

The 5th Workshop:
Quantum Contextuality in Quantum Mechanics and Beyond
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Abstracts (alphabetic order)

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Every non-local game leads to quantum advantage with shallow circuits

Theme: contextuality vs causality

An unconditional quantum advantage result for bounded depth and fan-in (shallow) circuits was proved by Bravyi, Gosset, and König. An interesting feature of their proof is the use of a non-local game. We show that every non-local game can be turned into an unconditional quantum advantage result with shallow circuits in a systematic way. Our main contribution is a technique that uses teleportation to distribute and measure the qudits of an entangled state at many different locations in a circuit.

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Combining contextuality and causality I

(joint work with Rui Soares Barbosa and Amy Searle)

Theme: contextuality vs causality

In this talk, we describe ongoing work which develops a unified framework for studying contextuality and causality. This extends the sheaf-theoretic framework for contextuality introduced by Abramsky and Brandenburger, using ideas from computer science, in particular concrete data structures (Kahn-Plotkin) and event structures (Plotkin-Nielsen-Winskel), which have been used to model the causal unfoldings of sequential and concurrent computations. Our framework accommodates general contextuality scenarios on causal backgrounds, and allows for adaptive processes, such as feed-forward in measurement-based quantum computation. It also subsumes the description of classical causal networks. We extend the treatment of Bell inequalities, contextual fraction, and resource notions for contextuality to this general setting.

This talk provides an introduction to these ideas. Further developments will be described in the talk by Amy Searle.

Rui Soares Barbosa

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Causal contextuality and adaptive MBQC

(joint work with Cihan Okay)

Theme: contextuality vs causality

Contextuality has been linked to quantum advantage in various setups. In particular, Raussendorf [1] considered a specific model of measurement-based quantum computation where a (possibly adaptive) classical control is restricted to performing \mathbb{Z}_2 -linear operations, being supplemented by access to a resource in the form of an empirical model (i.e. correlation table) in an $(n, 2, 2)$ Bell-type scenario (n sites, 2 measurement settings, 2 outcomes). It is shown that if the MBQC program *deterministically* implements a non-linear Boolean function then the resource correlation must be strongly contextual. However, in the presence of adaptivity – and if one is interested in a specific computation with a given dependency structure between sites rather than on at whole class of computations achievable by a single resource – there is arguably no reason to assume no-signalling to sites in the past, or to expect a classical counterpart to yield measurement outcomes independently of such prior measurements. For a fixed causal/adaptivity structure, we give sufficient conditions on the computed function that imply the resource is strongly causal in the sense of Gogioso and Pinzani [2]. This result generalises Raussendorf’s in the case of a flat causal order.

[1] R. Raussendorf, Contextuality in measurement-based quantum computation, *Physical Review A* 88: 022322 (2013).

[2] S. Gogioso and N. Pinzani, The sheaf-theoretic structure of definite causality, 18th International Conference on Quantum Physics and Logic (QPL 2021), M. Backens and C. Heunen (eds), *Electronic Proceedings in Theoretical Computer Science* 343: 301–324 (2021).

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Agreement and disagreement in a non-classical world

(joint work with Patricia Contreras-Tejada, Pierfrancesco La Mura, Giannicola Scarpa, and Kai Steverson)

Theme: negative probabilities

The Agreement Theorem (Aumann, 1976) states that if two Bayesian agents start with a common prior, then they cannot have common knowledge that they hold different posterior probabilities of some underlying event of interest. In short, the two agents cannot “agree to disagree.” This result applies in the classical domain where classical probability theory applies. But in non-classical domains (such as the quantum world), classical probability theory does not apply, and so we cannot assume that the same result holds when agents observe non-classical phenomena. Inspired by their use in quantum mechanics (Wigner, 1932; Dirac, 1942; Feynman, 1987; Wootters, 1987), we employ signed probability measures (“quasi-probabilities”) to investigate the epistemics of the non-classical world and ask, in particular: What conditions lead to agreement or allow for disagreement when agents may use signed probabilities?

- [1] Aumann, R., “Agreeing to Disagree,” *Annals of Statistics*, 4, 1976, 1236-1239.
- [2] Dirac, P., “The Physical Interpretation of Quantum Mechanics,” *Proceedings of the Royal Society of London (Series A: Mathematical and Physical Sciences)*, 180, 1942, 1-40.
- [3] Feynman, R., “Negative Probability,” in B. Hiley and F. Peat (eds.), *Quantum Implications: Essays in Honour of David Bohm*, Routledge and Kegan Paul, 1987, 235-248.
- [4] Wigner, E., “On the Quantum Correction for Thermodynamic Equilibrium,” *Physical Review*, 40, 1932, 749-759.
- [5] Wootters, W., “A Wigner-Function Formulation of Finite-State Quantum Mechanics,” *Annals of Physics*, 176, 1987, 1-21.

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Optimization for classical and quantum operations in time

(joint work with Mirjam Weilenmann and Miguel Navascués)

Themes: contextuality vs causality, negative probabilities

Motivated by the problem of temporal correlations generated by measurements on classical and quantum systems of bounded dimension, i.e., finite-state machines, we explore several optimization methods for sequential operations. First, we present simple methods to optimize over the set of classical and quantum finite-state machines to maximize the probability for finite sequences. These methods provide an explicit solution, i.e., a valid machine, and consequently a lower bound to the global maximum. In the classical case, upper bounds on the global maximum can be obtained via a modification of Lasserre's method of polynomial optimization, specialized to the case of sequential operations. Such methods can be extended to the quantum case either via quasiprobability representations or via noncommutative polynomial optimization methods. These methods have a broader range of applications - beyond the problem of temporal correlations - to general tasks involving sequential operations.

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What is the relation between AVN, PT, and FN?

Theme: contextuality vs causality

All-vs-nothing (AVN) nonlocality, pseudo-telepathy (PT), and full nonlocality (FN) are three extreme forms of Bell nonlocality. A matrix of correlations produces AVN nonlocality if it yields a list \mathcal{L} of predictions with certainty about a result of an experiment without in any way disturbing it such that, using Einstein-Podolsky-Rosen criterion of element of reality, all but one of the predictions in \mathcal{L} lead to a conclusion which is in contradiction with the remaining prediction in \mathcal{L} . PT allows two (or more) spacelike separated players to always win a game which is impossible to win classically unless they communicate, hence suggesting a seemingly telepathic connection between them. A matrix of correlations has FN if the maximum local weight of all its possible decompositions in terms of local and nonsignaling correlations is zero. Quantum mechanics allows for AVN, PT, and FN. It is known that neither bipartite AVN nor PT can be achieved with qubit-qubit entanglement. It is known that PT cannot be achieved using only two-outcome measurements or if one of the players only has two measurement settings. Similarly, it is known that bipartite FN cannot be achieved in the simplest Bell scenario. In addition, the simplest known examples of bipartite AVN, PT, and FN are based on the same matrix of correlations. However, an open question is: What is the relation between AVN, PT, and FN? Here, we will address this question.

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Free choice, causality, contextuality, and signed measures

Themes: free choice vs local causality, contextuality vs causality, negative probabilities

A hidden variable model (HVM) satisfies the assumption of Free Choice (and is denoted HVM-FC) if the settings chosen by experimenters are independent of the values of the hidden variable. An HVM satisfies the assumption of Context-Independence (HVM-CI) if the outcomes of measurements are independent of settings for other measurements. If the measurements are spacelike separated, CI assumption is known as Local Causality. An HVM satisfies Bell-type criteria of contextuality/nonlocality (HVM-B) if and only if it is both HVM-FC and HVM-CI. We show, in complete generality, for any system of random variables with or without disturbance, that any HVM-CI can be reformulated as an HVM-FC, and vice versa. Moreover, an HVM unconstrained by either of these assumptions (HVM-Gen) can always be reformulated as an HVM-CI or HVM-FC (so, by itself, neither of the two assumptions is constraining). It follows that measures of the degree of FC and the degree of CI in an HVM can always be made the same. Contextuality-by-Default (CbD) approach allows one to separate two components of context-dependence in any HVM-FC formulation: (A) direct context-dependence (overt, causal) and (B) contextuality proper (hidden, non-causal). These two components can be separately measured, and in CbD we have developed three distinct measures of contextuality. One of them, CNT3, is based on the use of signed measures, i.e. quasi-probabilities allowed to be negative. Any HVM has a quasi-coupling, in which the HVM is represented by a set of variables with joint signed distribution. CNT3 is the minimal total variation of this distribution.

[1] Dzhafarov, E.N. (2022). Context-independent mapping and free choice are equivalent: A general proof. *J. Phys. A: Math. Theor.* 55:305304. + Corrigendum Note in *J. Phys. A: Math. Theor.* (2022) 55:4095020. (arXiv:2110.15910)

[2] Dzhafarov, E.N., Kujala, J.V. (2020). Systems of random variables and the Free Will Theorem. *Phys. Rev. Res* 2:043288. (arXiv:2007.13229)

[3] Dzhafarov, E.N., Cervantes, V.H., Kujala, J.V. (2017). Contextuality in canonical systems of random variables. *Phil. Trans. Roy. Soc. A* 375:20160389. (arXiv:1703.01252)

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Contextuality, nonlocality, causality, and the incompleteness of quantum mechanics

Themes: contextuality vs causality, free choice vs local causality

With the Nobel Prize attributed to Aspect, Clauser, and Zeilinger, the international scientific community acknowledged the fundamental importance of the experimental violation of Bell's inequalities [1]. It is however still debated what fails in Bell's hypotheses, leading to these inequalities, and usually summarized as "local realism", or maybe more appropriately "classical local realism". The most common explanation is "quantum non-locality", that remains however fully compatible with relativistic causality; this makes wondering whether any non-local phenomenon is really involved in these experiments.

In this talk we recapitulate another option, called "predictive incompleteness", closely related to the idea that the usual state vector ψ is incomplete indeed [2,3], as it was claimed by Einstein, Podolsky and Rosen. We also argue that ψ should be completed, not by looking for any "hidden variables", but rather by specifying the measurement context, as it was claimed by Bohr [4]; this is required to define actual probabilities over a set of mutually exclusive physical events. Finally we propose a possible unified mathematical framework including both quantum and classical physics, appearing as required incommensurable facets in the description of nature [5, 6].

[1] For an overview of Bell-test experiments until 2015, see: Alain Aspect, "Closing the Door on Einstein and Bohr's Quantum Debate", *Physics* 8, 123 (2015) <https://physics.aps.org/articles/v8/123>

[2] P. Grangier, "Contextual inferences, nonlocality, and the incompleteness of quantum mechanics", *Entropy* 23, 1660 (2021); <https://www.mdpi.com/1099-4300/23/12/1660>

[3] P. Grangier, "Why ψ is incomplete indeed: a simple illustration", <https://arxiv.org/abs/2210.05969>

[4] N. Farouki and P. Grangier, "The Einstein-Bohr debate: finding a common ground of understanding?", *Found. Sci.* (2021); <https://doi.org/10.1007/s10699-020-09716-7>

[5] P. Grangier, "Completing the Quantum Formalism in a Contextually Objective Framework", *Found. Phys.* 51,76 (2021) [arXiv:2003.03121]

[6] M. Van Den Bossche and P. Grangier, "Contextual unification of classical and quantum physics", <https://arxiv.org/abs/2209.01463>

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An operational approach to bi-locality

(joint work with Kelvin Onggadinata, Paweł Kurzyński, and Valerio Scarani)

Theme: negative probabilities

We investigate a generic bi-locality scenario, presented in [1,2], using joint (quasi) probability distributions and (quasi) stochastic processes. Our approach reveals overlooked operational connections between bi-locality, (super) quantum theories and quasi-probabilities.

[1] A. J. Short, S. Popescu, and N. Gisin, Phys. Rev. A 73, 012101 (2006).

[2] P. Skrzypczyk, N. Brunner, and S. Popescu, Phys. Rev. Lett. 102, 110402 (2009).

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State-independent contextuality with nonunit rank

(joint work with Pascal Höhn, Zhen-Peng Xu, Xiao-Dong Yu)

Theme: contextuality vs causality

Almost all known instances of quantum contextuality rely on measurements where the projectors have rank one. Indeed, this is sufficient if one only aims to produce contextual correlations and one allows that the measurements as well as the quantum state are specifically tailored to the task. But when one aims to characterize contextuality as a feature of measurements alone, that is, for state-independent contextuality, it is no clear that all contextuality scenarios can be realized using only rank-one projectors. In this talk I will discuss the prospects of contextuality with nonunit rank and present generic construction methods for scenarios which only exhibit state-independent contextuality if the projectors have nonunit rank.

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Copula-based analysis of contextuality

Theme: contextuality vs causality

Following the notation of Contextuality-by-Default (CbD), we consider a system of random variables indexed by contents (properties) and contexts, with random variables sharing a context being jointly distributed (observed together). In CbD, one considers couplings of all random variables, with a certain condition imposed on the subcouplings corresponding to connections (random variables sharing the same content) to allow analysis of contextuality under inconsistent connectedness (when marginal distributions of random variables in a connection are not the same across contexts) generalizing the requirement of identity subcouplings implied in traditional analysis when the system is consistently connected.

In the present variant of CbD, we propose to analyze the contextuality of a system of real-valued random variables based on the copulas of random variables sharing context. The copula represents the interdependencies of random variables by representing them as variablewise increasing transformations of jointly distributed $[0, 1]$ -uniform random variables. These increasing transformations encode how the values of random variables sharing a content relate to each other in different contexts. Therefore, in effect, the original system is transformed in a certain well-defined way into a new system satisfying consistent connectedness (all marginals $[0, 1]$ -uniform) and contextuality analysis can then be performed essentially in the traditional way for this new, simpler system.

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Quasi-probability approach to Kochen-Specker theorem

(joint work with Kelvin Onggadinata and Dagomir Kaszlikowski)

Theme: negative probabilities

It is not possible to construct a joint probability distribution (JPD) for any set of Kochen-Specker (KS) measurements, simply because the KS theorem states that there is no joint set of outcomes over which such distribution can be constructed. However, we show it is possible to construct a joint quasi-probability distribution (JQD) over any KS set. The idea is to relax the completeness assumption. Interestingly, the completeness emerges at the level of measurable marginal probability distributions, therefore our construction is consistent with any observable data. The emergence of completeness occurs due to the negativity of the JQD. Our approach allows to treat state-independent and state-dependent contextuality on equal footing and to use the negativity as a measure of nonclassicality in both scenarios.

- [1] K. Onggadinata, D. Kaszlikowski, and P. Kurzynski, arXiv:2210.06822 (2022)

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Addressing the compatibility loophole in the absence of space-like separation

(joint work with Boris Bourdoncle, Pierre-Emmanuel Emeriau, Damian Markham, Andreas Fyrrillas, Boris Bourdoncle, Alexandre Mainos, Kayleigh Start, Pierre-Emmanuel Emeriau, Nico Margaria, Martina Morassi, Aristide Lemaitre, Isabelle Sagnes, Petr Stepanov, Thi Huong Au, Sebastien Boissier, Niccolo Somaschi, Nicolas Maring, Nadia Belabas)

Theme: contextuality vs causality

Contextuality is often defined in terms of hidden variables, for which it forces a contradiction with the assumptions of parameter-independence and determinism. The former can be justified by the empirical property of non-signalling or non-disturbance, and the latter by the empirical property of measurement sharpness. However in realistic experiments neither empirical property holds exactly, which leads to possible objections to accepting contextuality as a form of nonclassicality, which we will refer to as the compatibility and sharpness loopholes. The compatibility loophole is especially problematic for contextuality experiments which do not enforce spacelike separation between devices. Moreover, these introduce knock-on vulnerabilities for supposed quantum advantages that rely on contextuality.

In Part 1 of this talk we introduce measures to quantify both properties, and introduce quantified relaxations of the corresponding assumptions. We prove a continuity result on the contextual fraction measure of contextuality to ensure its robustness to noise. We then bound the extent to which these relaxations can account for contextuality, via correction terms to the contextual fraction (or to any noncontextuality inequality), culminating in a notion of genuine contextuality, which is robust to experimental imperfections.

In Part 2 of the talk we describe an integrated photonic demonstration with an effective 2-qubit device combining a solid-state emitter and a glass chip. We neither have nor suppose spacelike separation, and instead account for information leakage to address the compatibility loophole and witness genuine contextuality. The method is thus suitable for upcoming compact scalable devices. We finally discuss the prospect of using the approach to achieve quantum advantage in practical applications.

Cihan Okay

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Simplicial quantum contextuality

(joint work with Selman Ipek and Aziz Kharoof)

Theme: contextuality vs causality

In this talk, I will introduce a new framework for contextuality based on simplicial sets, combinatorial models of topological spaces that play a prominent role in modern homotopy theory. Our approach extends measurement scenarios to consist of spaces (rather than sets) of measurements and outcomes and thereby generalizes nonsignaling distributions, formulated using sheaf theory, to simplicial distributions that describe distributions on spaces. Strong contextuality can be generalized suitably for simplicial distributions, allowing us to define cohomological witnesses that extend the earlier topological constructions restricted to algebraic relations among quantum observables to the level of probability distributions. This also extends an earlier approach to quantum contextuality based on group cohomology.

[1] arXiv:2204.06648

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Qubit from classical collision entropy

(joint work with Paweł Kurzyński and Dagomir Kaszlikowski)

Theme: negative probabilities

A central problem in quantum foundations is to derive quantum theory from a simple and physically motivated principles. The standard formalism, one that is built upon the complex Hilbert space formalism, consists of axioms that are incredibly ad-hoc and unintuitive. A desire for a more intuitive set of postulates arises in the hope that a simple and minimal formulation is sufficient for the construction of quantum theories. In this work, we propose a simple information theoretic postulate where we look for a generalized dynamics in which certain Renyi entropies remain invariant for any given state. We find that the classical collision entropy is conserved under this and by demanding continuity of dynamical evolution, we can recover the full qubit system and its quantum dynamics. Therefore, constructing the basic elements of quantum theories without invoking Hilbert space at all.

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Contextuality as a precondition for entanglement

(joint work with Otfried Gühne)

Theme: contextuality vs causality

Quantum theory features several phenomena which can be considered as resources for information processing tasks. Some of these effects, such as entanglement, arise in a non-local scenario, where a quantum state is distributed between different parties. Other phenomena, such as contextuality, can be observed if quantum states are prepared and then subsequently measured. There is a clear distinction between non-local and sequential scenarios: Contextuality experiments have an explicit causal structure since preparations precede measurements, while in the case of entanglement the measurement events can be spacelike separated.

Here we provide an intimate connection between different resources by proving that entanglement in a non-local scenario can only arise if there is preparation & measurement contextuality in a sequential scenario derived from the non-local one by remote state preparation. Moreover, the robust absence of entanglement implies the absence of contextuality. As a direct consequence, our result allows to translate any inequality for testing preparation & measurement contextuality into an entanglement test. In addition, entanglement witnesses can be used to obtain novel noncontextuality inequalities which can be violated in the corresponding non-local experiment without any causal structure.



Figure 1: In the non-local scenario, one considers probability distributions of the type $p(a, b|\rho_{AB}) = \text{Tr}[(M_a \otimes N_b)\rho_{AB}]$, where M_a and N_b describe measurements. In the sequential scenario, quantum state σ is prepared and measured, leading to the probabilities $p(a|\sigma) = \text{Tr}(\sigma M_a)$. We show that entanglement in the non-local scenario is practically equivalent to noncontextuality in the sequential scenario, if the state $\sigma \sim \text{Tr}_B[(\mathbb{1}_A \otimes N_b)\rho_{AB}]$ is prepared remotely by performing a measurement on Bob's part of the bipartite state.

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Synchronous observation of Bell nonlocality and state-dependent contextuality

(joint work with Peng Xue, Lei Xiao, Gabriel Ruffolo, André Mazzari, Tassius Temistocles, Marcelo Terra Cunha)

Themes: contextuality vs causality, negative probabilities

Bell nonlocality and Kochen-Specker contextuality are two remarkable nonclassical features of quantum theory, related to strong correlations between outcomes of measurements performed on quantum systems. Both phenomena can be witnessed by the violation of certain inequalities, the simplest and most important of which are the Clauser-Horne-Shimony-Holt (CHSH) and the Klyachko-Can-Binicioglu-Shumovski (KCBS), for Bell nonlocality and Kochen-Specker contextuality, respectively. It has been shown that, using the most common interpretation of Bell scenarios, quantum systems cannot violate both inequalities concomitantly, thus suggesting a monogamous relation between the two phenomena. In this work, we show that the joint consideration of the CHSH and KCBS inequalities naturally calls for the so-called generalized Bell scenarios, which, contrary to the previous results, allows for their joint violation. In fact, this result is not a particular feature of such inequalities: we provide very strong evidence that there is no monogamy between nonlocality and contextuality in any scenario where both phenomena can be observed. Also, we have implemented a photonic experiment to test the synchronous violation of both CHSH and KCBS inequalities; our results agree with the theoretical predictions, thereby providing experimental proof of the coexistence of Bell nonlocality and contextuality in the simplest scenario, and shedding new light on further explorations of nonclassical features of quantum systems.

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Optimal measurement structures for contextuality applications

(joint work with Yuan Liu, Karol Horodecki, Monika Rosicka and Paweł Horodecki)

Theme: contextuality vs causality

Applications of the foundational Kochen-Specker (KS) theorem have attracted much interest recently. Here, we show that measurement structures within KS proofs termed gadgets provide an optimal toolbox for contextuality applications including (i) constructing classical channels exhibiting entanglement-assisted advantage in zero-error communication, (ii) finding optimal tests for contextuality-based semi-device-independent randomness generation, and (iii) identifying large separations between quantum theory and binary generalised probabilistic theories. Finally, we introduce a higher-order generalisation of gadgets that we show exist within general KS proofs, and use them to construct novel proofs of the KS theorem.

[1] Y. Liu, R. Ramanathan, K. Horodecki, M. Rosicka, P. Horodecki. *Optimal Measurement Structures for Contextuality Applications*. arXiv:2206.13139 (2022).

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Syntactic ambiguities of natural language in sheaf theoretic and CbD models of contextuality

(joint work with Samson Abramsky)

Theme: contextuality vs causality

Natural language ambiguities give rise to probability distributions that can be studied by the mathematics of Quantum contextuality. These ambiguities can be catalogued into two groups: syntactic and semantic. Previously, in CQMB 2020, we found sheaf theoretic and CbD contextual examples of instances of the semantic group. In this work, we focus on syntax and model the anaphoric ambiguities of discourse.

Anaphoric discourse ambiguities arise from the possibility of a pronoun in the current discourse referring to more than one noun of a previous discourse. Examples of these ambiguities have given rise to the famous Winograd Schema Challenge in Natural Language Processing [1,2], which improves on the Turing test. We construct a schema for these ambiguities in such a way that the instances exhibit logical quantum-like contextuality and conduct two experiments. In experiment (1), we take advantage of the neural word embedding engine BERT to instantiate the schemas to natural language and extract probability distributions for the instances. In experiment (2), we collect probabilities for an instance of the schema using the Amazon Turk crowd sourcing engine. The probabilities are analyzed in the CbD framework and in a recently developed signalling fraction of the sheaf-theoretic framework. Plenty of examples exhibited contextuality. Our hope is that these experiments will pave the way to use the quantum advantage in natural language processing.

[1] Winograd, Terry (1972). "Understanding Natural Language", *Cognitive Psychology*. 3 (1): 1-191.

[2] Levesque, Hector; Davis, Ernest; Morgenstern, Leora (2012). "The Winograd Schema Challenge", *Proceedings of the Thirteenth International Conference on Principles of Knowledge Representation and Reasoning*, 552-561.

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Combining contextuality and causality II

(joint work with Samson Abramsky and Rui Soares Barbosa)

Theme: contextuality vs causality

Following on from the work presented by Samson Abramsky, we describe how the framework developed therein can be used to classify the non-classicality of new examples in the literature as contextual. In doing so, we gain a deeper understanding of the assumptions of classicality which are implicitly made. In particular, we classify two types of event enablings (in the language of Plotkin-Nielsen-Winkel) which could be present: at the system level and at the level of the verifier or experimenter— together these enablings describe how measurements may unfold in time. In doing so, the contextual setups and the definite causal setups of Abramsky-Brandenburger and Gogioso-Pinzani, respectively, as well as examples in which outcomes of measurements are fed forward to determine the choice of future measurements, can be understood as special cases of this general framework. The measurement scenarios of contextual setups and definite causal setups are in fact those generated by repeated application of series and parallel composition operators to simple ‘building block’ measurement scenarios. We see that classicality, arising as a global section of the relevant presheaf, depends crucially on (i) whether measurements have total or partial access to those measurement which occurred in their history and (ii) to which combinations of measurements the experimenter has access, as might be enforced by feedforward of measurement outcomes.

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Impossibility theorem for extending contextuality to disturbing systems

(joint work with Matt Jones, Elie Wolfe, and Bárbara Amaral)

Theme: contextuality vs causality

Recently there has been interest, and impressive progress, in extending the definition of contextuality to systems with disturbance. We prove here that such an endeavor cannot simultaneously satisfy the following core principles of contextuality: (1) Measuring more information cannot change a contextual system to a noncontextual one. (2) Classical post-processing cannot create contextuality: appending new observables that are functions of given observables cannot change a noncontextual system to a contextual one. (3) The joint realization of two statistically independent noncontextual systems is noncontextual. (4) Determinism cannot create contextuality: Any deterministic system is noncontextual, and adding deterministic observables to a noncontextual system cannot yield a contextual one. We also prove the same result without Principle 3, under a slightly stronger version of Principle 4. Moreover, our results hold for restricted extensions of contextuality that apply only to systems of binary observables. Finally, we analyze several particular proposals and identify which of our axioms they obey and which they violate.

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Causality and signaling of garden-path sentences

(joint work with Samson Abramsky)

Theme: contextuality vs causality

Natural language ambiguities give rise to probability distributions and these can be studied in the mathematical frameworks of Quantum contextuality. Such ambiguities are manifold, but can be catalogued into two broad groups: syntactic and semantic. In [1], sheaf theoretic and CbD contextual examples of instances of the semantic group were discussed. In this work, we focus on syntax and model parsing ambiguities via the so-called garden-path effect.

Garden-path sentences first appeared in [2] and sparked a research program on the biological and cognitive bases of human interaction patterns. The 80's and 90's, saw a wave of Psycholinguistic experiments with the aim of classifying these ambiguities and measuring their cognitive dissonance levels. Recently, Natural Language Processing researchers have found correlations between the statistics learnt by deep neural network language models and human reading times [3]. They failed, however, when it came to predicting degrees of difficulty and reanalysis.

We show that garden-path sentences can be modelled and analysed by the signaling and causal fractions from the sheaf theoretic framework for contextuality, and by the degree of direct influences of the Contextuality-by-Default (CbD) model. We consider incrementally increasing subphrases as contexts, and study their local grammatical structures. The consistency of these local structures from one context to the other is calculated using the signaling and causal fractions, their degree of direct influences on one another is computed via CbD's Δ . Our goal is to use neural language models statistics and relate the predictions of the contextuality models to human reading times, as well as to the levels of difficulty of garden path sentences.

[1] Wang D., Sadrzadeh M., Abramsky S., Cervantes V. (2021). "Analysing Ambiguous Nouns and Verbs with Quantum Contextuality Tools". *Journal of Cognitive Science*, 22 (3) 391-420.

[2] Bever, T. (1970). "The Cognitive Basis for Linguistic Structures".

[3] Van Schijndel M., Linzen, T. (2021). "Single-stage prediction models do not explain the magnitude of syntactic disambiguation difficulty". *Cognitive science*, 45(6), e12988.