Quantity-with-Uncertainty Defines the Physics of Quantum Mechanics and Spacetime

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Quantification as Description

Physicists describe things by assigning numbers to them (Quantification)

One number is assigned for each property (Scalar Quantification)

Larger things are assigned larger numbers

Quantities as an Ontological Property

Classically, such quantities are thought of as Ontological **Properties** of an Object

1.5 volt battery

volts

6 volt battery

volts

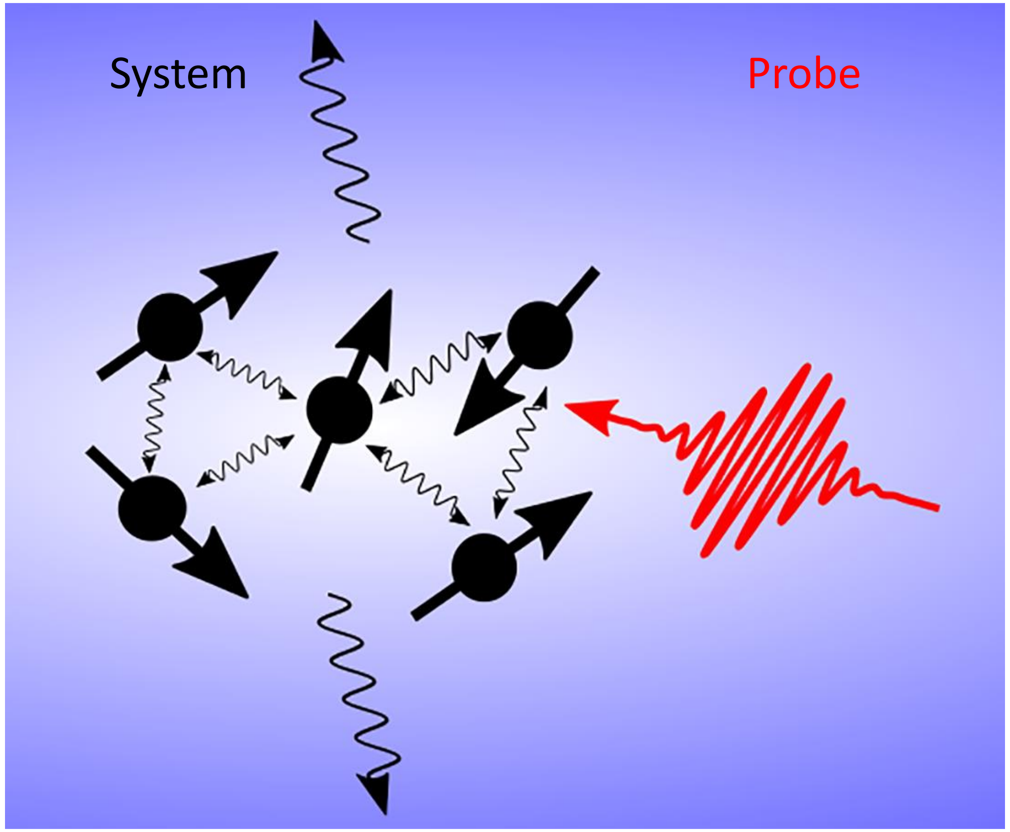
12 volt battery

volts

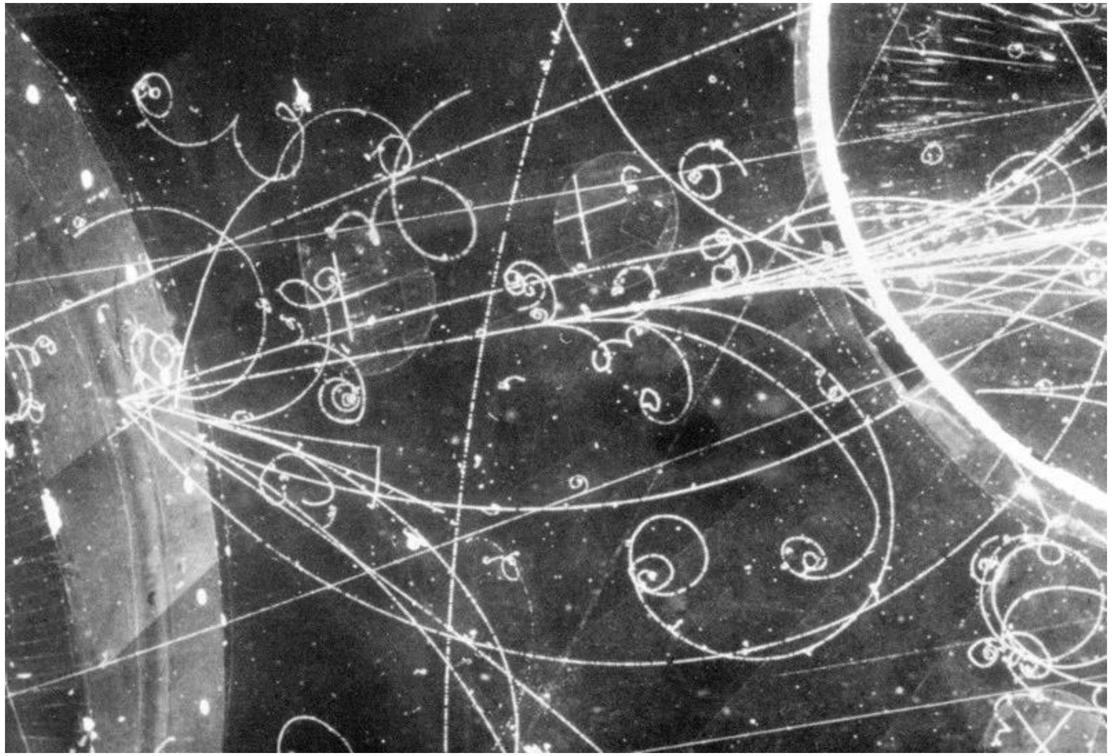
Quantities describing Interactions

For very small Objects, the Probes are equally small

Assigned Quantities describe **Interactions** rather than Properties



Quantum Interactions — Not Properties!



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An electron cannot possess a momentum
An electron cannot possess a position

Such quantities describe the relationship between the electron and the observer

Momentum is one description of the relationship between an electron and an observer. Position is another description.

The fact that the two descriptions are incommensurate is not unusual.

ONLY mysterious if one mistakenly thinks of these quantities as properties!

Quantum Interactions — Not Properties!

https

Contextuality: The Context (Details) of the Interaction Matters

Complementarity: Descriptions need not be commensurate

Quantification
How to Consistently Assign Quantities

Combining Stuff: The -with- Operator

A

-with-

B

Combining Stuff: The -with- Operator

A

-with-

B

||

A-with-B

Combining Stuff: The -with- Operator

Closure
Given stuff **A** and stuff **B**, then combining stuff **A-with-B** is still stuff.

A-with-B

A bunch of Taffy with a bunch of Taffy is still a bunch of Taffy

Combining Stuff: The -with- Operator

Commutativity
Order in which stuff is combined doesn't matter. **A-with-B** is the same as **B-with-A**

A

-with-

B

||

B

-with-

A

Combining Stuff: The –with- Operator

Closure
Given stuff **A** and stuff **B**, then combining stuff **A-with-B** is still stuff.

Commutativity
Order in which stuff is combined doesn't matter. **A-with-B** is the same as **B-with-A**

Associativity
Combination can be done in different but equivalent ways
(A-with-B)-with-C is the same as **A-with-(B-with-C)**

Reproducibility
We should be able to repeat experiments and have the results accumulate
so that **A** is different from **A-with-A** is different from **A-with-A-with-A**, etc.

Scalar Quantification

Does the operator -with- satisfy: ?

Quantify stuff with a scalar

Stuff **A** is represented by **a**
Stuff **B** is represented by **b**

How to represent **A-with-B**?

- ☑ Closure
- ☑ Commutativity
- ☑ Associativity
- ☑ Reproducibility

Associative Commutativity

Scalar Quantification

Does the operator -with- satisfy: ?

Quantify stuff with a scalar

Stuff **A** is represented by **a**
Stuff **B** is represented by **b**

How to represent **A-with-B**?

- ☑ Closure
- ☑ Commutativity
- ☑ Associativity
- ☑ Reproducibility

Associative-Commutativity implies:

Sum Rule

A-with-B	is represented by (up to isomorphism)	**a + b**

theorem

Combining Stuff: The -with- Operator

A

-with-

B

=

A-with-B

$$\boxed{3 + 10 = 13}$$

Replication: The -of- Operator

A

3 -of-

=

3 -of- A

Replication

Stuff can be replicated (multiple combinations)

Replication: The -of- Operator

Replication is subject to Right- and Left-Distributivity

$$4 \text{ –of– } (A \text{ –with– } B) = (4 \text{ –of– } A) \text{ –with– } (4 \text{ –of– } B)$$

$$(3 \text{ –with– } 5) \text{ –of– } A = (3 \text{ –of– } A) \text{ –with– } (5 \text{ –of– } A)$$

As well as Associativity

$$3 \text{ –of– } (4 \text{ –of– } 2) \text{ –of– } A = (3 \text{ –of– } 4) \text{ –of– } 2 \text{ –of– } \mathrm{A}$$

Associative-Distributivity implies:

Product Rule

A -of- B is represented by **a b**

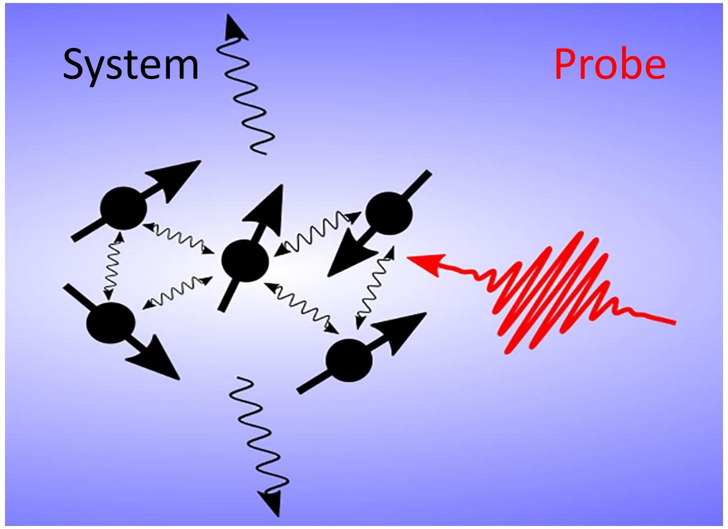
theorem

Quantum and Uncertainty

At some point, there are no smaller probes.
We must acknowledge that we are uncertain as to the value of the assigned quantity x

The relation may be more intimate than

$$x \pm \sigma$$



We assign a number pair $\binom{x_1}{x_2}$
and work to ***derive***
the appropriate relationship.

Associative Commutativity

Representing A with $\binom{x_1}{x_2}$

and

Representing B with $\binom{y_1}{y_2}$

Associative Commutativity implies that

A -with- B is represented (up to isomorphism) by

component-wise summation

$$\begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix} = \binom{x_1}{x_2} + \binom{y_1}{y_2}$$

Associative Distributivity

Upon interaction, Associative Distributivity gives

$$a \cdot (b+c) = a \cdot b + a \cdot c$$
$$(a+b) \cdot c = a \cdot c + b \cdot c$$

and

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

because summation has to remain linear regardless of probing.

The multiplication must be bilinear

$$\boxed{(\mathrm{x} \cdot \mathrm{y})_i = \sum_{j,k=1}^{2} \gamma_{ijk}\, x_j\, y_k \quad \text{or} \quad (\mathrm{x}\ \cdot)_{ik} = \sum_{j=1}^{2} \gamma_{ijk}\, x_j}$$

These algebraic symmetries are why so much of physics is fundamentally linear!

Three Multiplication Rules: A, B, C

Whereas scalar quantification results in one multiplication rule.
Pairwise quantification results in THREE possible multiplication rules!

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \underset{\text{A}}{\begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}} \text{ OR } \underset{\text{B}}{\begin{pmatrix} x_1 y_1 + x_2 y_2 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}} \text{ OR } \underset{\text{C}}{\begin{pmatrix} x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}}$$

or

$$(\mathrm{x}\ \cdot) = \underset{\text{A}}{\begin{pmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{pmatrix}} \text{ OR } \underset{\text{B}}{\begin{pmatrix} x_1 & x_2 \\ x_2 & x_1 \end{pmatrix}} \text{ OR } \underset{\text{C}}{\begin{pmatrix} x_1 & 0 \\ x_2 & x_1 \end{pmatrix}}$$

Three Multiplication Rules: A, B, C

Without loss of generality, we set

$$|\det(\mathrm{x}\,\cdot)| = 1$$

So that repeated application of $(\mathrm{x}\,\cdot)$ does not diverge to ∞ or collapse to 0

There is now only one free parameter ϕ related to ${}^{x_2}/_{x_1}$, which gives

$$(\mathrm{x}\ \cdot)\ =\ \underset{\text{A}}{\begin{pmatrix}\cos\phi & -\sin\phi \\ \sin\phi & \cos\phi\end{pmatrix}}\ \text{OR}\ \underset{\text{B}}{\begin{pmatrix}\cosh\phi & \sinh\phi \\ \sinh\phi & \cosh\phi\end{pmatrix}}\ \text{OR}\ \underset{\text{C}}{\begin{pmatrix}1 & 0 \\ \phi & 1\end{pmatrix}}$$

The Generators: A, B, C

The operator $(\mathrm{x}\ \cdot)$ can be written in terms of infinitesimal generators by

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

with

$$(\mathrm{x}\ \cdot) = \lim_{n \to \infty} \left(1 + \frac{\phi}{n} G\right)^n = \exp(\phi\, G)$$

where the Generator G is the matrix A or B or C.

Generator A

The first generator A is rotation by a phase angle ϕ

The sum and product rules in this case are those of complex arithmetic and the pairs are complex numbers:

$$\binom{x_1}{x_2} = r\, e^{i\phi}$$

Unit quantity is identified with unit modulus determinant, which is modulus-squared

$$|\mathrm{det}(\mathrm{x}\,\cdot)| = |\mathrm{x}|^2 = 1$$

The **inherent uncertainty** in a unit object refers to what remains undefined in x, namely phase , so that each new object brings with it an unknown phase!

Probability

We now must rely on probability theory to treat this inherent uncertainty properly.

Our ignorance of phase is uniformly distributed

$$\Pr(\phi) = \frac{1}{2\,\pi}, \qquad (0 \le \phi \le 2\pi)$$

To obtain the probability (likelihood) of a given outcome,
we must marginalize (integrate) over all of the unknown parameters (phases).

$$\langle |x_1 + x_2 + \cdots + x_n|^2 \rangle_{\phi_1,\phi_2,\cdots,\phi_n} = |x_1|^2 + |x_2|^2 + \cdots + |x_n|^2$$

This is why QM is a probabilistic theory.
The likelihood is additive because of the scalar sum rule, and it depends on the modulus-squared, which is the Born Rule!

Quantum Theory

This foundation of quantum theory is elementary and simple.

Quantity and uncertainty fuse together into complex numbers with uncertainty referring to phase.

Modulus-squared is the observable quantity (the Born rule for arbitrary amount) representing ensemble averages.

Quantum Mechanics and Bayes

Both Quantum Mechanics and Probability Theory are derived from the same symmetries. There can be no contradiction!

Physics makes predictions quantified probabilistically in terms of likelihoods

Quantum formalism is part of physics: it predicts the likely (probabilistic likelihoods) behavior of specified models of physical situations.

Given those likelihoods, Bayesian analysis then computes posterior probabilities, which assess the models in the light of outcomes as actually observed.

THIS is the relationship between Quantum Mechanics and Bayesian Inference.

Qubits

We now consider objects that can exist in two states ↑ and ↓

This gives us a quantification consisting of a pair of complex numbers

$$\psi = \begin{pmatrix} \psi_\uparrow \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \psi_\downarrow \end{pmatrix} = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \begin{pmatrix} \psi_0 + i\psi_1 \\ \psi_2 + i\psi_3 \end{pmatrix}$$

Qubits obey associative distributivity, so the representation in two-dimensional over the complex field.

Our generators now give us the Pauli matrices

$$\sigma_x = \underset{B}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} \quad \sigma_y = \underset{iA}{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}} \quad \sigma_z = \underset{BA}{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

Qubits

The Pauli matrices

$$\boldsymbol{\sigma_x} = \underset{B}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} \quad \boldsymbol{\sigma_y} = \underset{iA}{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}} \quad \boldsymbol{\sigma_z} = \underset{BA}{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

give us the scalar quantification

$$q_x = \psi^\dagger \boldsymbol{\sigma_x}\, \psi = 2\, \mathrm{Re}(\psi_\uparrow^* \, \psi_\downarrow)$$
$$q_y = \psi^\dagger \boldsymbol{\sigma_y}\, \psi = 2\, \mathrm{Im}(\psi_\uparrow^* \, \psi_\downarrow)$$
$$q_z = \psi^\dagger \boldsymbol{\sigma_z}\, \psi = |\psi_\uparrow|^2 - |\psi_\downarrow|^2$$

with

$$q_0 = \psi^\dagger \mathbf{1}\, \psi = \psi_0^2 + \psi_1^2 + \psi_2^2 + \psi_3^2 = |\psi_\uparrow|^2 + |\psi_\downarrow|^2$$

$$q_x^2 + q_y^2 + q_z^2 = q_0^2$$

Qubits

The Pauli matrices

$$\boldsymbol{\sigma_x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \boldsymbol{\sigma_y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \boldsymbol{\sigma_z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$B \qquad\qquad\qquad iA \qquad\qquad\qquad BA$$

give us the scalar quantification

$$q_x = \psi^\dagger \boldsymbol{\sigma_x}\, \psi = 2\,\mathrm{Re}(\psi_\uparrow^*\, \psi_\downarrow)$$
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with

$$q_0 = \psi^\dagger \mathbf{1}\, \psi = \psi_0^2 + \psi_1^2 + \psi_2^2 + \psi_3^2 = |\psi_\uparrow|^2 + |\psi_\downarrow|^2$$

$$q_x^2 + q_y^2 + q_z^2 = q_0^2$$

First clue that 3+1 spacetime emerges from QM !!!

Qubits

With complex coefficients, these generators define the 6-parameter Lorentz group

$$\psi' = \exp\left(\phi_x\, \boldsymbol{\sigma_x} + \phi_y\, \boldsymbol{\sigma_y} + \phi_z\, \boldsymbol{\sigma_z}\right)\psi$$

under which ensemble averages of independent samples result in

$$\langle q_0 \rangle^2 - \langle q_x \rangle^2 - \left\langle q_y \right\rangle^2 - \langle q_z \rangle^2$$

which is invariant.

Conclusion

The symmetries of associative-commutativity and associative-distributivity
along with consideration of uncertainty,

constrain our mathematical descriptions of the universe to a component-wise sum rule and three product rules, which give us:

Quantum Mechanics as a probabilistic theory
Spin and Pauli Matrices
Energy and Momentum
Phase and Action
3+1 dimensional Spacetime

These symmetries constrain our mathematical description of physics, which is why it works!

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The arithmetic of uncertainty unifies quantum formalism and relativistic spacetime

John Skilling, Kevin H. Knuth

The theories of quantum mechanics and relativity dramatically altered our understanding of the universe ushering in the era of modern physics. Quantum theory deals with objects probabilistically at small scales, whereas relativity deals classically with motion in space and time.
We show here that the mathematical structures of quantum theory and of relativity follow together from pure thought, defined and uniquely constrained by the same elementary "combining and sequencing" symmetries that underlie standard arithmetic and probability. The key is uncertainty, which inevitably accompanies observation of quantity and imposes the use of pairs of numbers. The symmetries then lead directly to the use of complex "$\sqrt{-1}$" arithmetic, the standard calculus of quantum mechanics, and the Lorentz transformations of relativistic spacetime. One dimension of time and three dimensions of space are thus derived as the profound and inevitable framework of physics.

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