# WITNESSING AND ENGINEERING QUANTUM GENERALIZED CONTEXTUALITY

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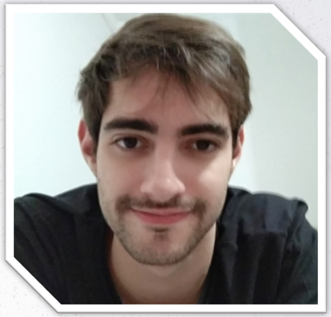

**Alisson Tezzin**

USP


**Rafael Wagner**

INL

**Roberto Baldijão**

UNICAMP

# Resource Theory

Powerful framework for the formal treatment of a physical property as an operational resource, adequate for its characterization, quantification, and manipulation.

B. Coecke, T. Fritz, R. W. Spekkens, **A mathematical theory of resources**, Information and Computation (2016).

# Resource Theory

- Objects and free objects;

C. Duarte and B. Amaral, **Resource theory of contextuality for arbitrary prepare-and-measure experiments**, Journal of Mathematical Physics (2018).

R. Wagner, R. D. Baldijão, A. Tezzin, B. Amaral, **Using a resource theoretic perspective to witness and engineer quantum generalized contextuality for prepare-and-measure scenarios**, arXiv:2102.10469.

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# Contextuality in preapare-and-measure experiments

R. Spekkens **Contextuality for preparations, transformations, and unsharp measurements**,Phys. Rev. A 71, 052108(2005).

Preparation $P_i$.

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Measurement $M_j$ with outcome $k$.

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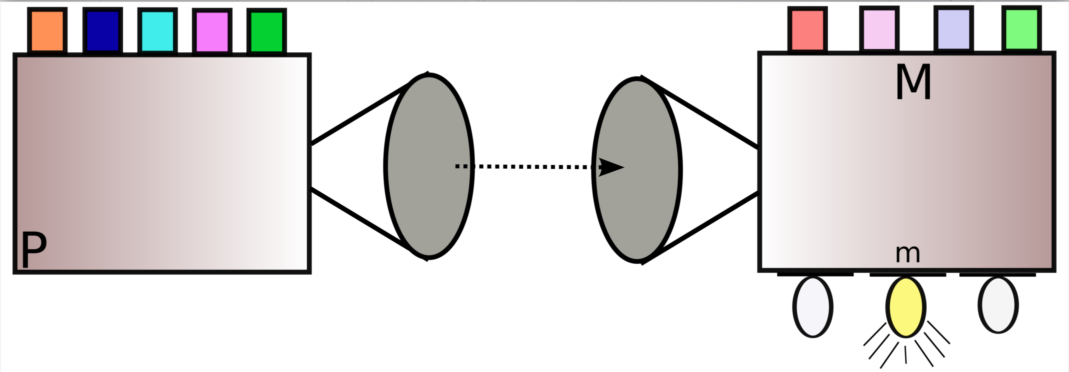
$$p\left(k|i,j\right)$$


P
M
m

# Preparation equivalence

Two preparations $P_1$ and $P_2$ are equivalent if

$$p\left(k|1,j\right) = p\left(k|2,j\right), \quad \forall j,k$$

# Measurement event equivalence

Two measurement events $k|m$ and $l|n$ are equivalent if

$$p\,(k|i,m) = p\,(l|i,n)\,, \quad \forall P_i$$

# Classical models

A set $\Lambda$, for each preparation $P_i$ a probability distribution $\mu_i$ over $\Lambda$, and a response function $\xi_{k|j}$ such that

$$p(k|i,j) = \sum_{\lambda \in \Lambda} \mu_i(\lambda)\, \xi_{k|j}(\lambda)$$

and $\mu$ and $\xi$ respect the operational equivalences:

$$P_1 \simeq P_2 \;\Rightarrow\; \mu_1 = \mu_2$$

$$k|m \simeq l|n \;\Rightarrow\; \xi_{k|m} = \xi_{l|n}$$

# The simplest scenario $\mathbb{B}_{si}$

- Four preparation procedures and two binary-outcome measurement procedures.

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- Four preparation procedures and two binary-outcome measurement procedures.
- Operational equivalences for preparation procedures:

$$\frac{1}{2}P_1 + \frac{1}{2}P_2 \simeq \frac{1}{2}P_3 + \frac{1}{2}P_4$$

- Preparation procedures $\{P_1, P_2, P_3, P_4, \ldots, P_J\}$ and binary-outcome measurement procedures $\{M_1, \ldots, M_{|I|}\}$.

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- Operational equivalences for preparation procedures:

$$\frac{1}{2}P_1 + \frac{1}{2}P_2 \simeq \frac{1}{2}P_3 + \frac{1}{2}P_4$$

- Other preparation equivalences that do not include $P_1, P_2, P_3, P_4$.

# Resource theory for contextuality in general preapare-and-measure experiments

# Objects

Objects: behaviors $B = \{p(k|i,j)\}$ .

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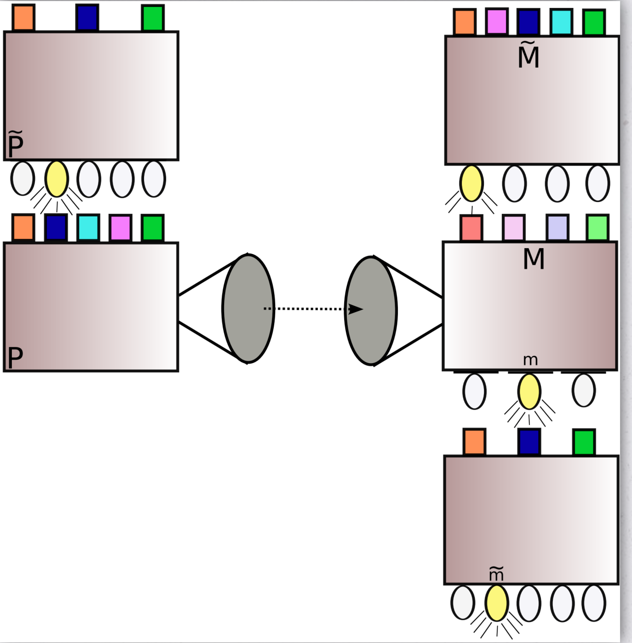
Free objects: classical behaviors.

# Free operations

C. Duarte and B. Amaral, **Resource theory of contextuality for arbitrary prepare-and-measure experiments** Journal of Mathematical Physics (2018).

$$p(k|M_i, P_j) = \sum_{i,j,k} q_O^i(\tilde{k}|k)p(k|M_i, P_j)q_M(i|\tilde{i})q_P(j|\tilde{j})$$



$\tilde{P}$
$P$
$\tilde{M}$
$M$
$m$
$\tilde{m}$

- Characterization of the set of classical behaviors in terms of linear programming.

D. Schimid, R. W. Spekkens, E. Wolfe, **All the nonlocality inequalities for arbitrary preapare-and-measure experiments with respect to any fixed sets of operational equivalence**, Physical Review A (2018).

C. Duarte and B. Amaral, **Resource theory of contextuality for arbitrary prepare-and-measure experiments** Journal of Mathematical Physics (2018).

- Characterization of the set of classical behaviors in terms of linear programming.
- Several quantifiers can be generalized to this notion of nonclassicality as well.

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# Our work

1. Resource theory framework of generalized contextuality as a tool for analyzing the structure of prepare-and-measure scenarios.

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1. Resource theory framework of generalized contextuality as a tool for analyzing the structure of prepare-and-measure scenarios.

2. Simplify proofs for quantum contextuality and strengthen known arguments regarding robustness of experimental implementations.

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# Our work

1. Resource theory framework of generalized contextuality as a tool for analyzing the structure of prepare-and-measure scenarios.
2. Simplify proofs for quantum contextuality and strengthen known arguments regarding robustness of experimental implementations.
3. Witness quantum contextuality associated with any nontrivial noncontextuality inequality for a class of scenarios by noticing a connection between the resource theory and measurement simulability.

R. Wagner, R. D. Baldijão, A. Tezzin, B. Amaral, **Using a resource theoretic perspective to witness and engineer quantum generalized contextuality for prepare-and-measure scenarios**, arXiv:2102.10469.

# Witneessing and engineeing quantum contextuality

# Measurement simulability

- Consider two sets of measurement procedures $\mathbb{I} = \{i\}$ and $\tilde{\mathbb{I}} = \{\tilde{i}\}$.

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- $\tilde{\mathbb{I}}$ is $\mathbb{I}$-simulable if

$$\left[\tilde{k}|\tilde{i}\right] = \sum_{i,k} q_i^O \left(\tilde{k}|k\right) [k|i]\, q\left(i|\tilde{i}\right).$$

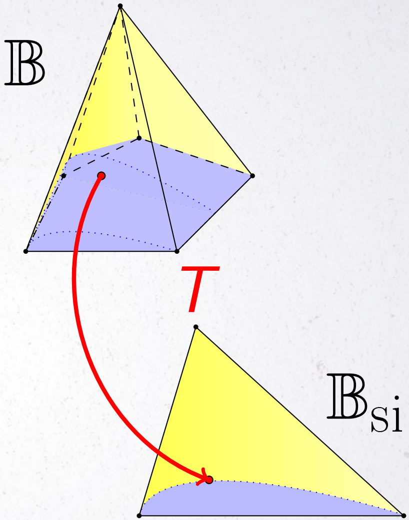
Measurement simulability states an equivalence between the measurements to be simulated and the measurements performing the simulation.

# Product

The binary operation $\boxplus$ is defined as the vertical stacking of vectors from the scenarios $\mathbb{B}_1, \mathbb{B}_2$ towards $\mathbb{B}$, i.e.,

$$B_1 \boxplus B_2 := \begin{pmatrix} p(k_1 | M_{i_1}, P_{j_1}) \\ p(k_2 | M_{i_2}, P_{j_2}) \end{pmatrix} \tag{1}$$

# Sufficient conditions for contextuality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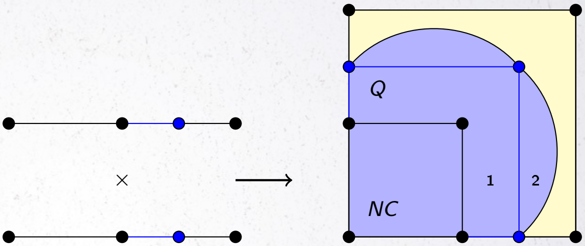
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NC
1
2

# Results

R. Wagner, R. D. Baldijão, A. Tezzin, B. Amaral, **Using a resource theoretic perspective to witness and engineer quantum generalized contextuality for prepare-and-measure scenarios**, arXiv:2102.10469.

Quantum contextual behaviors for $\mathbb{B}_n$.

For any scenario of the form $\mathbb{B} := \mathbb{B}_{\mathrm{si}}^{\boxplus n}$, $n \geq 1$, every tight and nontrivial noncontextuality inequality will be violated by some quantum contextual behavior.

# Quantum advantages

- Consider a task that has a success rate defined by a function $g : \mathbb{B} \to \mathbb{R}_+$, for $\mathbb{B}$ of the form of $\mathbb{B}_{\mathrm{si}}^{\boxplus n}$.

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# Quantum advantages

- Consider a task that has a success rate defined by a function $g : \mathbb{B} \to \mathbb{R}_+$, for $\mathbb{B}$ of the form of $\mathbb{B}_{\mathrm{si}}^{\boxplus n}$.
- Supose that he noncontextual bound for the success, $g^{NC}(B) \leq \delta$, can be expressed as a linear combination of the noncontextuality inequalities of $NC(\mathbb{B})$.
- Then, there exists a quantum behavior $B^Q$ such that $g(B^Q) > \delta$.

# Quantum cloning inherits contextuality from $\mathbb{B}_6$

The scenario $\mathbb{B}_{\mathrm{qc}}$ related to the state-dependent quantum cloning task, can be written as

$$\mathbb{B}_{\mathrm{qc}} = \mathbb{B}_6 \boxplus \mathbb{B}_6 \boxplus \mathbb{B}_6 . \qquad (2)$$

M. Lostaglio and G. Senno, **Contextual advantage for state-dependent cloning**, Quantum (2020).

# Perspectives

- Finding other physically appealing scenarios to apply these tools.

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- Use of resource-theoretic perspectives to contextuality to find quantum advantages in practical tasks.

# QUANTUM FOUNDATIONS @ USP

Thank you!