Measuring locality vs free choice

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joint work with Emmanuel Pothos, James Yearsley, Christoph Gallus and Ewa Borsuk

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Violations of locality and free choice are equivalent resources in Bell experiments

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Goals for today

- *Which is **more costly** **locality** or **free choice**?*
- ***Calculate** and **compare** measures of both in **QM** and **not only**.*

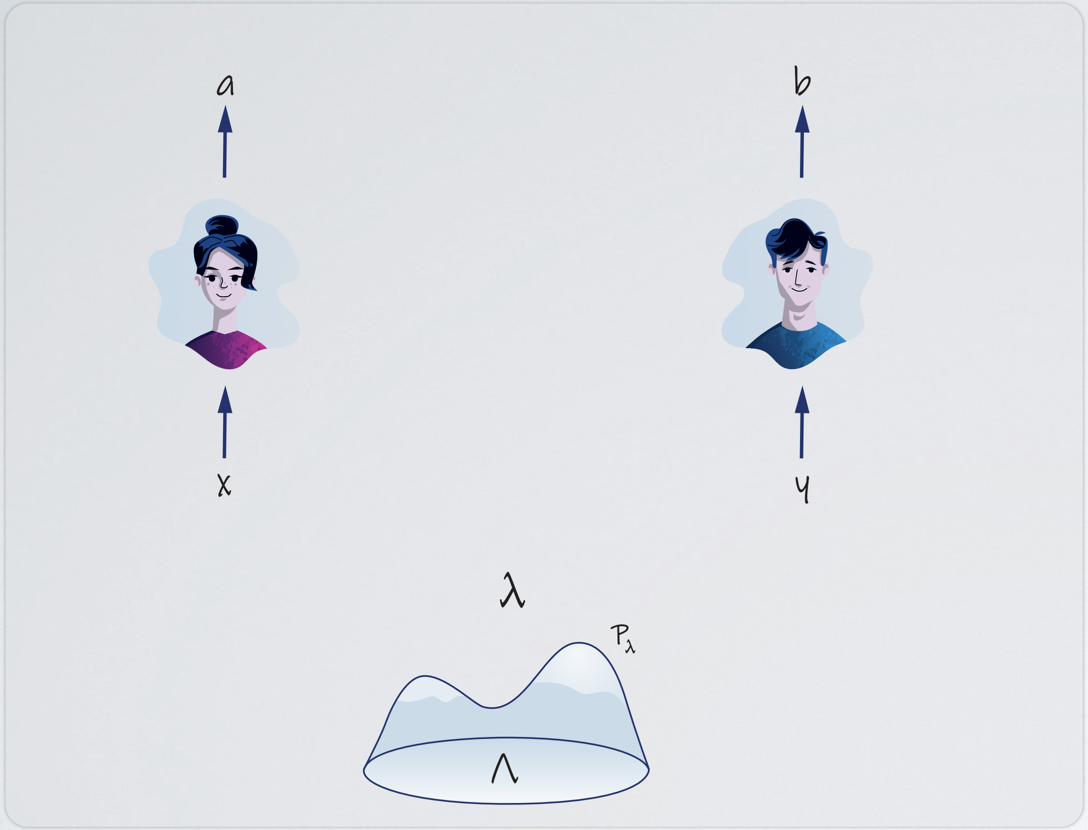
Bell experiment — recap (I)

After many trials Alice and Bob collect experimental statistics for ***outcomes*** *a,b under different* ***choices of settings*** *x,y.*

- *Observed* $\{P_{ab|xy}\}_{xy}$ *Experimental behaviour*

  P_{xy} *Distribution of settings*

Bell experiment — recap (I)



- *Observed* $\{P_{ab|xy}\}_{xy}$ *Experimental behaviour*

  P_{xy} *Distribution of settings*

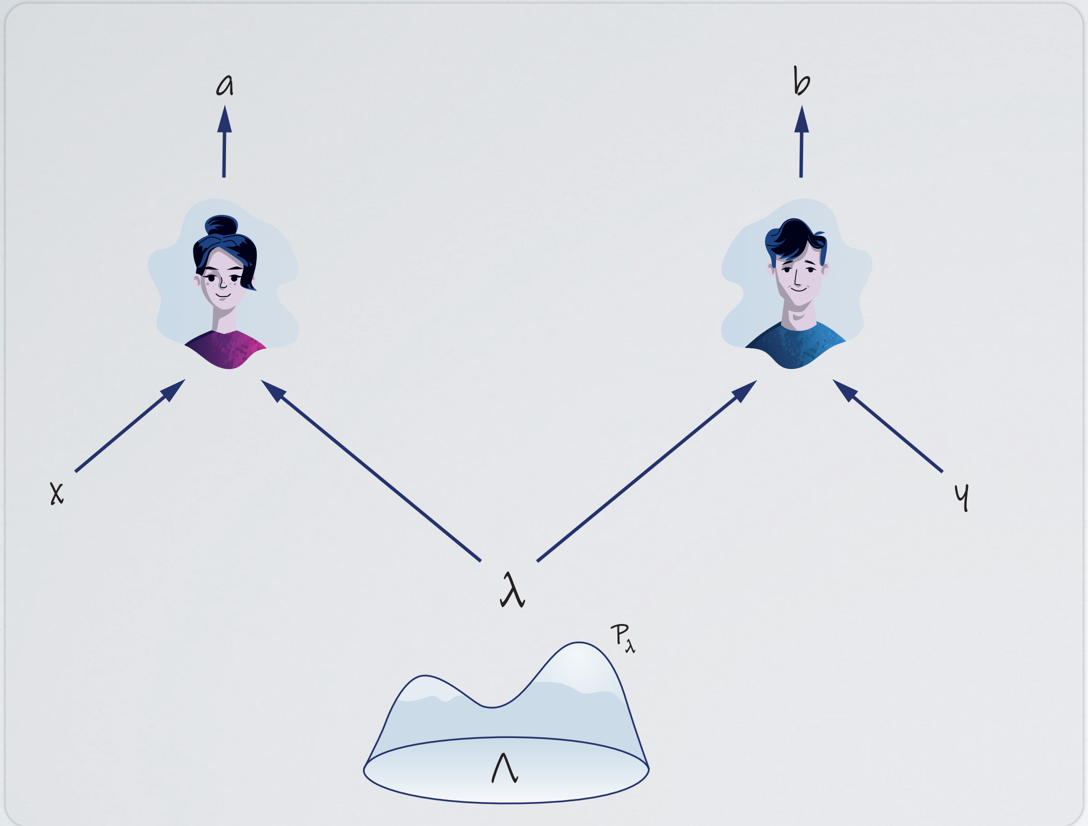
- *Realist (causal) framework*

$$P_{ab|xy} = \sum_{\lambda \in \Lambda} P_{ab|xy\lambda} \cdot P_{\lambda|xy}$$

$$P_{xy} = \sum_{\lambda \in \Lambda} P_{xy|\lambda} \cdot P_{\lambda}$$

Hidden variable model

Bell experiment — recap (I)



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Hidden variable model

- ***Locality*** $P_{ab|xy\lambda} = P_{a|x\lambda} \cdot P_{b|y\lambda}$

- ***Free choice*** $P_{\lambda|xy} = P_{\lambda}$ *(or equiv.* $P_{xy|\lambda} = P_{xy}$ *)*

- *Arrow of time (no retro-causality)*

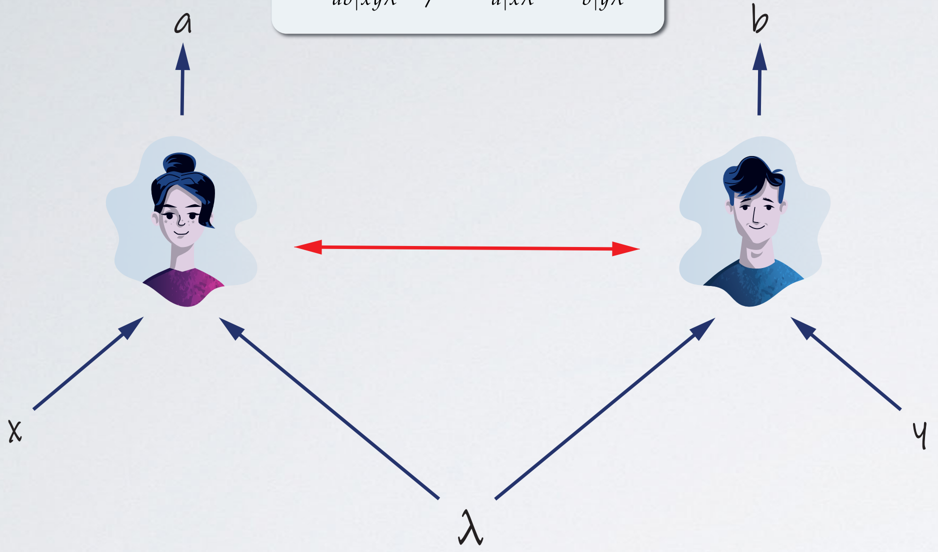
Bell's theorem

Those assumptions entail ***testable constraints*** *on correlations called Bell inequalities (violated in QM).*

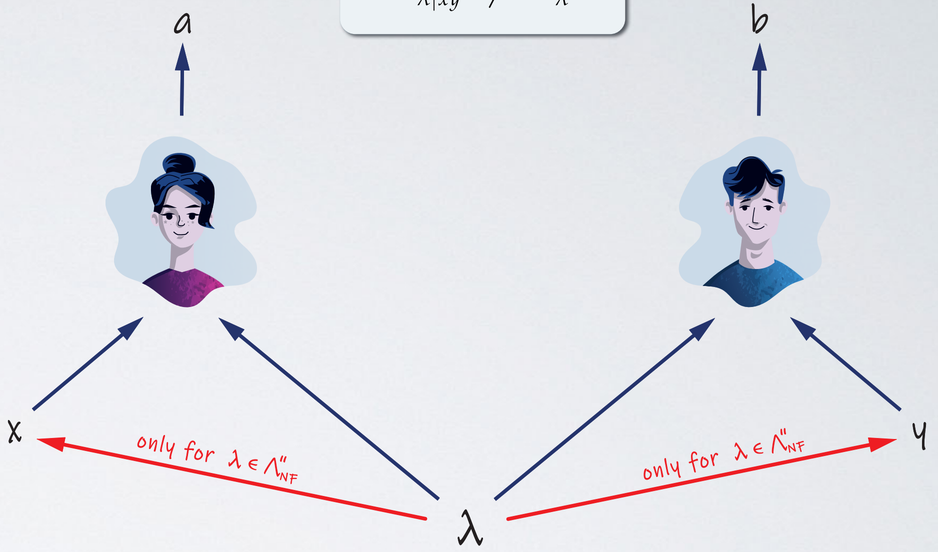
To be continued …

Violation of Locality vs. *Violation of Free choice*

$$P_{ab|xy\lambda} \neq P_{a|x\lambda} \cdot P_{b|y\lambda}$$



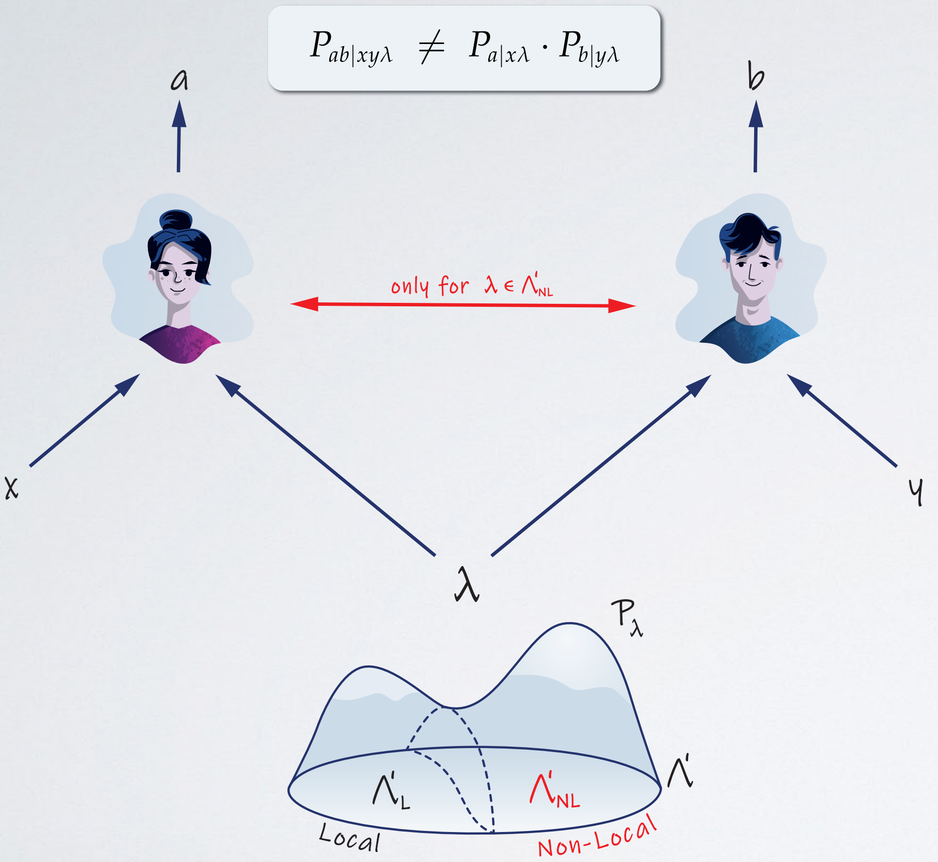
$$P_{\lambda|xy} \neq P_{\lambda}$$

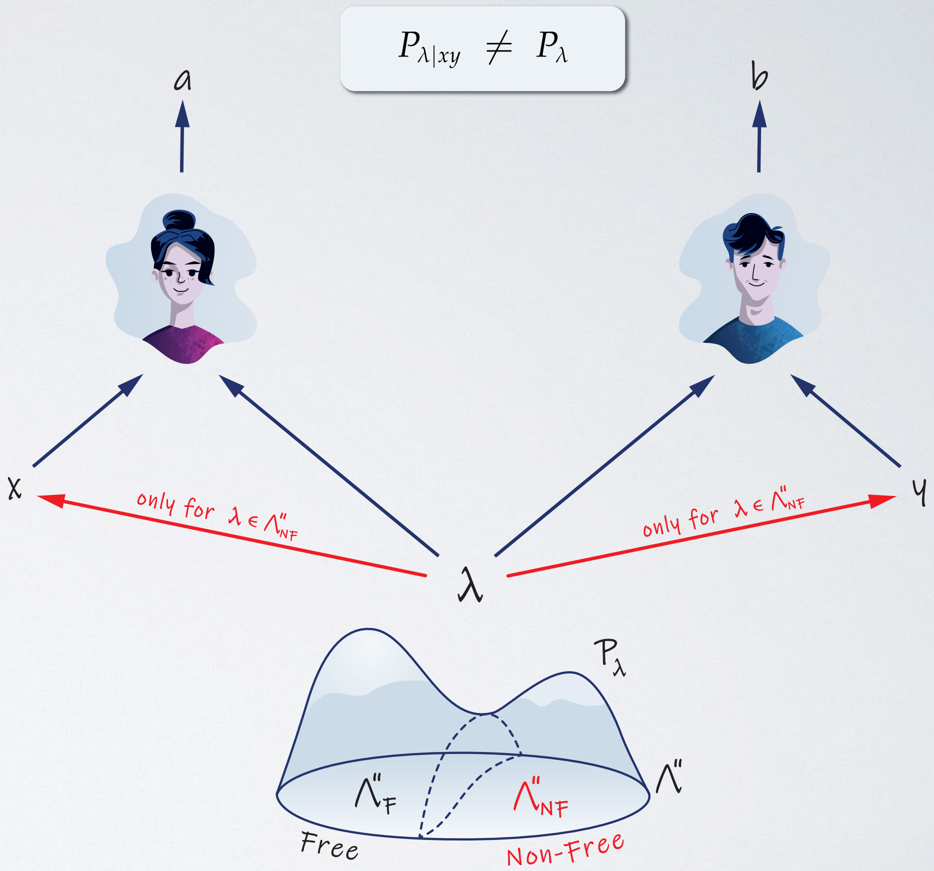


Heuristic idea

How often *a given assumption, i.e.* ***locality*** *or* ***free choice****, can be* ***retained****, while* ***safeguarding the other assumption****, in order to fully reproduce some given experimental statistics within a standard causal (or realist) approach?*

Violation of *Locality* vs. Violation of *Free choice*





$\lambda \in \Lambda'_L \quad \Leftrightarrow$ **locality** **holds** for *all* x, y

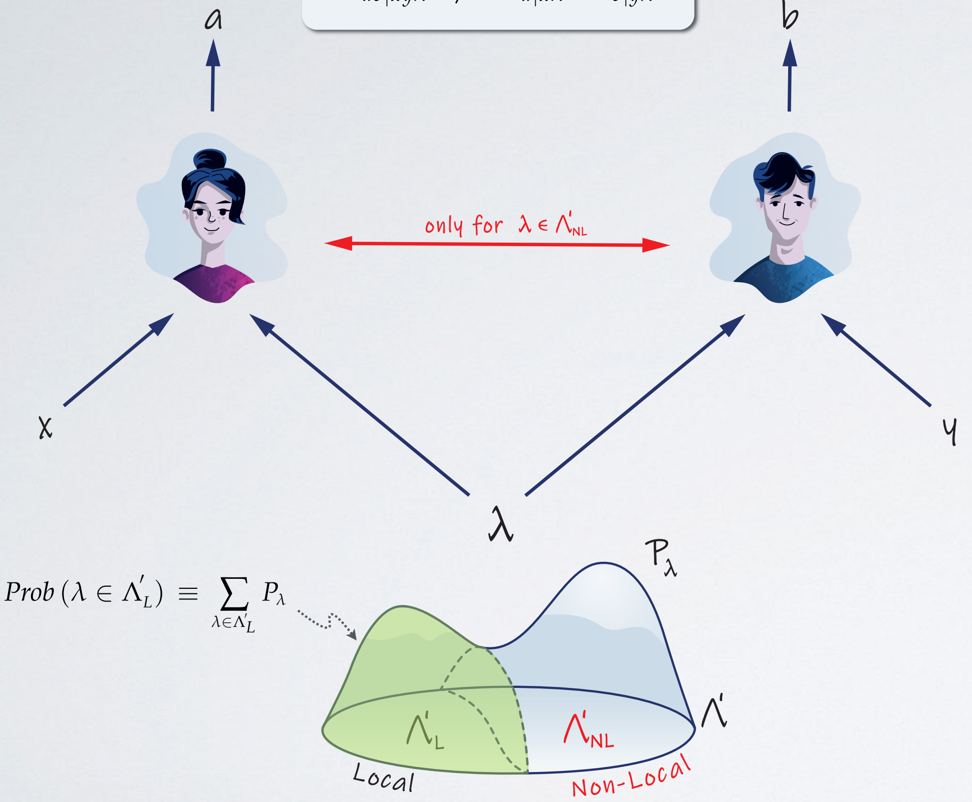
$\lambda \in \Lambda'_{NL} \quad \Leftrightarrow$ **locality** **fails** for *some* x, y

$\lambda \in \Lambda''_F \quad \Leftrightarrow$ **free choice** **holds** for *all* x, y

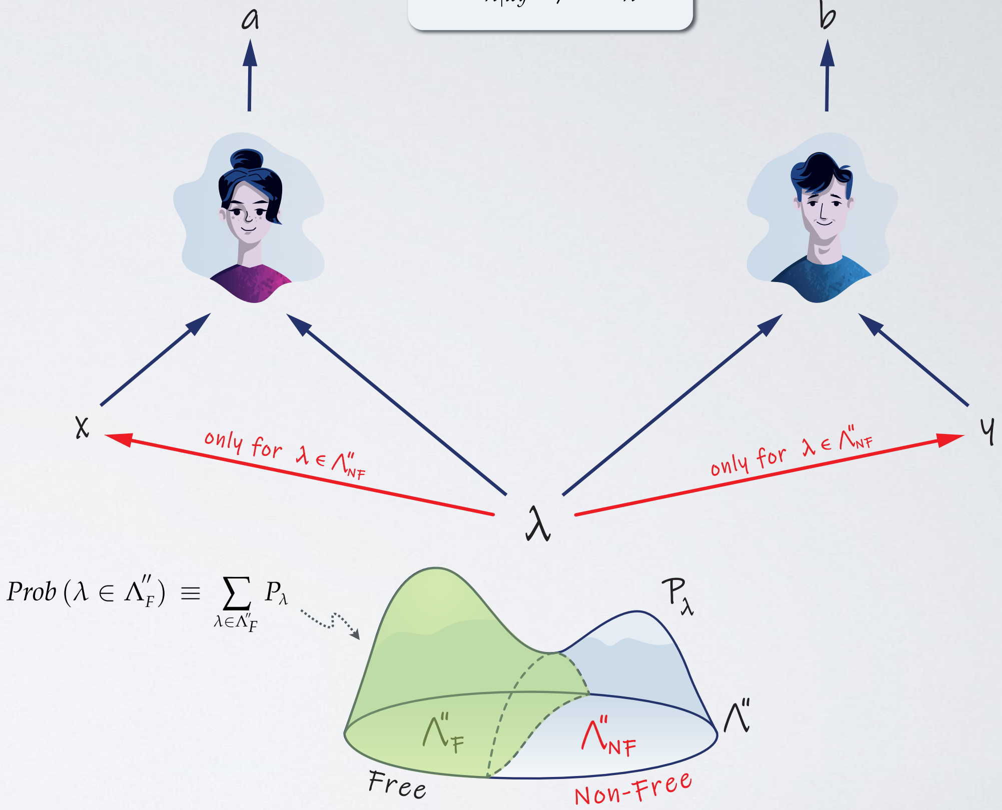
$\lambda \in \Lambda''_{NF} \quad \Leftrightarrow$ **free choice** **fails** for *some* x, y

Violation of *Locality* vs. Violation of *Free choice*

$$P_{ab|xy\lambda} \neq P_{a|x\lambda} \cdot P_{b|y\lambda}$$



$$P_{\lambda|xy} \neq P_\lambda$$



$\lambda \in \Lambda'_L \quad \Leftrightarrow \quad$ **locality holds** for <u>*all*</u> x, y

$\lambda \in \Lambda'_{NL} \quad \Leftrightarrow \quad$ **locality fails** for <u>*some*</u> x, y

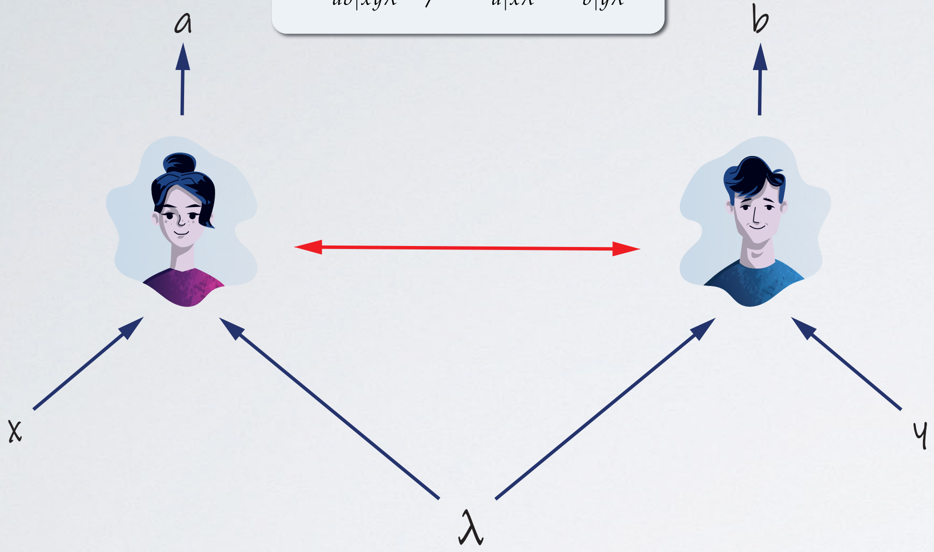
$\lambda \in \Lambda''_F \quad \Leftrightarrow \quad$ **free choice holds** for <u>*all*</u> x, y

$\lambda \in \Lambda''_{NF} \quad \Leftrightarrow \quad$ **free choice fails** for <u>*some*</u> x, y

Violation of Locality vs. Violation of Free choice

Violation of Locality

$$P_{ab|xy\lambda} \neq P_{a|x\lambda} \cdot P_{b|y\lambda}$$

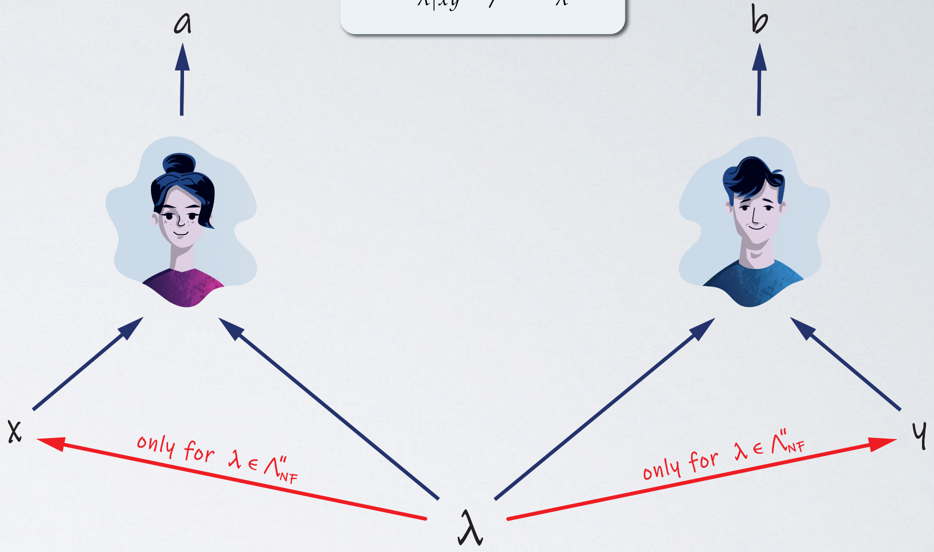


Measure of locality

*the **maximal fraction** of trials in which Alice and Bob **do not** need to **communicate** trying to simulate a given behaviour $\{P_{ab|xy}\}_{xy}$ for <u>any</u> distribution of settings P_{xy}, optimised over <u>all</u> conceivable strategies with **freely chosen** settings.*

Violation of Free choice

$$P_{\lambda|xy} \neq P_{\lambda}$$

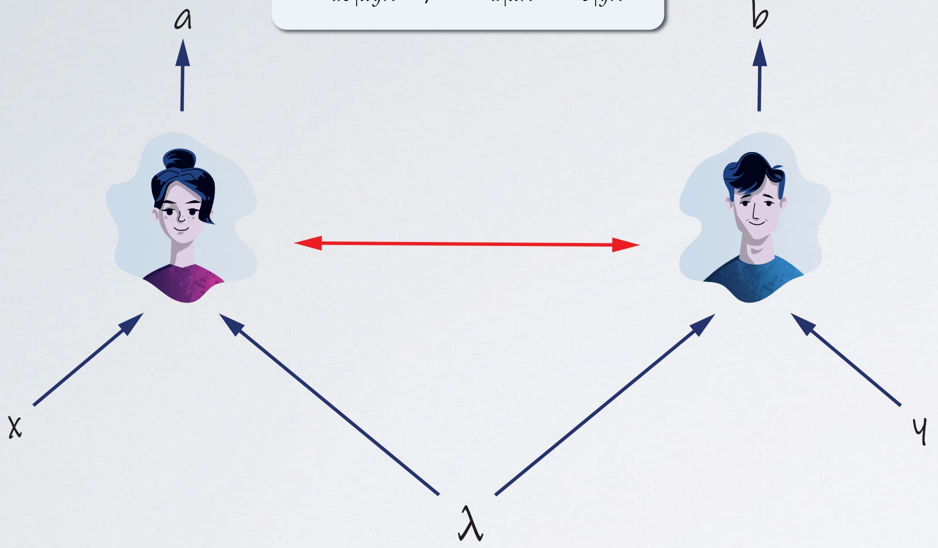


Measure of free choice

*the **maximal fraction** of trials in which Alice and Bob **can grant freedom of choice** of settings in trying to simulate a given behaviour $\{P_{ab|xy}\}_{xy}$ for <u>any</u> distribution of settings P_{xy}, optimised over <u>all</u> conceivable **local strategies**.*

Violation of Locality vs. *Violation of Free choice*

$$P_{ab|xy\lambda} \neq P_{a|x\lambda} \cdot P_{b|y\lambda}$$

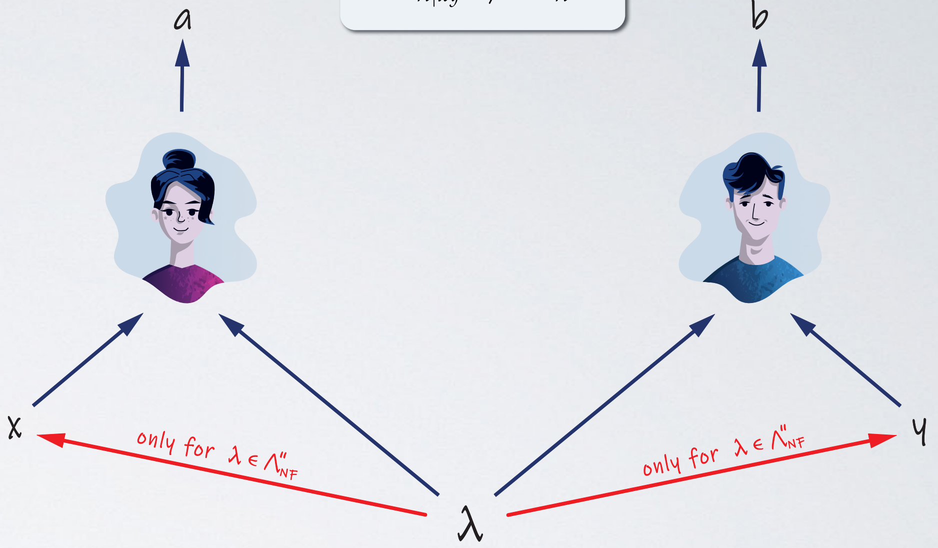


Measure of locality

$$\mu_L := \min_{P_{xy}} \max_{FHV} \sum_{\lambda \in \Lambda_L} P_\lambda$$

*Hidden variable model with **free choice***

$$P_{\lambda|xy} \neq P_\lambda$$



Measure of free choice

$$\mu_F := \min_{P_{xy}} \max_{LHV} \sum_{\lambda \in \Lambda_F} P_\lambda$$

*Hidden variable model with **locality***

Comparison

Theorem:

For a given behaviour $\{P_{ab|xy}\}_{xy}$ *the degree of* ***locality*** *and* ***free choice are the same****, i.e.*

$$\mu_L = \mu_F$$

- *Any number of settings and outcomes*
- *Readily extends to any number of parties* $\{P_{abc\ldots|xyz\ldots}\}_{xyz\ldots}$

- *Sketch of proof*

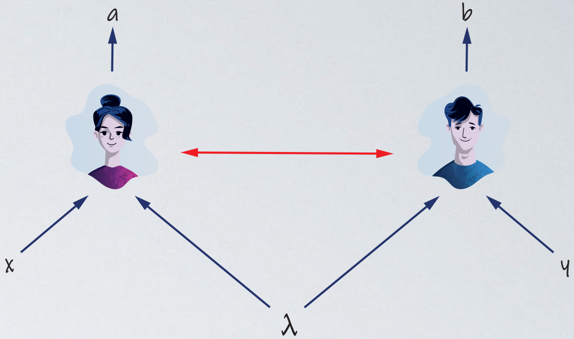
$$P_{ab|xy} = \sum_{\lambda\in\Lambda} P_{ab|xy\lambda}\cdot P_{\lambda|xy}$$

Hidden variable model

$$P_{xy} = \sum_{\lambda\in\Lambda} P_{xy|\lambda}\cdot P_{\lambda}$$

- Get rid of the mins (warning)

- Bijective construction



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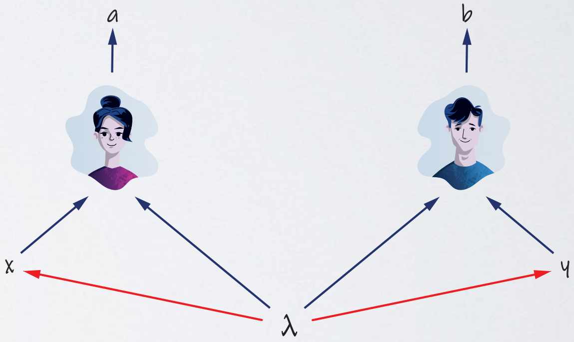

Measure of free choice

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Comparison

Theorem:

> *For a given behaviour* $\{P_{ab|xy}\}_{xy}$ *the degree of* ***locality*** *and* ***free choice are the same****, i.e.*
>
> $$\mu_L \;=\; \mu_F$$

- *Any number of settings and outcomes*
- *Readily extends to any number of parties* $\{P_{abc\ldots|xyz\ldots}\}_{xyz\ldots}$

- *Sketch of proof*

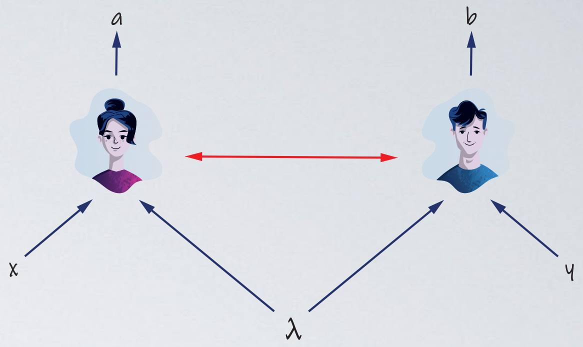
$$P_{ab|xy} \;=\; \sum_{\lambda\in\Lambda} P_{ab|xy\lambda}\cdot P_{\lambda|xy}$$

$$P_{xy} \;=\; \sum_{\lambda\in\Lambda} P_{xy|\lambda}\cdot P_{\lambda} \quad \Rightarrow \quad \tilde{P}_{xy} \;=\; \sum_{\lambda\in\Lambda} \tilde{P}_{xy|\lambda}\cdot P_{\lambda}$$

- Get rid of the mins (warning)

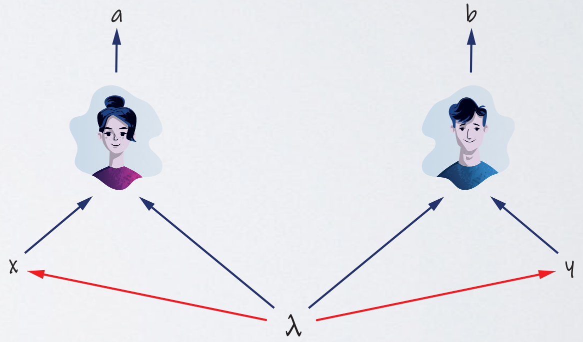
- Bijective construction

$$\tilde{P}_{xy|\lambda} \;=\; \frac{P_{\lambda|xy}\cdot\tilde{P}_{xy}}{P_\lambda} \overset{?}{\leqslant} 1$$



Measure of locality

$$\mu_L \;:=\; \cancel{\min_{P_{xy}}}\,\max_{FHV} \sum_{\lambda\in\Lambda_L} P_\lambda$$



Measure of free choice

$$\mu_F \;:=\; \cancel{\min_{P_{xy}}}\,\max_{LHV} \sum_{\lambda\in\Lambda_F} P_\lambda$$

Democritus
(460 - 370 BC)

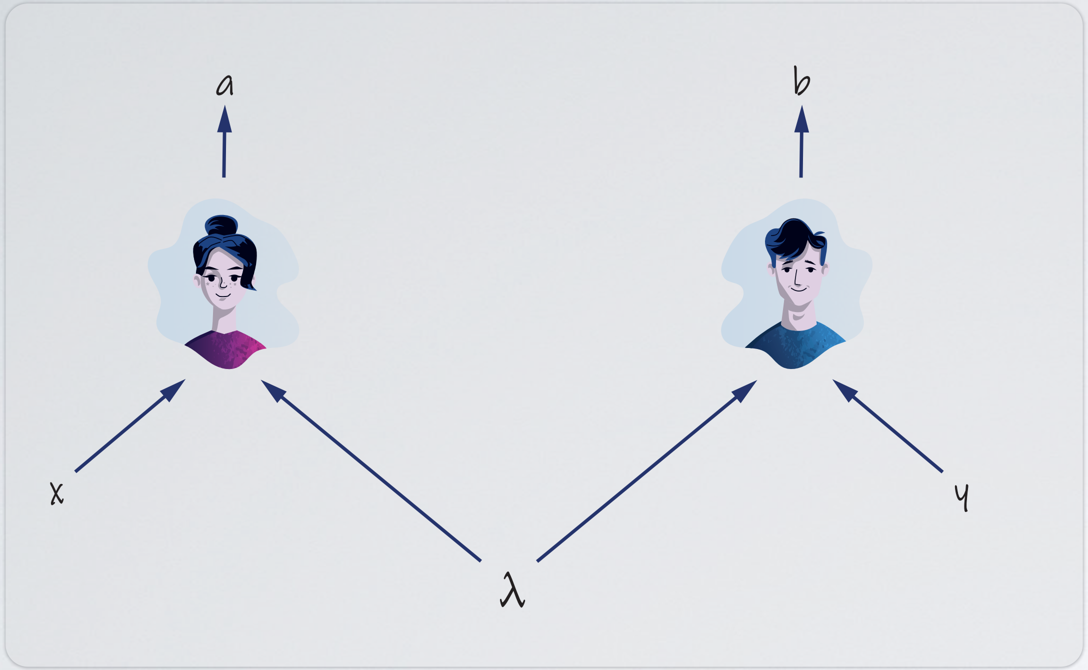
"I would rather discover one true cause than gain the kingdom of Persia."

$$\mu_L = \mu_F = ?$$

"Measure what is measurable, and make measurable what is not so."

Galileo GALILEI
(1564 - 1642)

Bell experiment — recap (II)



Bell's theorem

Realism + Locality + Free choice: $|S_i| \leqslant 2$

whereas in QM it can be: $|S_i| \leqslant 2\sqrt{2} \leqslant 4$

Tsirelson *PR- box*

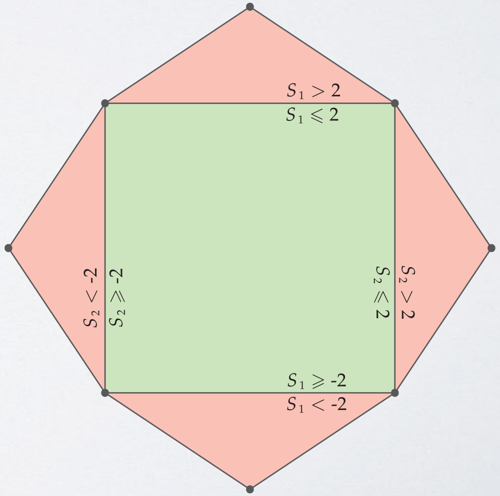
- *Bell-CHSH expressions*

$$\begin{aligned}
S_1 &= \langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} - \langle ab\rangle_{11} \\
S_2 &= \langle ab\rangle_{00} + \langle ab\rangle_{01} - \langle ab\rangle_{10} + \langle ab\rangle_{11} \\
S_3 &= \langle ab\rangle_{00} - \langle ab\rangle_{01} + \langle ab\rangle_{10} + \langle ab\rangle_{11} \\
S_4 &= -\langle ab\rangle_{00} + \langle ab\rangle_{01} + \langle ab\rangle_{10} + \langle ab\rangle_{11}
\end{aligned}$$

where: $\langle ab\rangle_{xy} = \sum_{a,b} ab\, P_{ab|xy}$

- *Non-signalling* $P_{b|0y} = P_{b|1y}$ for all b, y; $P_{a|x0} = P_{a|x1}$ for all a, x

- *Non-signalling polytope (<u>free choice assumed</u>)*



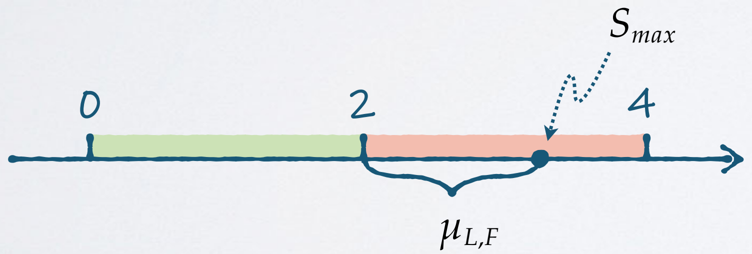
Explicit measure for Bell scenario

Theorem:

For a given ***non-signalling*** *behaviour* $\{P_{ab|xy}\}_{xy}$ *with* ***binary settings*** *both measures of locality* μ_L *and free choice* μ_F *are equal to*

$$\mu_L = \mu_F = \begin{cases} \frac{1}{2}(4 - S_{max}), & \text{if } S_{max} > 2, \\ 1, & \text{otherwise}, \end{cases}$$

where $S_{max} = \max\{|S_i| : i = 1, \dots, 4\}$ *is the maximum absolute value of the four CHSH expressions.*



- *Convex decomposition*

$$P_{ab|xy} = p_L \cdot P^L_{ab|xy} + (1 - p_L) \cdot P^{NL}_{ab|xy}$$

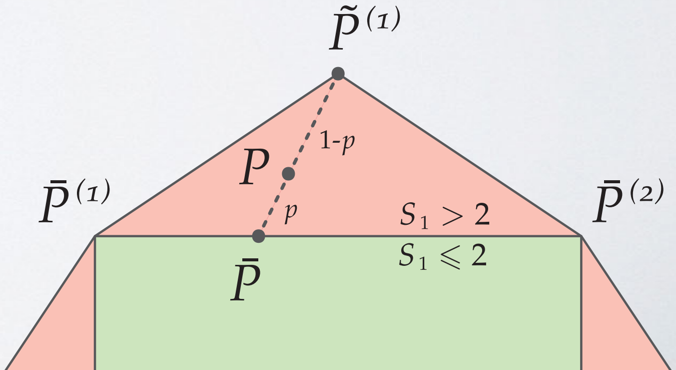
$$\mu_L = \max_{\text{decomp.}} p_L$$

- *Upper bound*

$$p_L \leqslant \frac{1}{2}(4 - |S_i|) \quad \Rightarrow \quad \mu_L \leqslant \frac{1}{2}(4 - S_{max})$$

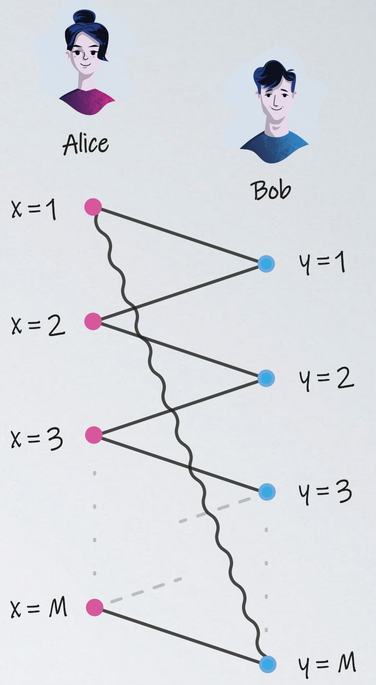
- *Saturation of the bound*

$$P_{ab|xy} = \sum_{j=1}^{16} p_j \cdot \bar{P}^{(j)}_{ab|xy} + \sum_{k=1}^{8} q_k \cdot \tilde{P}^{(k)}_{ab|xy}$$



S. Pironio, PRA 68, 062102 (2003)
S. Abramsky et al., PRL 119, 050504 (2017)

Quantum statistics



- ***Binary settings***

  Tsirelson's bound

$$\mu_L = \mu_F = 2 - \sqrt{2} \approx 0.59$$

- ***Bell state & infinite number of settings***

  A. Elitzur et al., Phys. Lett. A 162, 25 (1992)
  J. Barrett et al., PRL 97, 170409 (2006)

$$\mu_L = \mu_F \xrightarrow[M \to \infty]{} 0$$

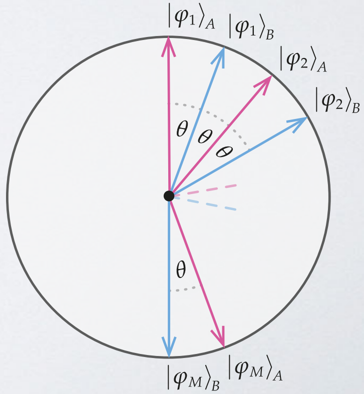
- ***Two-qubit state & arbitrary settings***

$$|\psi\rangle = \cos\tfrac{\theta}{2}\,|00\rangle + \sin\tfrac{\theta}{2}\,|11\rangle$$

$$\theta \in [0, \tfrac{\pi}{2}]$$

$$\mu_L = \mu_F = \cos\theta$$

S. Portman et al., PRA 86, 012104 (2012)



Summary of results

		Locality	Free choice
Any statistics	*any no. settings*	$\mu_L \quad = \quad \mu_F$	
	non-signalling two settings	$\mu_L = \frac{1}{2}(4 - S_{max})$	$\mu_F = \frac{1}{2}(4 - S_{max})$
Quantum statistics	*Bell state infinite no. settings*	$\mu_L \xrightarrow[M \to \infty]{} 0$ (*)	$\mu_F \xrightarrow[M \to \infty]{} 0$
	two-qubit state any no. settings	$\mu_L = \cos\theta$ (*)	$\mu_F = \cos\theta$


Thank you for your attention
QCQMB'21, 20 May 2021