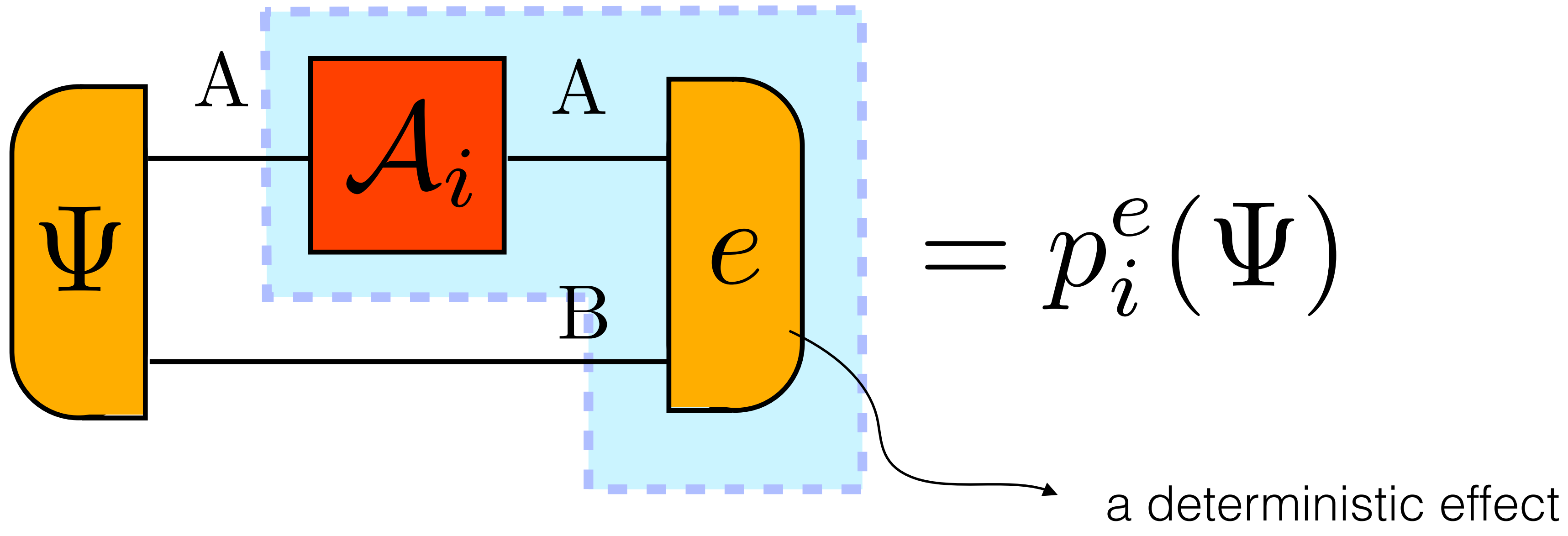

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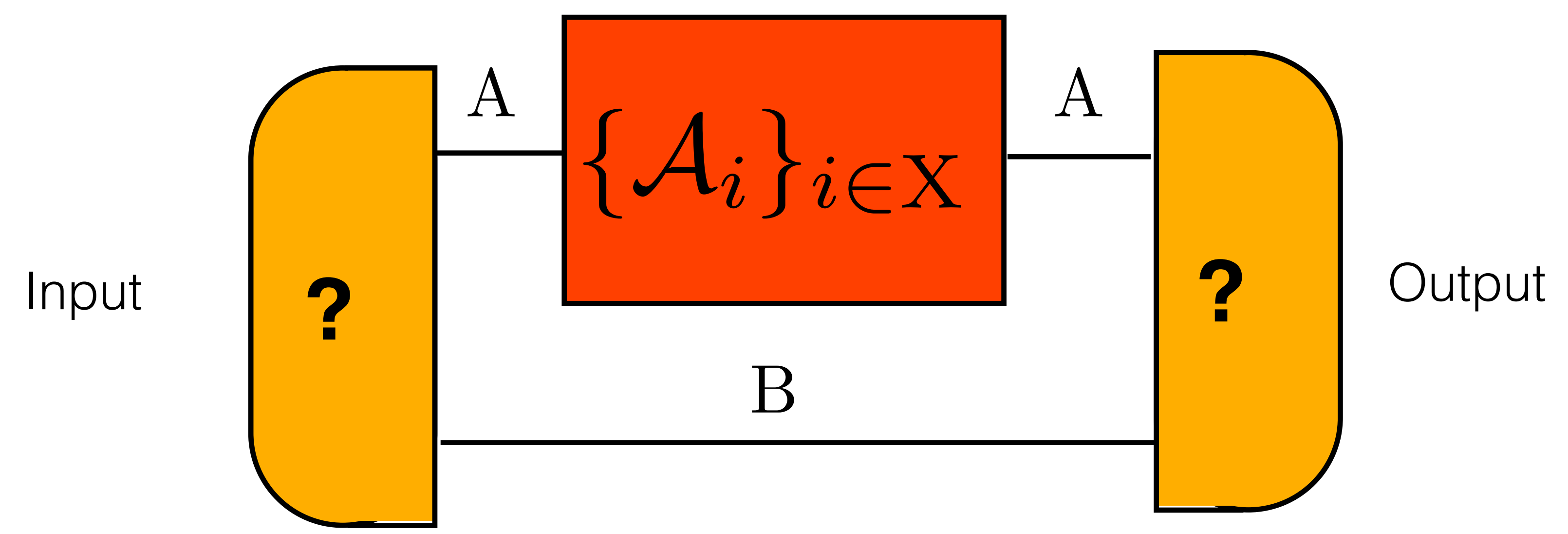

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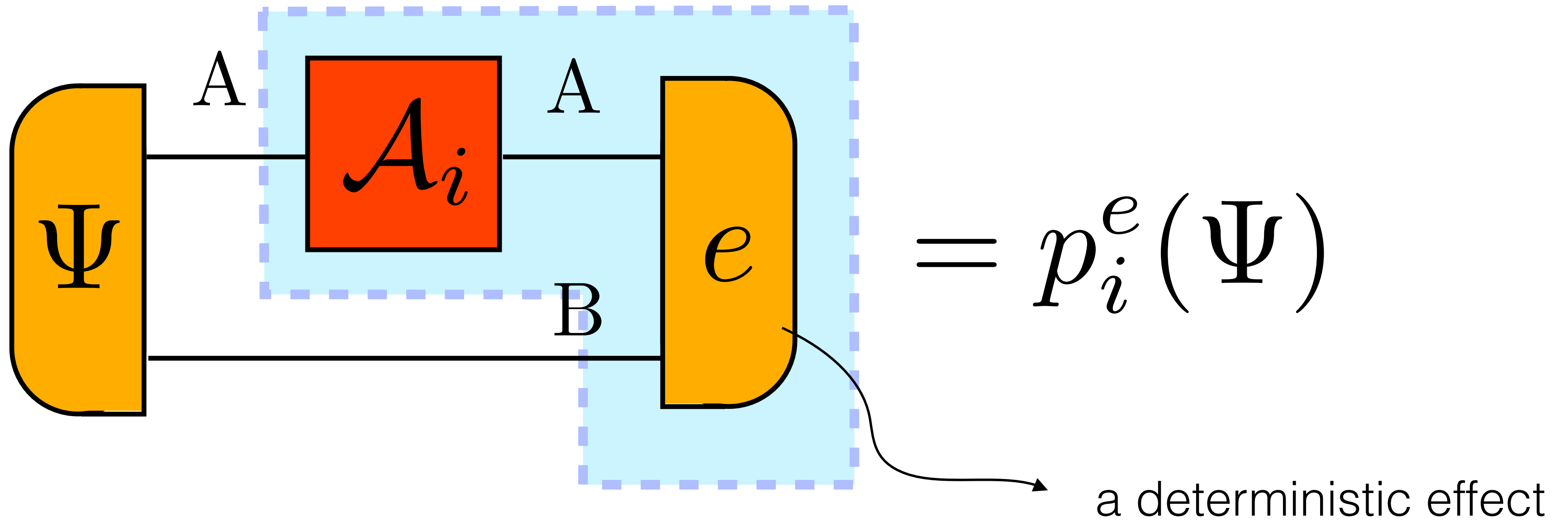
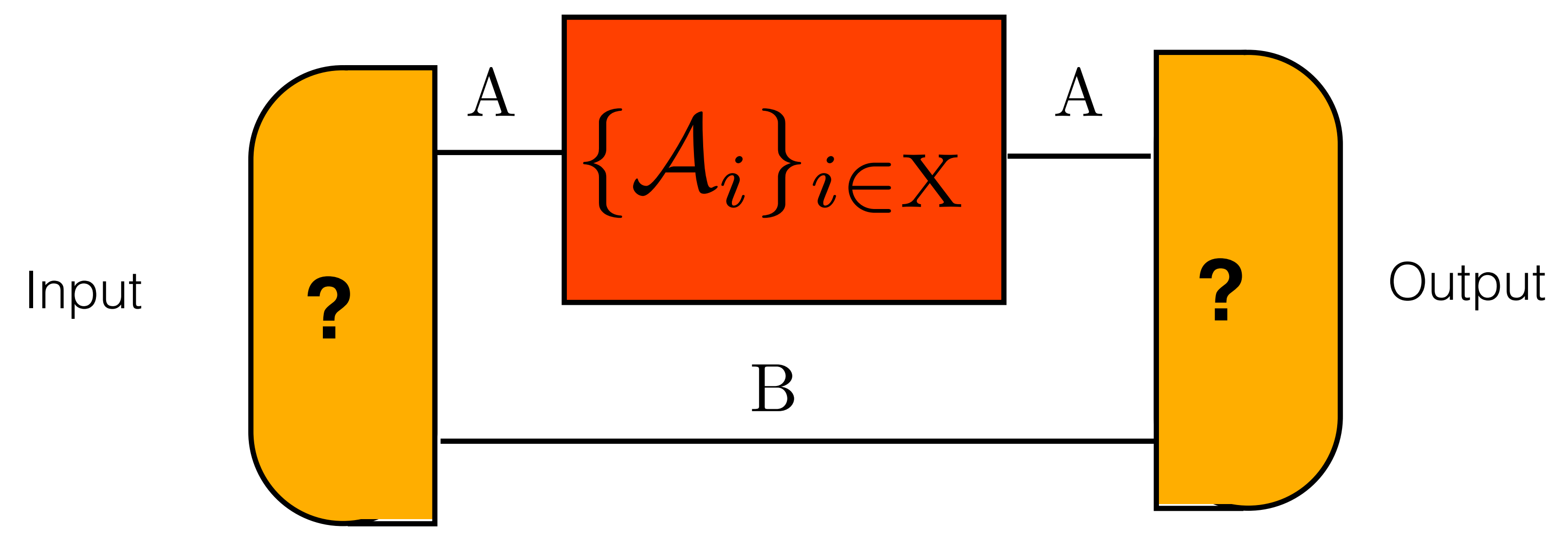


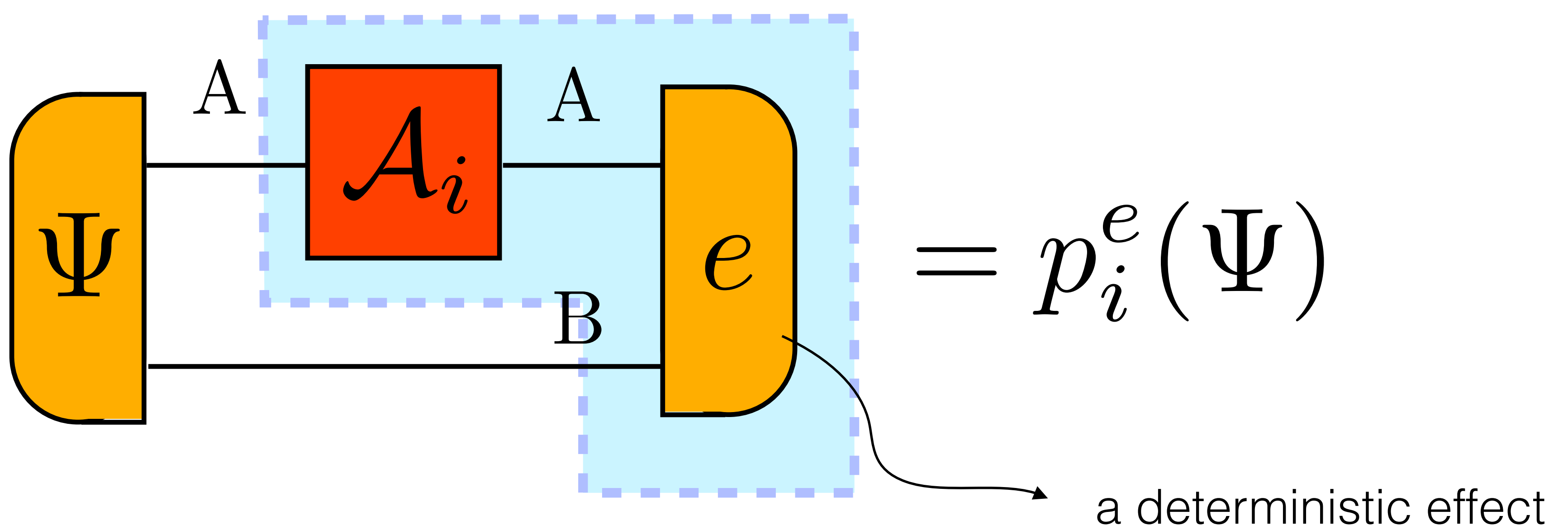
- When does the test provide info on the output?

INFORMATION FROM A TEST

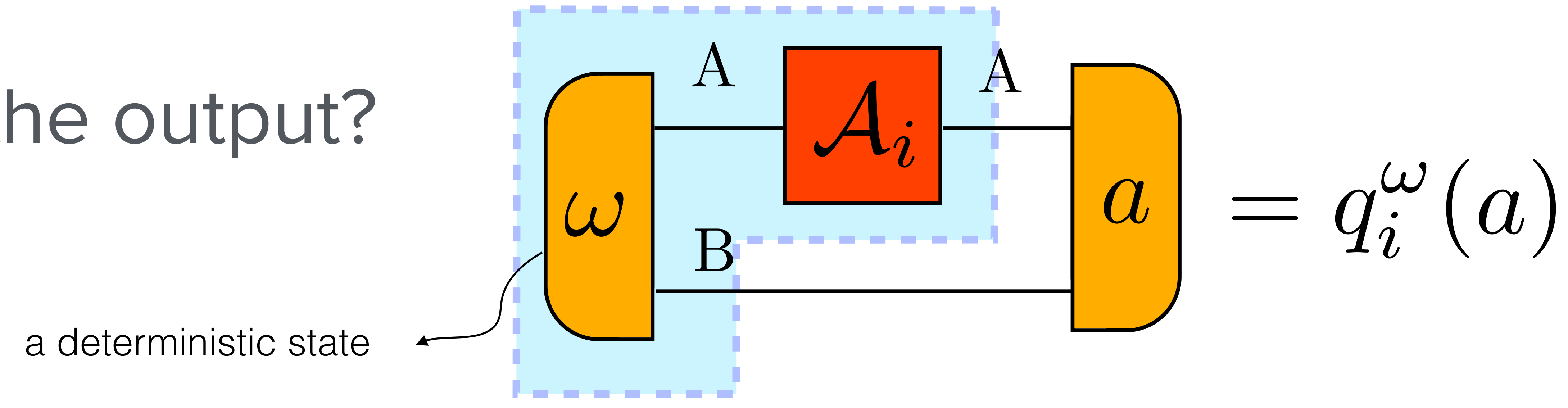
- Consider a test of a theory



- When does the test provide info on the input?



- When does the test provide info on the output?



NO-INFORMATION TEST

Definition: Given the test 

we say that **it does not provide information** if

G. M. D'Ariano, PP, and A. Tosini, Quantum 4, 363 (2020).

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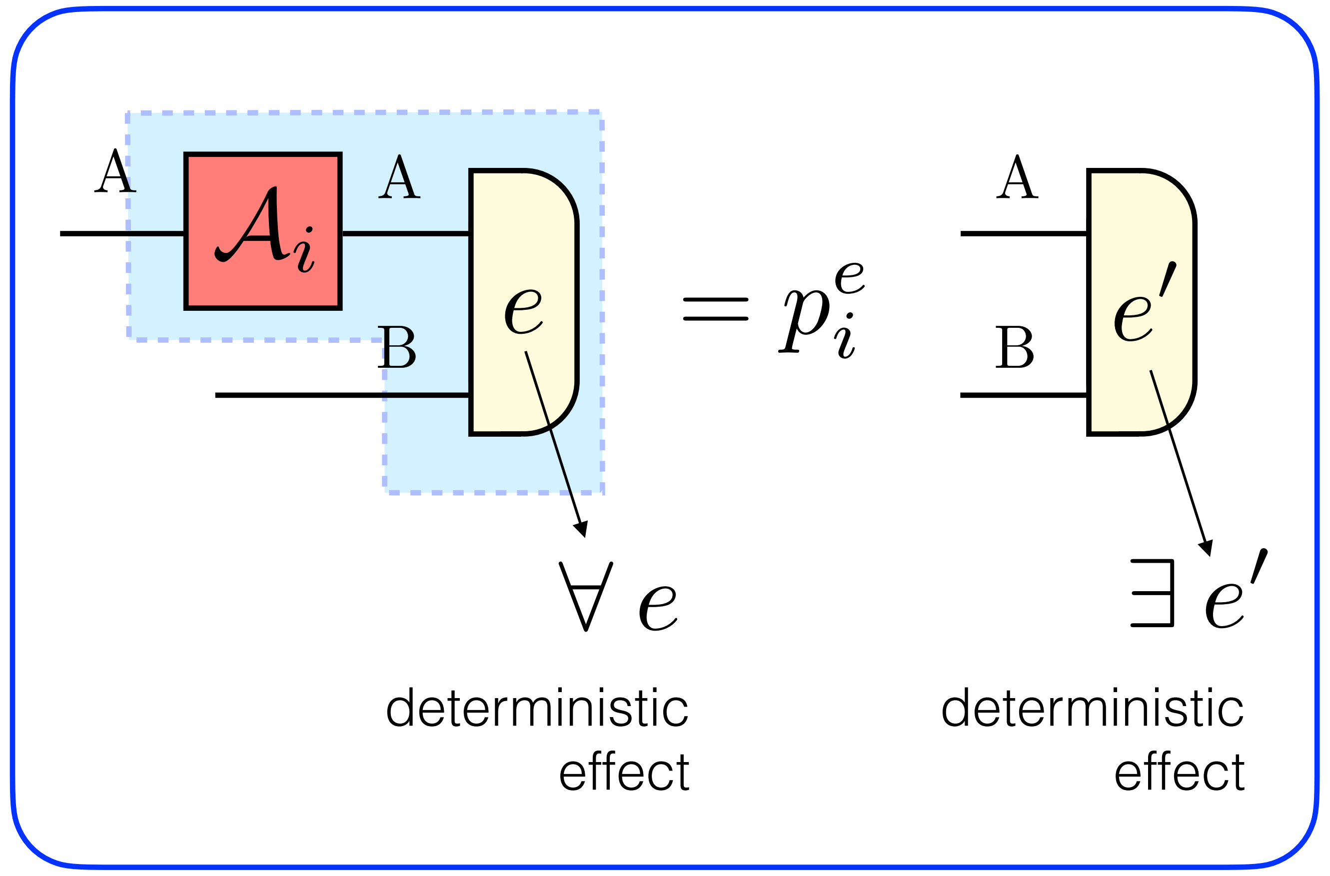

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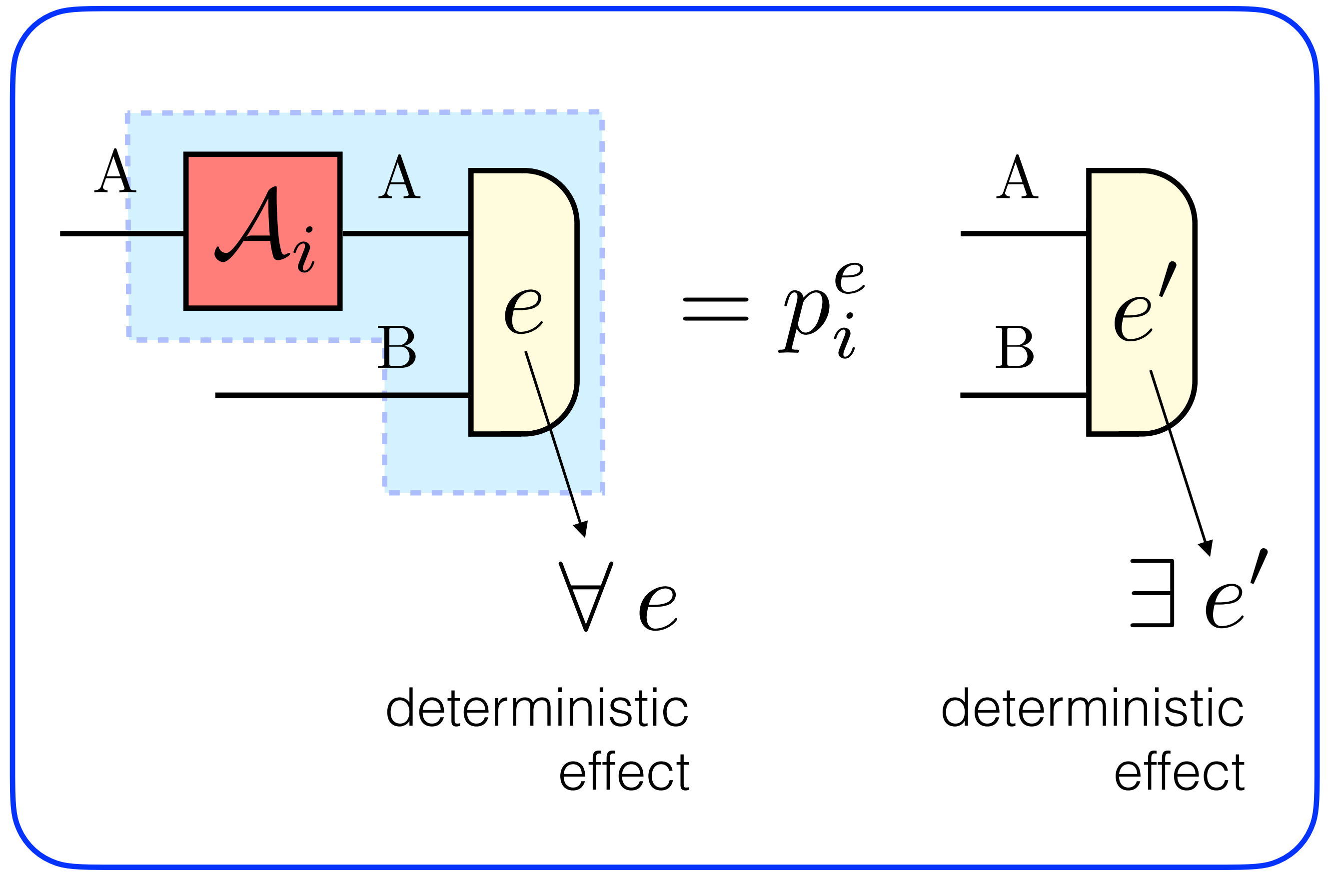
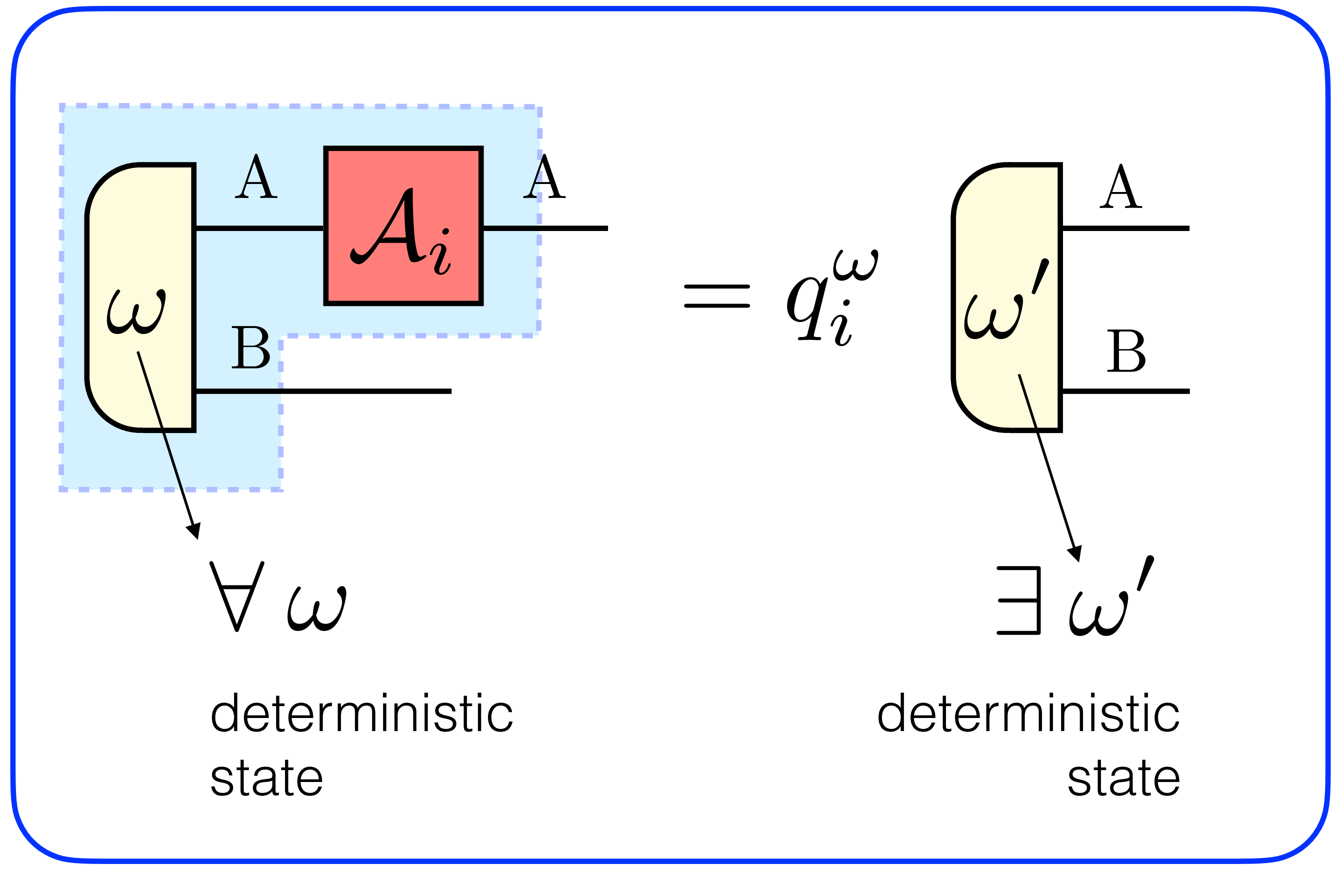


and

no-information on the output



G. M. D'Ariano, PP, and A. Tosini, Quantum 4, 363 (2020).

NO INFORMATION WITHOUT DISTURBANCE

- We say that a theory has no information without disturbance if

$$\{\mathcal{A}_i\}_{i\in\mathrm{X}} \quad \textbf{non-disturbing} \;\Rightarrow\; \{\mathcal{A}_i\}_{i\in\mathrm{X}} \quad \textbf{no-information}$$

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- Theorem: a theory has NIWD iff the identity transformation is atomic for every system

$$\mathcal{I}_{\mathrm{A}} = \sum_i \mathcal{A}_i \Rightarrow \mathcal{A}_i \propto \mathcal{I}_{\mathrm{A}}$$

G. M. D'Ariano, PP, and A. Tosini, Quantum 4, 363 (2020).

OTHER CONDITIONS FOR NIWD

- A theory has NIWD $\Leftrightarrow$ for every system there exists a reversible atomic transformation
- Sufficient condition for NIWD: existence of purification

G. M. D'Ariano, PP, and A. Tosini, Quantum 4, 363 (2020).

INFORMATION WITHOUT DISTURBANCE

- What if the identity map is not atomic?

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INFORMATION WITHOUT DISTURBANCE

- What if the identity map is not atomic?
- Theorem: for every system the atomic decomposition of the identity is “unique”, and

$$\mathcal{I}_{\mathrm{A}} = \sum_i \mathcal{A}_i \quad \Rightarrow \quad \mathcal{A}_i \mathcal{A}_j = \delta_{ij} \mathcal{A}_i$$

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- The information that can be extracted without disturbance is classical information

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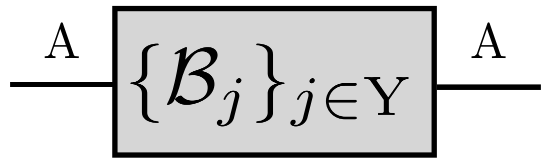
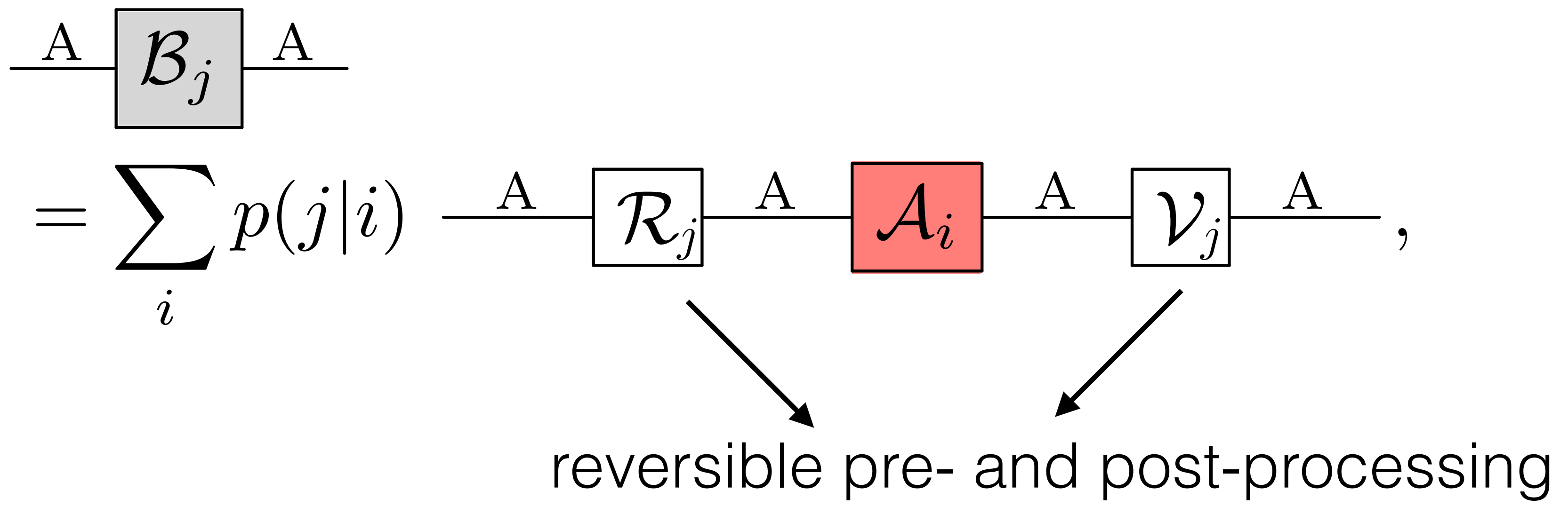
FULL INFORMATION WITHOUT DISTURBANCE

- Definition: a theory satisfies full-information without disturbance if

  for every test 

  there exists a non-disturbing test 

  such that 

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FULL INFORMATION WITHOUT DISTURBANCE

- Definition: a theory satisfies full-information without disturbance if

  for every test $\xrightarrow{\mathrm{A}} \boxed{\{\mathcal{B}_j\}_{j\in\mathrm{Y}}} \xrightarrow{\mathrm{A}}$

  there exists a non-disturbing test $\xrightarrow{\mathrm{A}} \boxed{\{\mathcal{A}_i\}_{i\in\mathrm{X}}} \xrightarrow{\mathrm{A}}$

  such that

$$\xrightarrow{\mathrm{A}} \boxed{\mathcal{B}_j} \xrightarrow{\mathrm{A}} = \sum_i p(j|i) \xrightarrow{\mathrm{A}} \boxed{\mathcal{R}_j} \xrightarrow{\mathrm{A}} \boxed{\mathcal{A}_i} \xrightarrow{\mathrm{A}} \boxed{\mathcal{V}_j} \xrightarrow{\mathrm{A}} ,$$

  reversible pre- and post-processing

- Theorem: if an theory is full-information without disturbance then every system of the theory is classical

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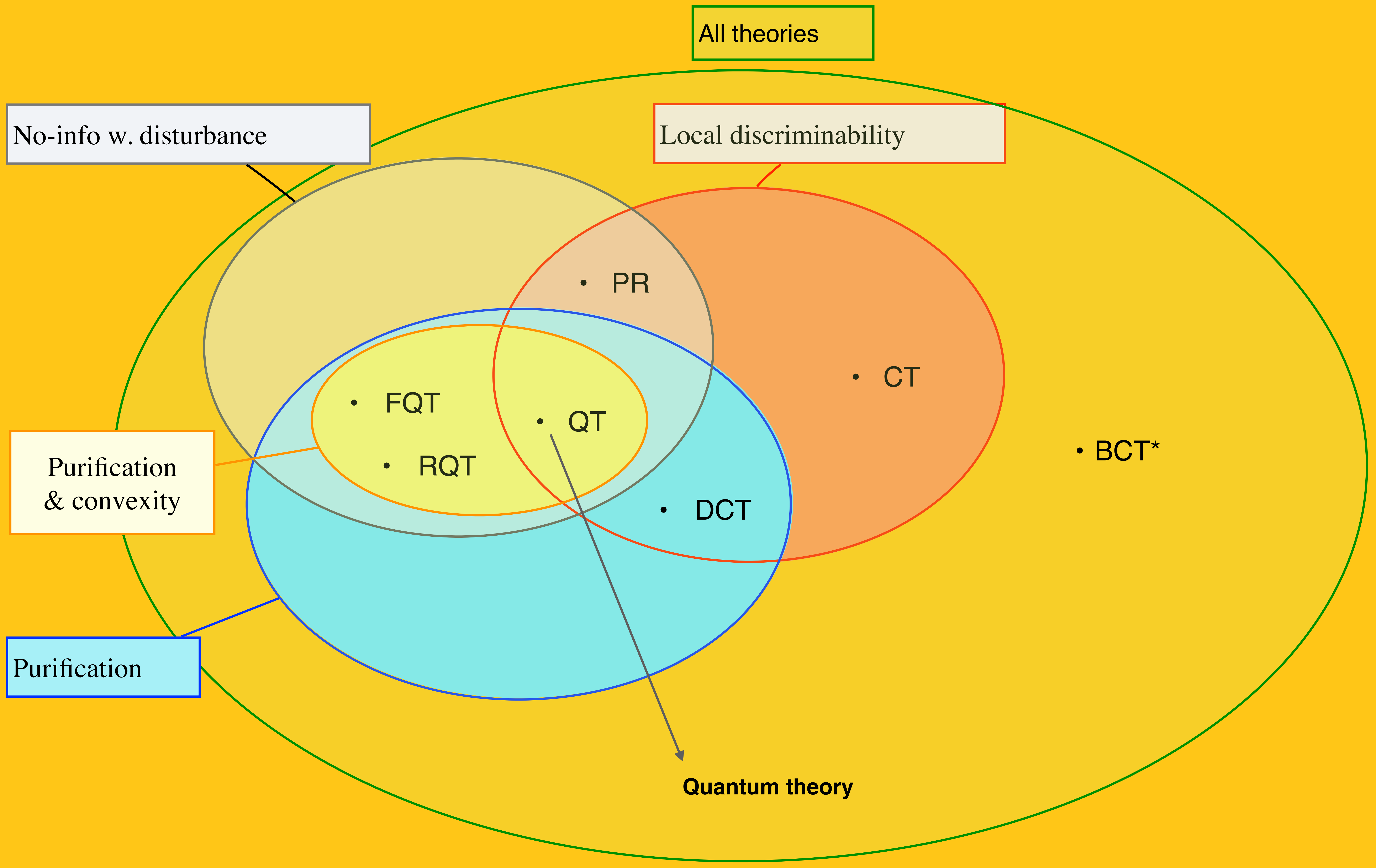

*Giacomo Mauro D'Ariano, Marco Erba, and PP, Phys. Rev. A 102, 052216 (2020)