A structure theorem for transformations in noncontextual models

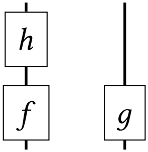
Matthew F. Pusey <m@physics.org>

joint work with John Selby, David Schmid and Rob Spekkens

Part I

Diagrams

Operational diagrams



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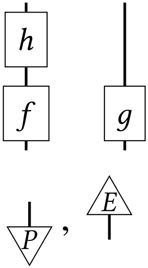

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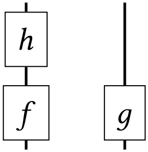
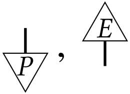
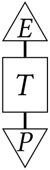
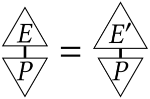






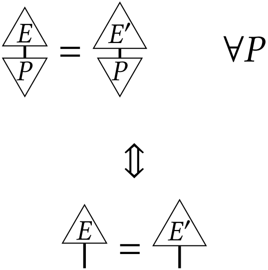
$$= p(E|P,T)$$

Operational equivalence

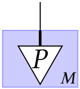


$\forall P$

Operational equivalence



Ontological diagrams



$$= p(\lambda|P)$$

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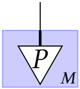

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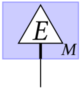

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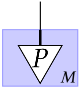

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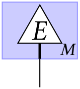

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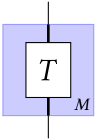
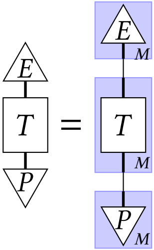


$= p(\lambda'|\lambda, T)$

Ontological diagrams

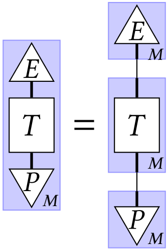
$$p(E|P,T) = \int\int p(E|\lambda')p(\lambda'|\lambda,T)p(\lambda|P)d\lambda' d\lambda$$



Part II

Statement of structure theorem

Assumption 1: Diagram preservation (uncontroversial cases)



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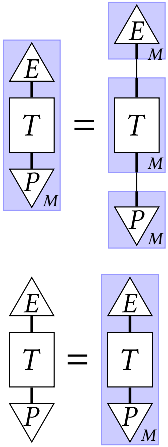

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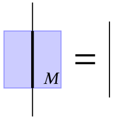

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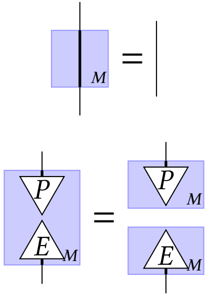

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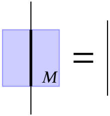
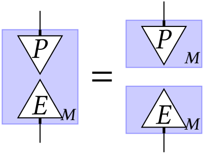
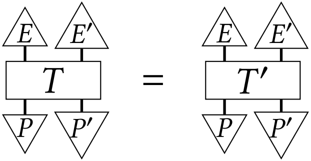


$$= \delta(\lambda' - \lambda)$$

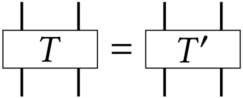


$$= p(\lambda'|P)p(E|\lambda)$$

Assumption 2: Local tomography

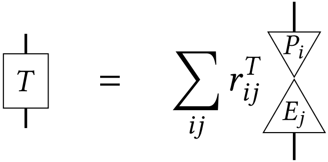


$\forall E, E', P, P' \iff$



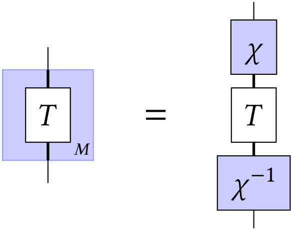
Assumption 2: Local tomography (equivalent condition)

For any T there exist r_{ij}^T such that

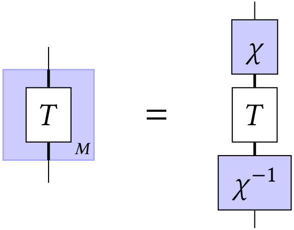


Structure theorem

A diagram-preserving noncontextual model M of a locally tomographic theory can be written



What is χ?



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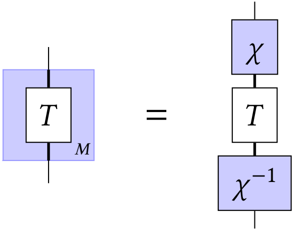
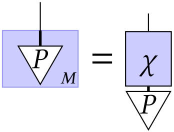
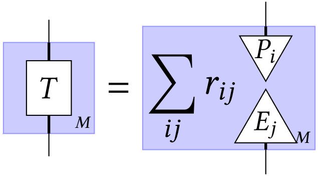




Part III

Proof of structure theorem

Step 1



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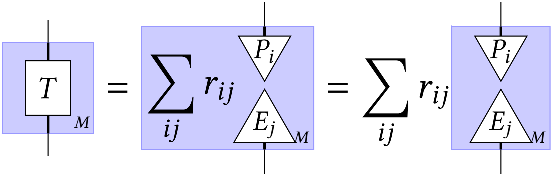

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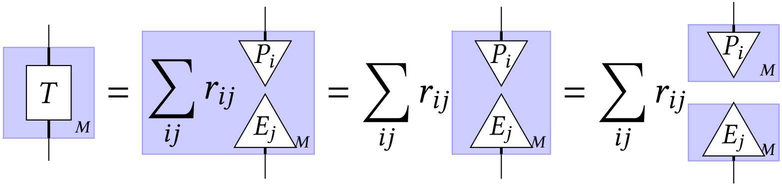
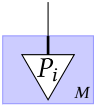


Step 2



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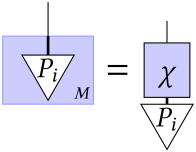

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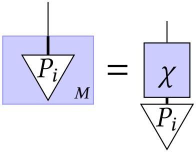

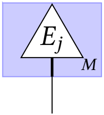

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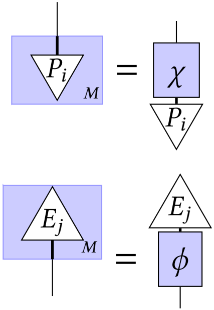
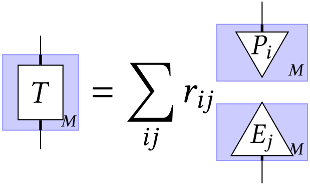


Step 1 + Step 2



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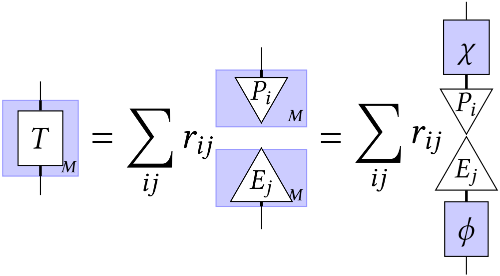

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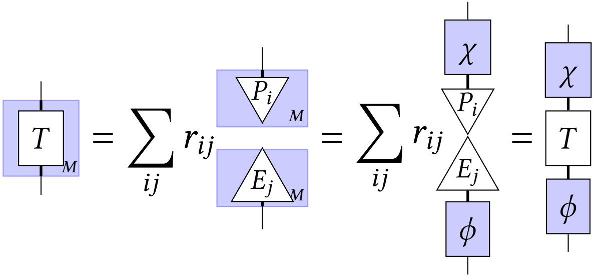

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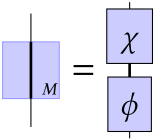

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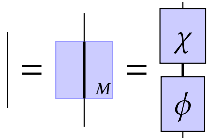

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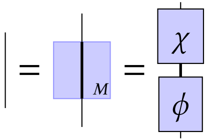

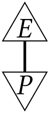

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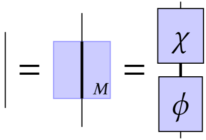

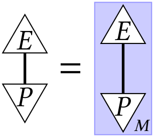

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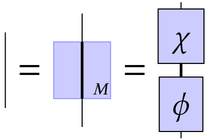

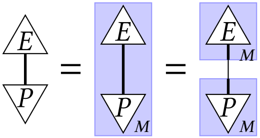

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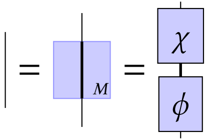

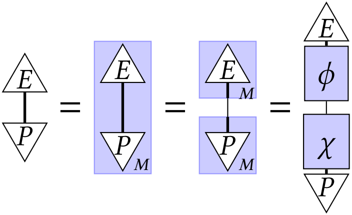

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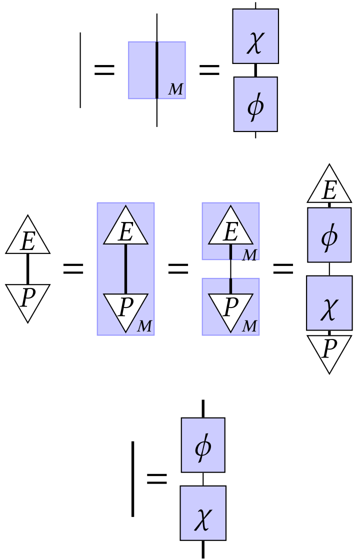
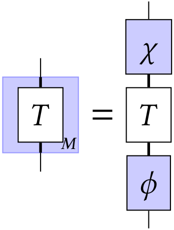


QED



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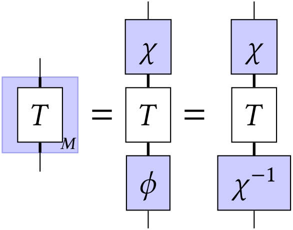

Part IV

Consequences and other versions

Number of ontic states

Number of ontic states = dimension of state space

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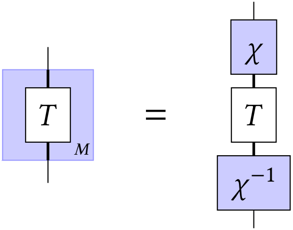
c.f. Wallman & Bartlett, arXiv:1203.2652

Quasiprobability representations

A diagram-preserving quasiprobability representation M of a locally tomographic theory can be written



Stabilizer subtheory

The only noncontextual model of the stabilizer subtheory is

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Haoxing Du
Department of Physics, University of California, Berkeley, Berkeley, California 947

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Matthew F. Pusey
Department of Mathematics, University of York, Heslington, York YO10 5DD, United

We prove that there is a unique nonnegative and diagram-preserving quasi
representation of the stabilizer subtheory in all odd dimensions, namely Gross's discre
function. This representation is equivalent to Spekkens' epistemically restricted toy theor
consequently singled out as the unique noncontextual ontological model for the stabilizer
Strikingly, the principle of noncontextuality is powerful enough (at least in this setting
out *one particular* classical realist interpretation. Our result explains the practical utility
representation, e.g. why (in the setting of the stabilizer subtheory) negativity in this
representation implies generalized contextuality, and hence sheds light on why negativ
particular representation is a resource for quantum computational speedup. It also al
prove that generalized contextuality is a necessary resource for universal quantum comp
the state injection model. In all even dimensions, we prove that there does not exist any n
and diagram-preserving quasiprobability representation of the stabilizer subtheory, and,
the stabilizer subtheory is contextual in all even dimensions. Together, these results c
complete characterization of the (non)classicality of all stabilizer subtheories.

Conclusions and outlook

- Noncontextual models (and quasiprobability representations) of locally tomographic theories have a very rigid structure
  - Fixed number of ontic states
  - Only freedom is representation of preparations
- Any model without this structure can immediately be declared contextual
- Computational searches for noncontextual models can leverage this structure
- First step towards unique charachterization of noncontextual models of the Stabilizer subtheory