From contextuality to quantum advantage with shallow circuits

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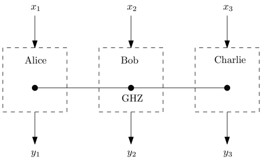
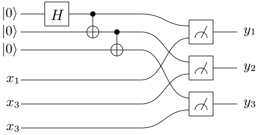
- Quantum advantage with shallow circuits, Bravyi et al. (Science 2018)
  - Unconditional separation for circuits of bounded depth and fan-in.
  - Introduced a shallow quantum circuit $\{Q_n\}_{n\in\mathbb{N}}$ and *relational* problems $\{A_n\}_{n\in\mathbb{N}}$.
- The GHZ non-local game



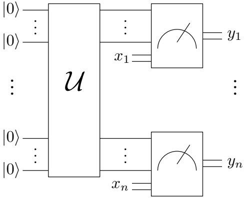


Overview

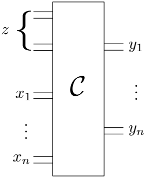
- Quantum non-locality can be recast with circuits
- Shallow classical circuits extend local hidden variables in a "limited sense".
- Detecting shallow circuits with simulations.
- Constructing examples.

Nonlocality and shallow circuits

- Circuits for empirical model $e(x_1, \ldots, x_n)(y_1, \ldots, y_n)$:

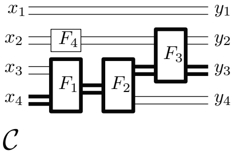


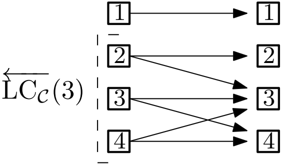
quantum circuit



"hidden variable model"

- "Shallow": bounded lightcones:





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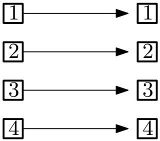
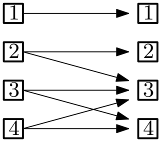
- Local vs shallow hidden variable:



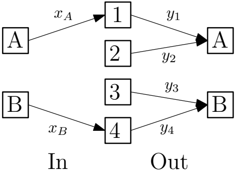
$\mathrm{LC}_{\mathcal{C}} = \mathrm{id}$



$|\overleftarrow{\mathrm{LC}}_{\mathcal{C}}(i)| << n$

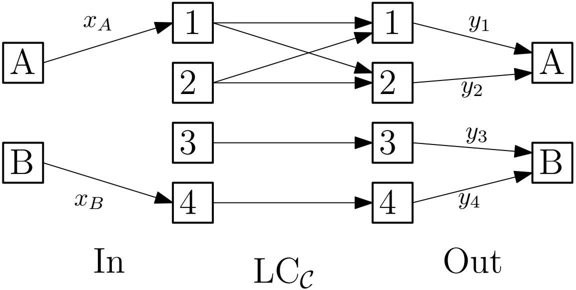
Simulations

- Simulation from n-partite e to m-partite e:
  - Communicate inputs from $[m]$ to $[n]$ and outputs from $[n]$ to $[m]$:



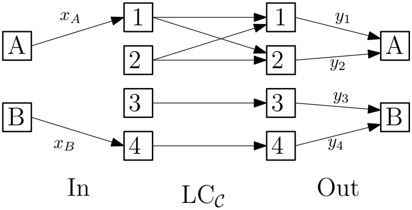
  - "Communication relations": $\mathrm{In} \subset [m] \times [n]$, $\mathrm{Out} \subset [n] \times [m]$
  - No-communication: $\mathrm{In}; \mathrm{Out} \subset \mathrm{id}_{[m]}$.

Case: A particular circuit $\mathcal{C}$



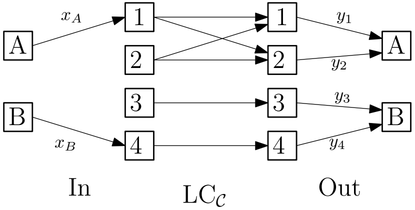
- If
  - e simulates e'
  - $\mathcal{C}$ “generates” e
  - $\mathrm{In}; \mathrm{LC}_{\mathcal{C}}; \mathrm{Out} \subset \mathrm{id}_{[m]}$
- Then e' is non-contextual.

Case: A class of circuits



- Probabilistic simulation: In and Out are sampled randomly from a probability distribution.
- If:
  - e simulates e'
  - e is generated by *some* circuit $\mathcal{C}$
  - For all circuits $\mathcal{C}$: $\mathrm{Prob}(\mathrm{In}; \mathrm{LC}_{\mathcal{C}}; \mathrm{Out} \subset \mathrm{id}) \leq \epsilon$
- Then $\mathrm{CF}(e') \leq \epsilon$.

Detecting shallow circuits



- Lemma (adapted from BGK): If
  - e simulates e'
  - e is generated by shallow circuit $\mathcal{C}$: $|\overleftarrow{\mathrm{LC}}_{\mathcal{C}}(i)| \leq K$.
  - $|\overleftarrow{\mathrm{Out}}(i)| \leq L$ is small.
  - $\overleftarrow{\mathrm{In}}(i)$ is "sufficiently uniformly distributed" when $\overleftarrow{\mathrm{Out}}(i')$ is fixed
- Then $\mathrm{CF}(e') \leq m^2 K L \epsilon$.

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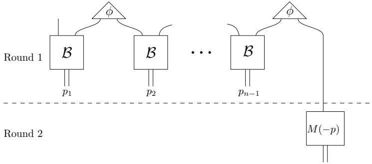
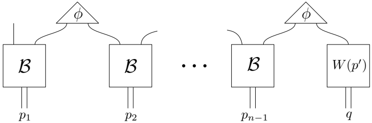
Teleportation on a line



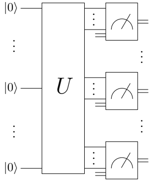
Two-round teleportation protocol



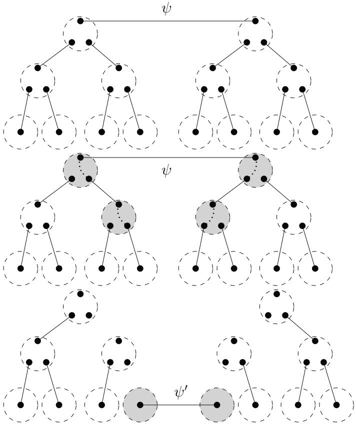
Single-round teleportation protocol.

- $\phi, \mathcal{B}$: maximally entangle state, Bell basis
- $M(-p) := W(-p)MW(-p)^\dagger$
  - $W(-p)$ is a Weyl operator
  - $p := p_1 + \dots p_{n-1}$.
- $q' := q - [p, p']$:
  - $[p, p']$: Symplectic product

Teleportation in shallow circuits



- U: Prepare ψ and many maximally entangled states.
- Measurement gates: Bell basis or Weyl operator measurements.



Result

- e is generated by Weyl measurements on an n-qudit state
- Theorem:
  - $\mathcal{Q}(\psi, G)$ has "small depth and fan-in" (independent of $|G|$
  - Any equivalent classical circuit has depth D and fan-in K such that:

$$K^D \geq \mathrm{CF}(e) \frac{|G|}{\mathrm{rad}(G) n^2}$$

Beyond Weyl operator measurements

- Use two-round teleportation protocol.
- Interactive shallow circuits: