Contextuality-by-Default (CbD) description of Bell tests

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http://arxiv.org/abs/2104.11555

(121 references)

Araujo M., M. T. Quintino M.T., Budroni C.,Cunha M.T., and Cabello A., All noncontextuality inequalities for the n-cycle scenario. Phys. Rev. A 88, 022118 (2013)

Kujala, J.V., Dzhafarov, E.N., Larsson, J-Å (2015). Necessary and sufficient conditions for extended non-contextuality in a broad class of quantum mechanical systems. Physical Review Letters 115:150401

Kupczynski M.2017 Is Einsteinian no-signalling violated in Bell tests? *Open Physics* 15 , 739-753, DOI: https://doi.org/10.1515/phys-2017-0087

Motivation: troubling anomalies

- Adenier and Khrennikov 2007 discovered the dependence of marginal single count frequencies **on distant settings** in Weihs et al. data.
- Anomalies were confirmed by De Raedt, Jin and Michielsen 2012, 2013
- Adenier and Khrennikov 2017 and Bednorz 2015 reported similar anomalies in Hensen et al. data .
- Liang, at FQMT2017, reported that p-value of observing some data points under the assumption of non-signaling was smaller than 3.17×10^{-55} (Schalm et al. data)
- We explain using **CbD** , why in spite of these anomalies, **Einsteinian non-signaling is not violated** .

Plan of our talk

- Introduction: statistical populations, random variables and their measurements
- Bohr- and Kochen –Specker- contextuality
- Non-contextual inequalities for n-cycle scenarios (NCI)
- CbD approach and modified NCI
- Ideal EPR-B experiment and spin polarization correlation experiments (SPCE)
- Inconsistent connectedness and a modified CHSH inequality.
- New tests of contextuality in SPCE.

Statistical populations

- A set of physical systems, objects, animals or people (whose properties, behaviour and opinions at a given moment of time) we want to investigate
- An infinite set of outcomes which might have been obtained in subsequent repetitions of a random experiment

The information about statistical populations is inferred from properties of finite simple random samples drawn from these populations

Properties and random variables

- Macroscopic objects have predetermined/non-contextual properties.
- Non-contextual properties can be 'measured' jointly or in any order.
- Measurement outcomes are coded using a set of random variables.
- In mathematical statistics populations are described by joint probability distributions of non-contextual random variables.

Measurements in classical physics

- Non-contextual properties "exist" independently of the fact of being measured (length of a table, colour of eyes, Corona virus ADN code, …).
- In statistical physics we use joint probability distributions of, impossible to measure, positions, linear momenta and energies of invisible molecules in order to describe thermodynamics of materials.
- Classical Filters are selectors of preexisting properties.

Measurements in quantum physics

- We observe only the macroscopic effects of interactions between **invisible** physical systems and measuring instruments or environment:

  traces in ionization chambers, clicks on detectors

- Contextual properties **are created** in the interaction of a physical system with a measuring instrument in a well-defined experimental context .(e.g. spin projection)
- Values of complementary properties may not be assigned 'to a physical system at the same time'

Three Principles

1. Bohr-contextuality: The output of any quantum observable is indivisibly composed of the contributions of the system and the measurement apparatus.
2. Bohr-complementarity: There exist incompatible observables (complementary experimental contexts).
3. Kochen-Specker (KS)-contextuality:
   A measurement of an observable does not need to yield the same value independently of what other measurements may be made simultaneously

Khrennikov A., Can there be given any meaning to contextuality without incompatibility? Int. J. Theor. Phys., 2020; https://doi.org/10.1007/s10773-020-04666-z .'

Contextuality as the rule not an exception

- KS- contextuality is not limited to quantum mechanics (QM)
- In behavioral sciences answers to Yes-or-NO questions depend on which other questions and in which order are asked at the same time
- Therefore in CbD random variables are labelled not only by a 'measured' content but also by a context of the experiment.

Dzhafarov, E.N., Kujala, J.V. (2014). Contextuality is about identity of random variables. Physica Scripta T163:014009.

Kujala, J.V., Dzhafarov, E.N., Larsson, J-Å (2015). Necessary and sufficient conditions for extended noncontextuality in a broad class of quantum mechanical systems. Physical Review Letters 115:150401

Contextuality and non-contextuality

- A set X of random variables is contextual, if a studied population may not be described by a joint probability distribution of all these variables. Otherwise X is non-contextual.
- A set X of dichotomous random variables, taking values ±1, is contextual, if and only if non-contextuality inequalities (NCI) are violated.

Cabello A, Experimentally testable state-independent quantum contextuality. Physical Review Letters, 101(21):210401, 2008.

Cabello A., Simple explanation of the quantum violation of a fundamental inequality. Physical Review Letters, 110:060402, 2013.

NCI for n-cycle scenarios

- A simple inequality is satisfied by $x_i=\pm 1$:

$$x_0x_1+x_1x_2+_{\dots}x_{n-2}x_{n-1}-x_{n-1}x_0\leq n-2$$

the maximal value of the blue sum is n-1

- NCI are satisfied by pair-wise expectations :

$$<X_0X_1>+<X_1X_2>\ldots-<X_{n-1}X_0>\leq \mathrm{n}-2$$

- For n=3 we have one of Boole or Suppes-Zanotti-Legett-Garg (SZLG) inequalities . For n=4 we obtain Clauser-Horn-Shimony-Holt (CHSH) inequality and for n=5 we obtain Klyachko-Can-Binicioglu-Shumovsky (KCBS) inequality

Generalized NCI

Generalized n-cyclic NCI for $n \geq 3$ and $\gamma_i = \pm 1$:

$$\sum_{i=0}^{n-1} \gamma_i < X_i X_{i+1} > \leq n-2$$

where $X_n = X_0$ and such that **the number of γ_i = -1 is odd.**

Araujo M., M. T. Quintino M.T., Budroni C.,Cunha M.T., and Cabello A., All noncontextuality inequalities for the n-cycle scenario. Phys. Rev. A 88, 022118 (2013)

Dzhafarov, E.N., & Kujala, J.V., & Larsson, J.-Å. (2015). Contextuality in three types of quantum-mechanical systems. Foundations of Physics 7, 762-782.

Contextuality –by- default

- Let us consider **n-cycle scenario** of binary random variables $X=\{X_0,\ldots X_{n-1}\}$ such that **only** all **successive pairs** $\{X_i, X_{i+1}\}$ are **commeasurable** and $X_n = X_0$.

- Each pair of random variables defines a **different experimental context**.

- **In CbD** : X_i and X'_i measure the same content in two contexts ,thus we have a system containing **2n** binary random variables $X'=\{X_0, X'_{0\ldots} X_{n-1}, X'_{n-1}\}$.

M.K, CbD description of Bell Tests, arXiv:2104.11555 [quant-ph]

Inconsistent connectedness

- We have still ***n* measurable** pairwise expectations **$< X_i X'_{i+1} >$,** but random variables **$X_i \neq X'_i$** are **stochastically unrelated** (our **system is not cyclic**) and **we may not derive NCI**.

$$X_0 \text{--} X'_1 \quad X_1 \text{--} X'_2 \quad \ldots . X_{n-2} \text{--} X'_{n-1} \quad X_{n-1} \text{--} X'_0$$

- If marginal expectation values of random variables measuring the same content depend on the experimental context: $< X_i >_m \neq <X'_i>_m$, then they are *inconsistently connected* (NCC).
- CbD proposes a new method for studying the contextuality of NCC system.

New extended 2n-cyclic system

- CbD **imposes** a counterfactual joint probability distribution upon the system X'
- If such probability distribution is imposed, the expectations $< X_i\ X'_i >$ **are defined** and we have a **new <u>2n</u>-cyclic system/scenario** :

$$--X_0--X'_1--X_1--X'_2-- \ \ldots.--X_{n-2}--X'_{n-1}-- \ X_{n-1}--X'_0--X_0$$

$$\sum_{i=0}^{n-1} \gamma_i < X'_i X_{i+1} > + \sum_{i=0}^{n-1} < X_i X'_i > \leq 2\mathrm{n} - 2$$

Maximal and identity couplings

- Random variables X_i and X'_i **should be as similar as possible** what imposes constraints on $\langle X_i X'_i \rangle$.

- In CbD we impose ***the maximal coupling*** on each pair of random variables $\{X_i, X'_i\}$ **replacing $\langle X_i X'_i \rangle$ by its maximal value :**

$$1 - |\langle X_i \rangle_m - \langle X'_i \rangle_m|$$

- *A counterfactual joint probability distribution* of 2n variables, consistent the constraints and experimental data, does not always exist.

Modified NCI in CbD

$$< X_0 X_0' > \; + \; < X_0' X_1 > \ldots - < X_{n-1}' X_0 > \leq 2\mathrm{n} - 2$$

$$\sum_{i=0}^{n-1} \gamma_i < X'_i X_{i+1} > + \sum_{i=0}^{n-1} < X_i X'_i > \leq 2\mathrm{n} - 2$$

$$\sum_{i=0}^{n-1} \gamma_i < X'_i X_{i+1} > + \sum_{i=0}^{n-1} [1 - | \langle X_i \rangle_m - \langle X'_i \rangle_m | \leq 2\mathrm{n} - 2$$

$$S_n = \sum_{i=0}^{n-1} \gamma_i < X'_i X_{i+1} > + \sum_{i=0}^{n-1} | \langle X_i \rangle - \langle X'_i \rangle | \leq \mathrm{n} - 2$$

EPR-B (1951) Experiment

- Twin-electron or twin-photon pairs are prepared in a singlet state

- ‘Particles’ are allowed to separate

- Spin components on different directions ,measured by Alice and Bob, are strongly correlated

$$\Psi = (|+>_{\hat{p}} \otimes |->_{\hat{p}} - |->_{\hat{p}} \otimes |+>_{\hat{p}}) / \sqrt{2}$$

where $|+>_{\hat{p}}$ *and* $|->_{\hat{p}}$ *are state vectors corresponding to the particle states in which the spin is "up" or "down" in the direction of* $\hat{p}$ *respectively.*

Experimental Protocol of ideal EPR-B

- A stationary flow of pairs of entangled spins .
- No losses of pairs, the outcomes are coded by two random variables (A ,B) where A=±1 and B=±1 .
- An experimental run is described by 3 samples $S_{A=}\{a_1,\dots,a_n\}$, $S_{B=}\{b_1,\dots,b_n\}$, $S_{AB=}\{a_1 b_1,\dots, a_n b_n\}$
- One may estimate : $P(A=a, B=b|x, y)$ and marginal distributions $P(A=a|x, y)$, $P(B=b|x,y)$ do not depend on distant settings.

Non-signaling is strictly obeyed.

Non-signaling = marginal distributions do not depend on distant settings

$$\mathrm{P}(A=a\,|\,\mathrm{x},\mathrm{y})=\sum_b \mathrm{P}(A=a,B=b\,|\,\mathrm{x},\mathrm{y})=P(A=a\,|\,x)$$

$$\mathrm{P}(\mathrm{B}=b\,|\,\mathrm{x},\mathrm{y})=\sum_a \mathrm{P}(A=a,B=b\,|\,\mathrm{x},\mathrm{y})=P(B=b\,|\,y)$$

Spin polarization correlation experiments (**SPCE**)

Bell-Tests with twin-photon-beams

- Two correlated signals are sent to Alice and Bob in distant laboratories

- The signals after passing by PBS-detector modules produce **two time series of clicks** registered by Alice's and Bob's online computers .

- One has to identify the clicks produced by entangled 'photon-pair'. Correlated clicks are rare events

3 step experimental protocol for (x,y)

1. Raw time-tagged data ($a_k = \pm 1$ and $b_m = \pm 1$):

$S_A(x, y) = \{(a_k, t_k) | k=1,..n_x\}$, $S_B(x, y) = \{(b_m, t'_m) | j=1,..n_y\}$

2. Using synchronized time-windows of width W and keeping only those with no click at all or a click on one of Alice's or/and Bob's detectors new samples are constructed:

$S_A(x, y, W) = \{a_s \mid s=1,\ldots N_x\}$, $S_B(x, y, W) = \{b_t \mid t = 1,\ldots N_y\}$

$\mathbf{a_s = 0, \pm 1}$ **and** $\mathbf{b_t = 0, \pm 1.}$

3. Keeping only synchronized time-windows in which both Alice and Bob observed a click a new sample is constructed:

$S'_{AB}(x, y, W) = \{(a_r, b_r) \mid r=1,\ldots N_{xy}\}$ $\mathbf{a_r = \pm 1}$ **and** $\mathbf{b_r = \pm 1.}$

Data analysis and non–signaling

- If samples constructed in the step 2 are used:

$\langle A \,|x, y, W\rangle \approx \langle A|\, x, y', W\rangle$; $\langle B \,|x ,y, W\rangle \approx \langle B|\, x, y, W\rangle$

Einsteinian non -signaling (parametric independence) is not violated in SPCE

- To test CHSH inequality we have to estimate :

$$\langle A'B'|\, x, y, W\rangle,\ \ \langle A'\, B'|\, x, y', W\rangle,$$

$$\langle A'\, B'|\, x', y, W\rangle \text{ and } \langle A'B'|\, x', y', W'\rangle$$

using samples constructed in the step 3.

Now $\langle A'\rangle$ and $\langle B'\rangle$ depend on distant settings:

$$\langle A'|\, i, j\rangle \neq \langle A'|\, i, j'\rangle \quad \text{and} \quad \langle B'|\, i, j\rangle \neq \langle B'|\, i, j\rangle$$

where i=x or x' and j=y or y'

Standard CHSH

- If there exists a joint probability distribution of dichotomic random variables (A_1,A_2,B_1,B_2) then:

$$S = < A_1 B_1 > + < A_1 B_2 > + < A_2 B_1 > - < A_2 B_2 > \leq 2$$

Since $\left\langle A_i B_j \right\rangle = \left\langle B_j A_i \right\rangle$, **CHSH** inequality is **NCI for a 4-cyclic system** :

$$B_2 - A_1 - B_1 - A_2 - B_2$$

$$S = < B_2 A_1 > + < A_1 B_1 > + < B_1 A_2 > - < A_2 B_2 > \leq 2$$

CbD description of SPCE

- We have **4 experimental contexts** **(11)…(22)**:

  $A'_1B'_1$, $A'_1B'_2$, $A'_2B'_1$ and $A'_2B'_2$

Thus: $A'_1B'_2 = A_{12}B_{12}$ …where 'red labels' indicate the contents measured in the context (**12**).

- The variables : A_{11} and A_{12} ; A_{21} and A_{22} ; B_{11} and B_{21} ; B_{12} and B_{22} **are *stochastically unrelated***

- Moreover : **we have a system of *inconsistently connected random variables*** (NCC)

$$\langle A_{ij} \rangle_m \neq \langle A_{ij'} \rangle_m, \quad \langle B_{ij} \rangle_m \neq \langle B_{i'j} \rangle_m$$

For a NCC system CHSH does not exist

- **Instead of a 4-cyclic system** (A_1, A_2, B_1, B_2), we have a **non-cyclic system X'** of **8 different random variables** labelled by their contexts:

$A_{11} – B_{11}$; $A_{12} – B_{12}$; $A_{21} – B_{21}$; $A_{22} – B_{22}$

- **The only inequality** which may be derived without additional assumptions is:

$$S' = \langle A_{11}B_{11} \rangle + \langle A_{12}B_{12} \rangle + \langle A_{21}B_{21} \rangle - \langle A_{22}B_{22} \rangle \leq 4$$

Maximal coupling

- CbD imposes a counterfactual joint probability distribution upon the system X'
- If such probability distribution is imposed, the **expectations** $< A_{11} A_{12} >$, $<A_{21} A_{22}>$, $<B_{11}B_{21}>$, $<B_{12}B_{22}>$ are defined and we have a new **8-cyclic system**:

$$A_{11} - B_{11} -- B_{21} - A_{21} -- A_{22} - B_{22} -- B_{21} - A_{21} -- A_{11}$$

- CbD imposes maximal couplings between stochastically unrelated variables

$$\langle A_{11} A_{12} \rangle = 1 - |\langle A_{11} \rangle_m - \langle A_{12} \rangle_m| \text{ etc}$$

Modified CHSH in CbD

- **We start with NCI for our** 8-cyclic system:

$$S_4 = S' + \langle A_{11}A_{12} \rangle + \langle A_{21}A_{22} \rangle \ldots + \langle B_{12}B_{22} \rangle \leq 8 - 2$$

- After **replacing $\langle A_{11}A_{12} \rangle, \ldots \langle B_{12}B_{22} \rangle$ by maximal couplings** we obtain the modified CHSH inequality:

$$S_4 = \langle A_{11}B_{11} \rangle + \langle A_{12}B_{12} \rangle + \langle A_{21}B_{21} \rangle - \langle A_{22}B_{22} \rangle + D_4 \leq 2$$

$$D_4 = |\langle A_{11} \rangle - \langle A_{12} \rangle| + \ldots + |\langle B_{12} \rangle - \langle B_{22} \rangle|$$

Testing the contextuality of X'

- Following Kujala-Dzhafarov-Larsson we construct a conservative **Bonferroni confidence interval for S_4**:

$$I_{\alpha}(S_4) = I_{\frac{\alpha}{8}}(<A_{11}B_{11}>) + \ldots I_{\frac{\alpha}{8}}(<A_{21}B_{21}>) - I_{\frac{\alpha}{8}}(<A_{22}B_{22}>) + I_{\frac{\alpha}{2}}(\mathrm{D}_4) \leq 2$$

- If ***I_α (S_4)=[l_α,u_α]*** is an estimated (1-α)100% confidence interval then there is **(1-α)100% chance** that the value of S_4 is included in I_α (S_4).
- **[a, b] + [c, d]=[a + c, b+ d] , -[a, b]= [-b, -a]**

Kujala, J.V., Dzhafarov, E.N., Larsson, J-Å (2015). Necessary and sufficient conditions for extended noncontextuality in a broad class of quantum mechanical systems. Physical Review Letters 115:150401.

Significance test

- If the **lower bound** of I_α $(S_4)=[l_\alpha,u_\alpha]$ **is greater than *2***, then with $(1-\alpha)100\%$ confidence, we conclude that **X'** is not only inconsistently connected (NCC) but also **contextual** (**does not allow a maximal non-contextual description**) .
- If the **upper bound** of I_α (S_4) is **smaller than 2** then we conclude with $(1-\alpha)$ 100% confidence that X' **allows the maximal non-contextual description.**
- **Inconsistent connectedness is already the first manifestation of contextuality as we define it.**

Contextuality as the rule not the exception

- In SPCE the context dependent step 3 of experimental protocol does not depend on how signals are correlated at the source.

- Bohr- *contextuality* does not depend on particular chosen experimental settings.

- New SPCE experiments using non-entangled photonic sources should give the answer to the following question :

  What is more important cause of the violation of Bell-type inequalities: an entanglement of the incoming signals and a choice of particular settings or Bohr-contextuality and context dependent experimental protocols?

Contextuality confirmed by an Italian group

The results of the experiment performed by Iannuzzi, Francini, Messi and Moricciani seem to confirm our intuition .

> *"We present a **Bell-type polarization experiment using two independent sources** of polarized optical photons, and detecting the temporal coincidence of pairs of uncorrelated photons which have never been entangled in the apparatus. **The outcome of the experiment gives evidence of violation of the Bell-like inequalities**."*

M. Iannuzzi, R. Francini, R. Messi, D. Moricciani:Bell-type Polarization Experiment With Pairs Of Uncorrelated Optical Photons, Physics Letters A 384 (2020) 126200;Doi:10.1016/j.physleta.2019.126200 (arXiv:2002.02723 [quant-ph])

Realism versus contextuality

- Realism or counterfactual definiteness (CDF): measuring devices register values of physical observables existing independently whether they are measured or not.(They are ontic)

- Contextuality: the values of contextual physical observables such as a spin projections are created in the interaction of the physical system with the measuring apparatus and they do not exist before the measurement. (They are epistemic)

Conclusions

- The results of Bell tests , **analyzed using CbD** , confirm the importance of contextuality
- **Einsteinian non-signaling is not violated** .
- **The inconsistent connectedness and the violation of Bell-type inequalities** does not allow to make any statements about local causality, non-locality of Nature, *superdeterminism* and experimenters' freedom of choice.

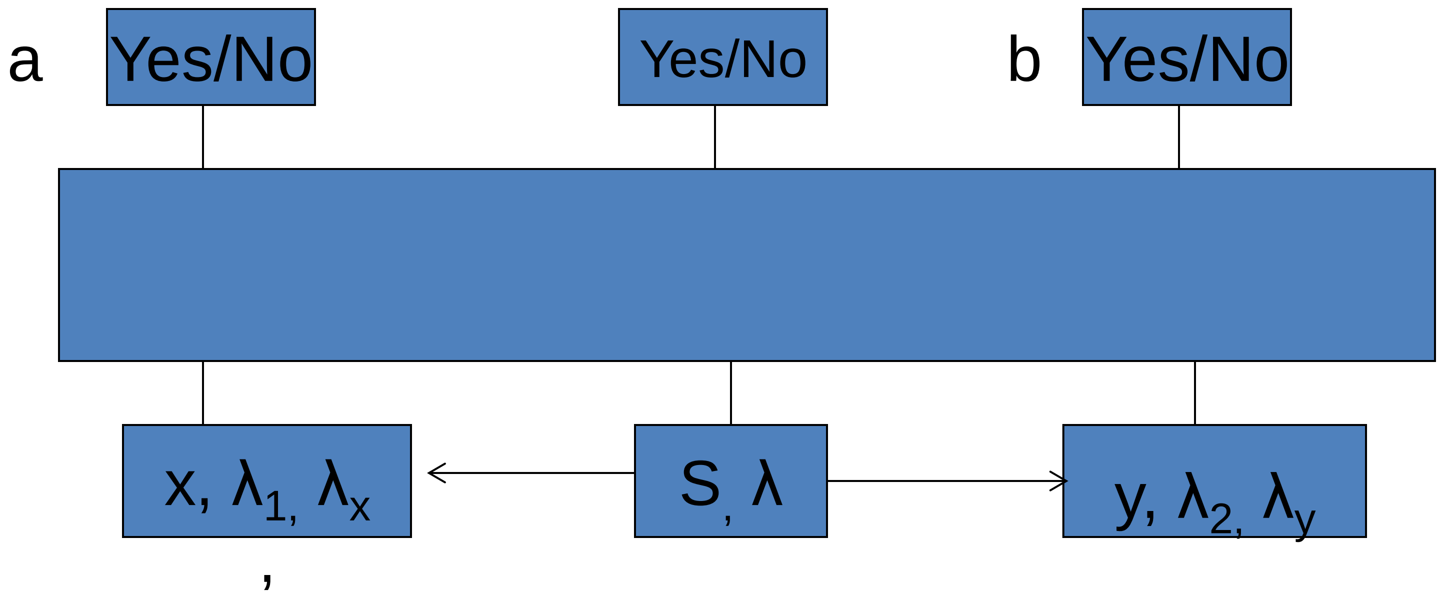
Kupczynski M., Can we close the Bohr-Einstein quantum debate?, Phil.Trans.R.Soc.A., 2017, 20160392., DOI: 10.1098/rsta.2016,0392

Kupczynski M., Is the Moon there when nobody looks: Bell inequalities and physical reality, Front. Phys., 23 September 2020 | https://doi.org/10.3389/fphy.2020.00273

Thank you

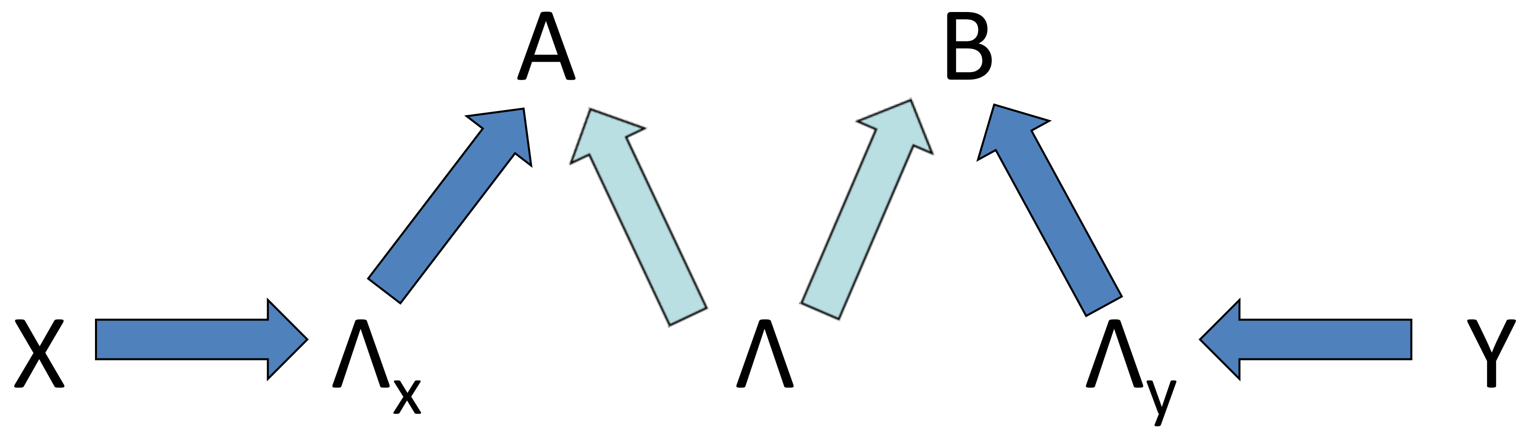
Supplementary material

Contextual **causally local** explanation of correlations



$a=\pm 1$ or 0 is <u>determined</u> locally by the values of λ_1 and λ_x describing the signal S_1 and the measuring device x. Similarly $b = \pm 1$ or 0 is <u>determined</u> by λ_2 and λ_y .

Contextual Causal Diagram



Experimenter has a free will: the settings (x, y) are chosen in any way he wants but parameters Λx and Λy depend on settings (x , y) chosen.

Simple contextual probabilistic model

- (λ_1 , λ_2) describe correlated signals arriving to PBS-D
- (λ_x , λ_y) describe PBS-D modules at a moment of interaction.
- Click or no click are produced in locally causal way and are coded by: $A_x(\lambda_1 ,\lambda_x)=0, \pm 1$ and $B_y (\lambda_2 ,\lambda_y)=0, \pm 1$

Setting-dependent expectation values :

$$E(A,B \,|\, \mathrm{x},\mathrm{y}) = \sum_{\lambda \in \Lambda_{xy}} \mathrm{A}_x(\lambda_1,\lambda_x)\mathrm{B}_y(\lambda_2,\lambda_y)P_x(\lambda_x)P_y(\lambda_y)\mathrm{P}(\lambda_1,\lambda_2)$$

where $\lambda=(\lambda_1,\lambda_2,\lambda_x,\lambda_y)$, $\lambda \in \Lambda_{xy} = \Lambda_1 \times \Lambda_2 \times \Lambda_x \times \Lambda_y$

CHSH-BELL PROOFS

FATAL CONTEXTUALITY LOOPHOLE

THE EXISTENCE OF A COMMON PROBABILITY IS TAKEN FOR GRANTED **BUT**

$$\Lambda_{xy} \neq \Lambda_{x'y} \neq \Lambda_{xy'} \neq \Lambda_{x'y'} \neq \Lambda$$

In our model experiments in different settings are described by **disjoint** probability spaces and **dedicated** probability distributions

Describing the data using our model

Samples obtained in **the step 2** of the protocol

Setting independent single counts of Alice

(1) $$E(A|\mathrm{x}) = \sum_{\lambda_1,\lambda_x\lambda_2} \mathrm{A}_x(\lambda_1,\lambda_x)P_x(\lambda_x)\mathrm{P}(\lambda_1,\lambda_2)$$

and Bob

(2) $$E(\mathrm{B}|\mathrm{y}) = \sum_{\lambda_1,\lambda_y\lambda_2} \mathrm{B}_y(\lambda_2,\lambda_y)P_y(\lambda_y)\mathrm{P}(\lambda_1,\lambda_2)$$

No violation of Einsteinian non-signaling.

Inconsistent connectedness of the data obtained in the step 3 explained

Setting dependent marginal expectations

(1')

(2') $$E(A\,|\,\mathrm{x},\mathrm{y}) = \sum_{\lambda\in\Lambda'_{xy}} \mathrm{A}_x(\lambda_1,\lambda_x)P_x(\lambda_x)P_y(\lambda_y)\,\mathrm{P}(\lambda_1,\lambda_2)$$

$$E(B\,|\,\mathrm{x},\mathrm{y}) = \sum_{\lambda\in\Lambda'_{xy}} B_y(\lambda_2,\lambda_y)P_x(\lambda_x)P_y(\lambda_y)\,\mathrm{P}(\lambda_1,\lambda_2)$$

where $\Lambda'_{xy} = \{\lambda \in \Lambda_{xy} \,|\, A_x(\lambda_1, \lambda_x) \neq 0 \text{ and } B_y(\lambda_2, \lambda_y) \neq 0\}$.

In general **$E(A|x,y) \neq E(A|x,y')$** and **$E(B|x,y) \neq E(B|x',y)$**

Our model contains a sufficient number of **free parameters** to explain any correlations.

Details may be found in

Kupczynski M., Can we close the Bohr-Einstein quantum debate?, Phil.Trans.R.Soc.A., 2017, 20160392., DOI: 10.1098/rsta.2016,0392

Kupczynski M., Is the Moon there when nobody looks: Bell inequalities and physical reality, Front. Phys., 23 September 2020 | https://doi.org/10.3389/fphy.2020.00273