An operational approach to bi-locality

A) A few remarks about negative probabilities

B) Bi-locality as a process

C) Immediate future research

$\boxed{\text{Prague, December '22}}$

A

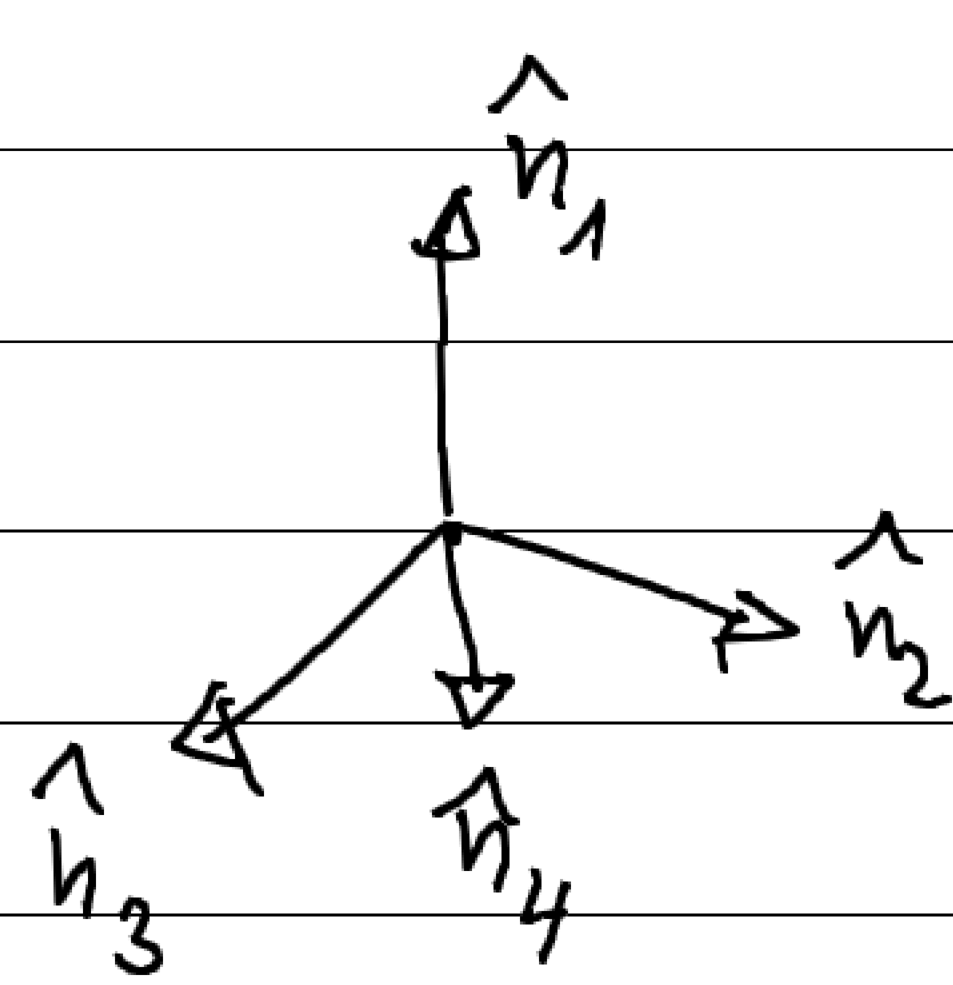
Quantum mechanics

$\Psi \longrightarrow \vec{p}(\Psi) \geq 0$ (always!)

$U \longrightarrow S(U)$, bi-stochastic with negative trans. prob.

Example: qubit $\longrightarrow \vec{s}$, Bloch vector

$\vec{s} \longrightarrow p_k(\vec{s}) = \frac{1}{4}(1 + \hat{n}_k \cdot \vec{s})$



$$U|\psi\rangle \longrightarrow \hat{O}\vec{S}$$

$$\hat{O} \longrightarrow S'_{kl}(\hat{O}) = 1/4 + 3/4\,\hat{n}_k \cdot (\hat{O}\hat{n}_l)$$

$$S_{\vec{\rho}} \longrightarrow \left[\begin{array}{c|c} S_+^{(1)} & 0 \\ \hline 0 & S_+^{(2)} \end{array}\right] \vec{\rho} \otimes \begin{bmatrix} 1+\delta \\ -\delta \end{bmatrix}$$

(nebit)

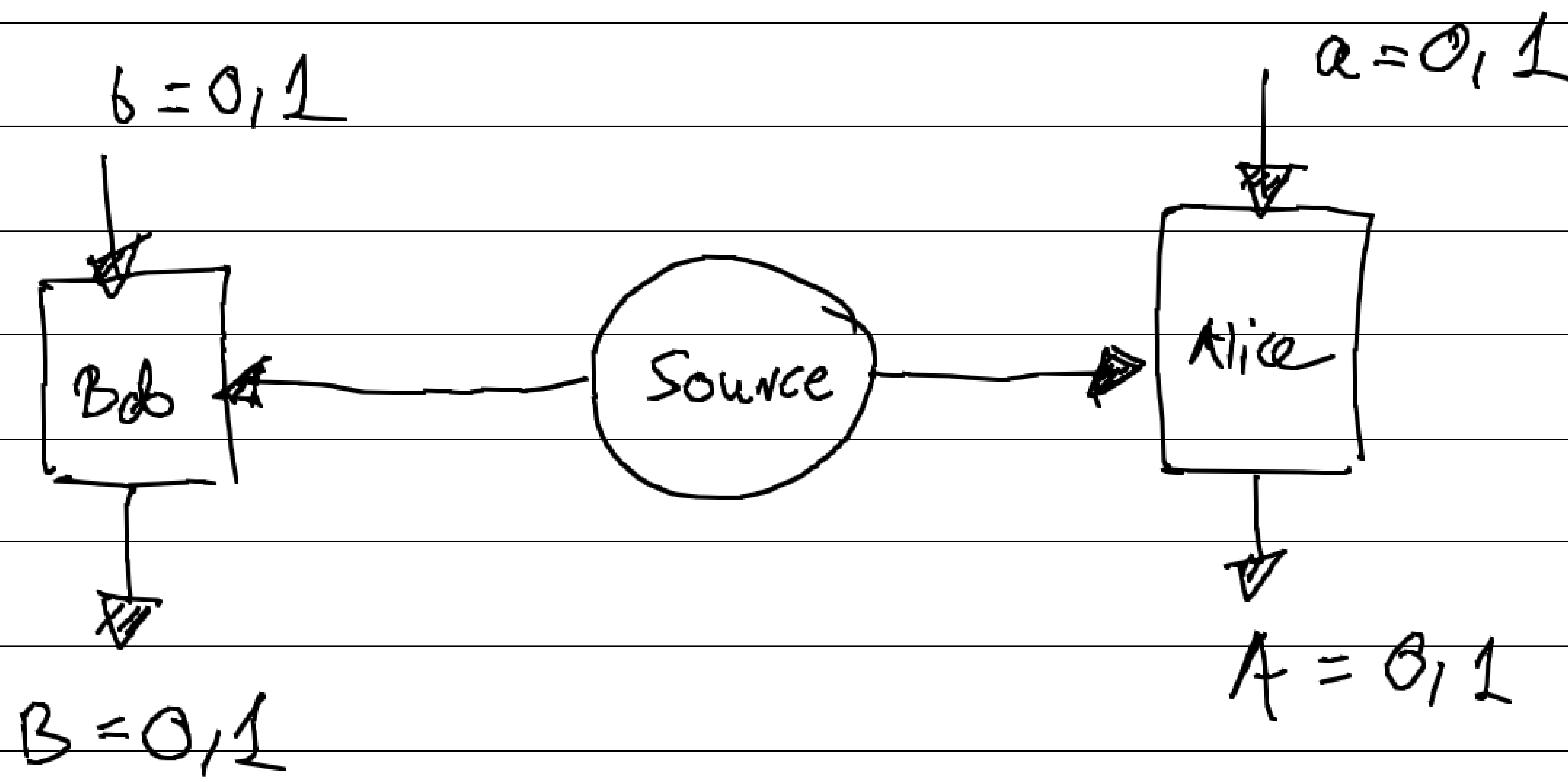
$$0 \leq \vec{\rho} \longrightarrow \boxed{\text{neg. } S'} \longrightarrow \vec{\rho}\,' \geq 0$$

<u>Great Smokey Dragon</u>

Nebits

↓

quantum non-locality and beyond



LHV ✓

$$\Downarrow$$

$$p_{LHV}(A_0 A_1 | B_0 B_1) \geq 0$$

$\boxed{\text{How to get quantum correlations?}} \equiv \boxed{p_{QM}(A_a | B_b)}$

Alice (Bob) chooses a box a (b) with a <u>nebit inside</u>

nebit

Alice: $\boxed{a} \equiv S_A^{(a)}(A_a|\tilde{A}_a)$

nebit

Bob: $\boxed{b} \equiv S_B^{(b)}(B_b|\tilde{B}_b)$

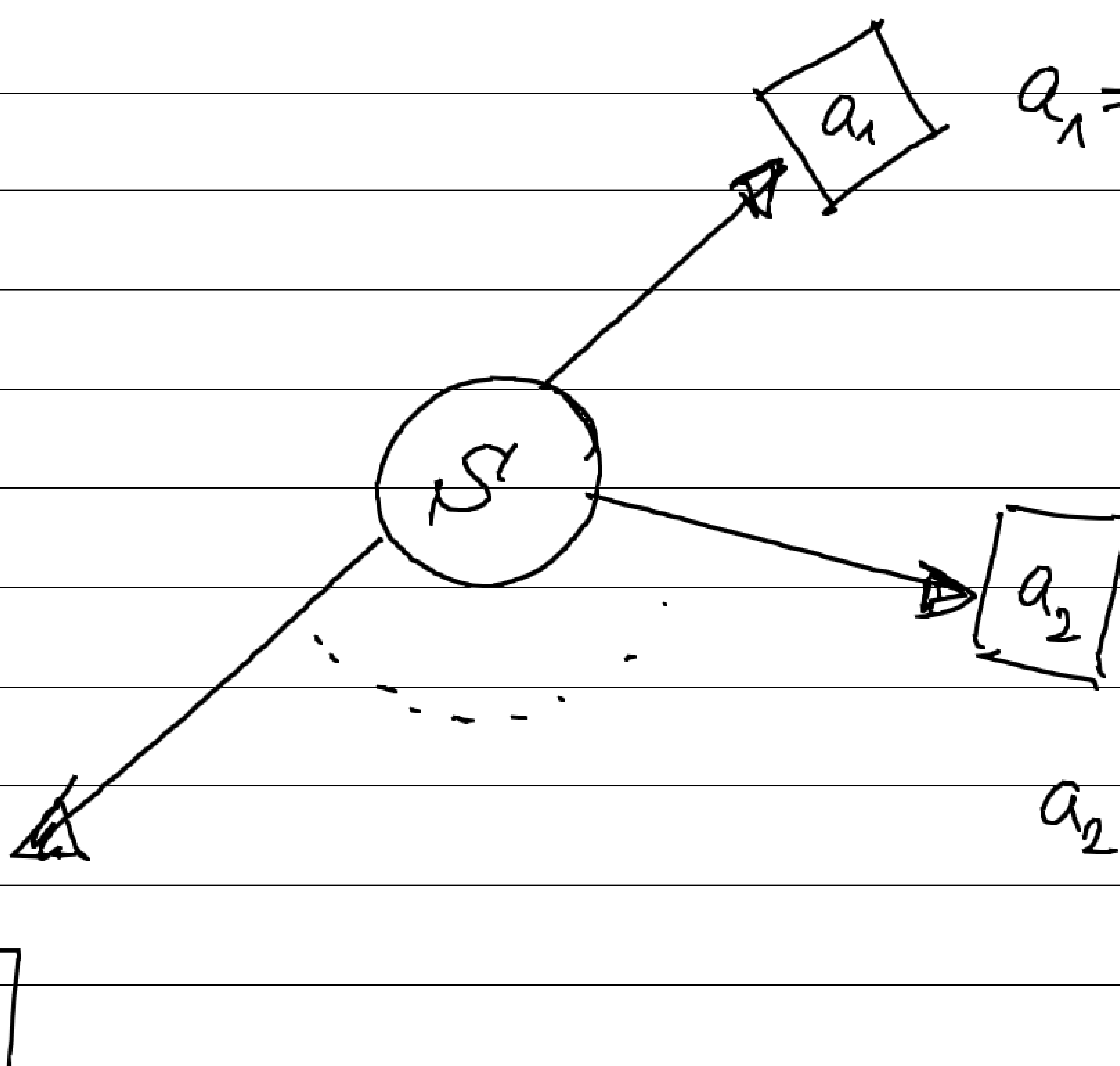
$$\boxed{P_{QM}(A_a, B_b) = \sum_{\tilde{A}_a,\tilde{B}_b} S_A^{(a)}(A_a|\tilde{A}_a)\, S_B^{(b)}(B_b|\tilde{B}_b) \times p_{LHV}(\tilde{A}_a,\tilde{B}_b)}$$

Example:

$$p_{LHV}(A_0 A_1, B_0 B_1) = \mathrm{Tr}\left[\rho_{AB}\, \overline{\Pi}_{A_0 A_1} \otimes \overline{\Pi}_{B_0 B_1} \right]$$

$$\left\{ \overline{\Pi}_{A_0 A_1}, \overline{\Pi}_{B_0 B_1} \right\} \leftarrow \boxed{\text{Bush POVMs}}$$

Generalization:



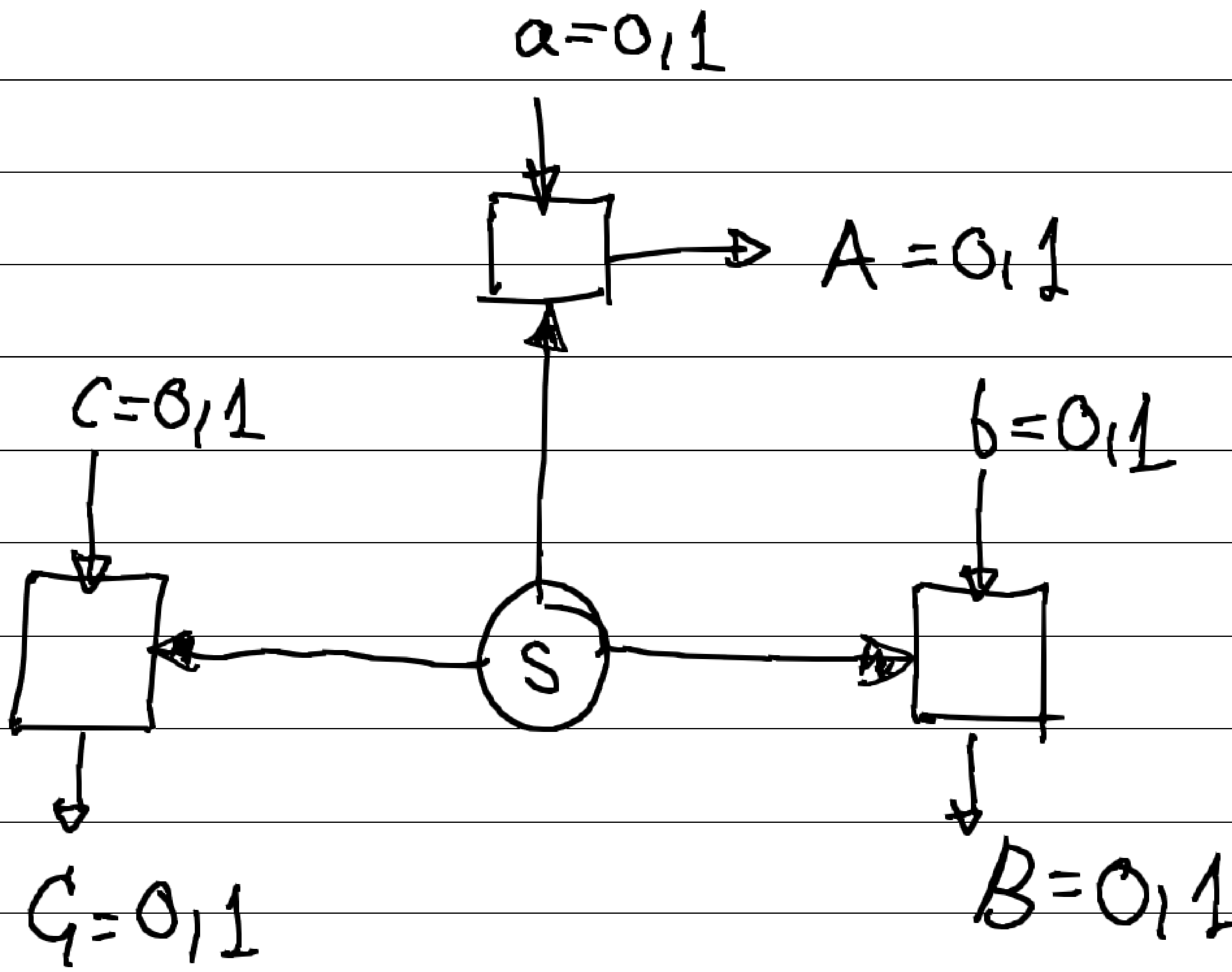
$a_1 = 0, 1, 2, \dots D_1$

$a_2 = 0, 1, 2, \dots, D_2$

$a_N = 0, 1, 2, \dots, D_N$

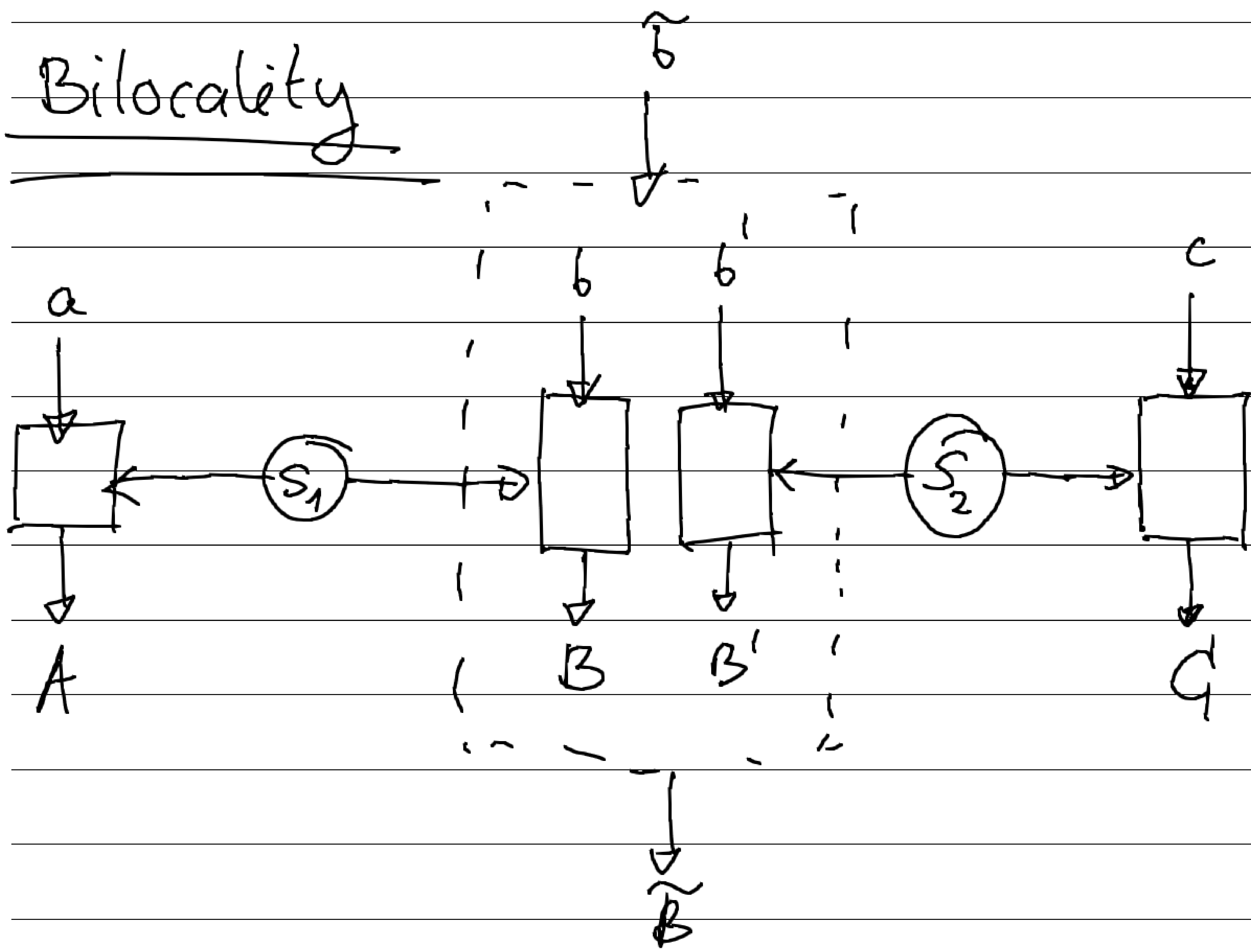
B

Locality



$$p_{LHV}(A_0 A_1, B_0 B_1, C_0 C_1)$$

Bilocality



$$p_{LHV}(A_0A_1, B_0B_1)\, p_{LHV}(B'_0B'_1, C_0C_1)$$

$$\Big\downarrow \quad \boxed{\text{data processing}}$$

$$p_{LHV}(\underbrace{A_0A_1}_{\vec{A}}, \underbrace{\tilde{B}_0\tilde{B}_1}_{\vec{\tilde{B}}}, \underbrace{C_0C_1}_{\vec{C}})$$

The only choice $\longmapsto$ stochastic process

$$p_{LHV}(\vec{A}, \vec{\tilde{B}}, \vec{C}) = \sum_{\vec{B}, \vec{B}'} S(\vec{\tilde{B}} \mid \vec{B}, \vec{B}') \times$$

$$\times\, p_{LHV}(\vec{A}, \vec{B})\, q_{LHV}(\vec{B}', \vec{C})$$

$$\Downarrow$$

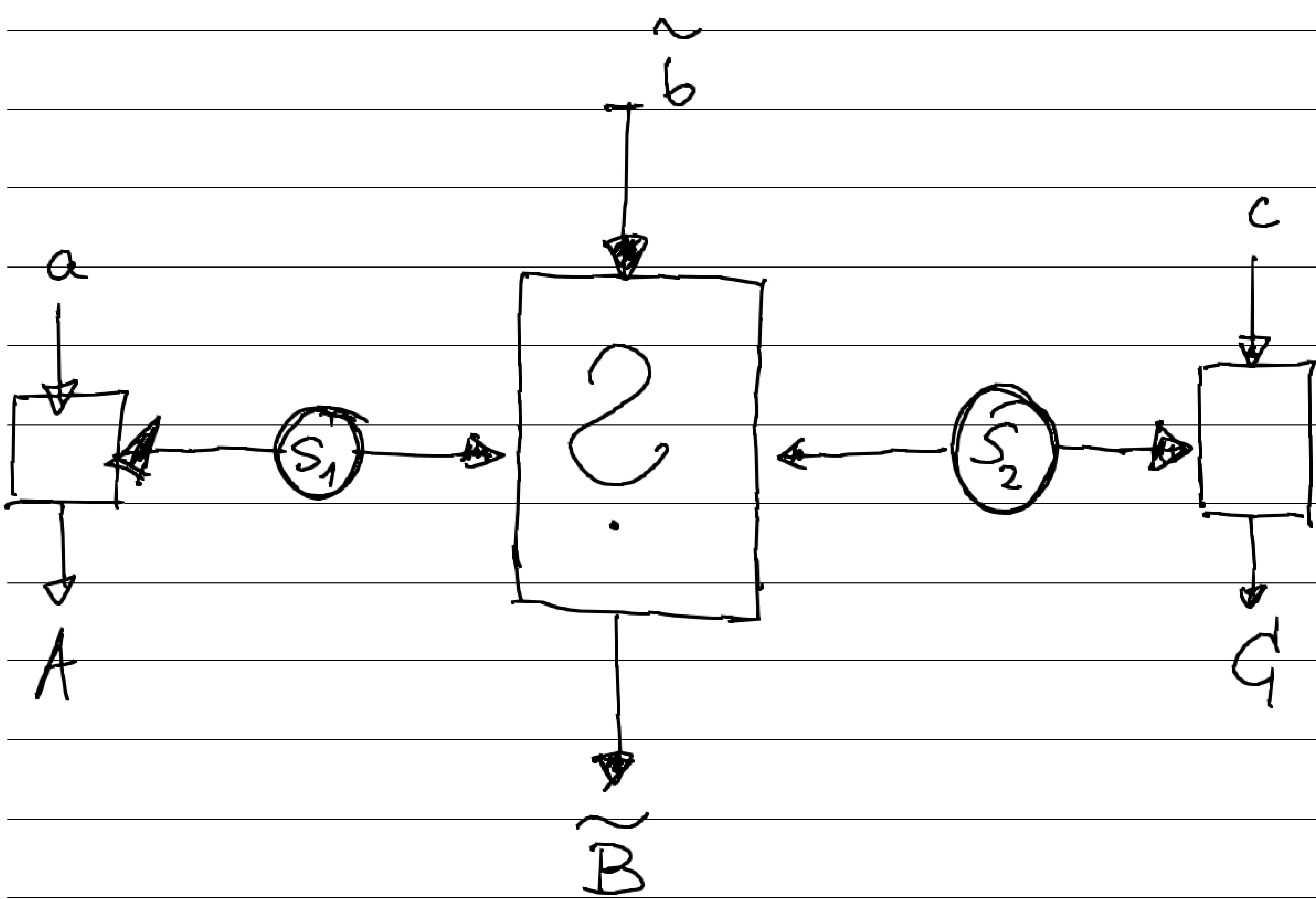
$$\sqrt{|\langle I \rangle|} + \sqrt{|\langle J \rangle|} \leq 1$$

Branciard et. al.

$$\langle I \rangle = \frac{1}{4} \sum_{ij} \langle A_i \tilde{B}_0 C_j \rangle$$

$$\langle J \rangle = \frac{1}{4} \sum_{ij} (-1)^{i+j} \langle A_i \tilde{B}_1 C_j \rangle$$

Mysterious data processing



Violation!

1. $S(\vec{\tilde{B}} \mid \vec{B}, \vec{B}') \leq 0$

2. $p(\vec{A}, \vec{B}) \leq 0$

or/and

$q(\vec{B}', \vec{C}) \leq 0$

3. 1 and 2

$$p(A_0A_1, B_0B_1) = \frac{1}{16}\left[1 + \mu_1(a_0b_0 + a_0b_1 + a_1b_0 - a_1b_1)\right]$$

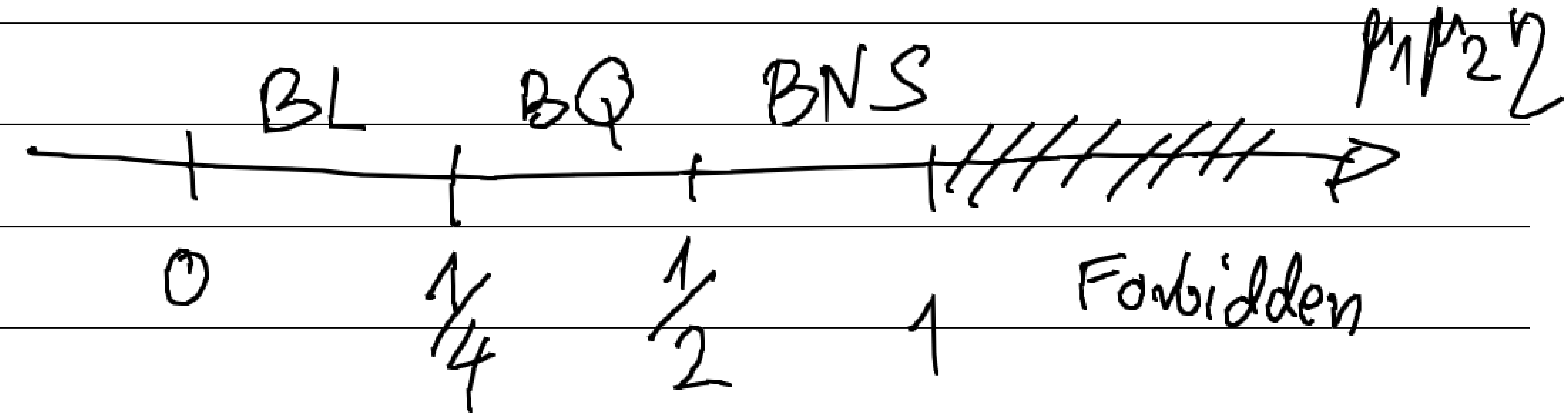
$$q(B_0'B_1', C_0C_1) = \frac{1}{16}\left[1 + \mu_2(b_0'c_0 + b_0'c_1 + b_1'c_0 - b_1'c_1)\right)$$

$$S(\tilde{B}_0\tilde{B}_1 \mid B_0B_1, B_0'B_1') = \frac{1}{2}\left[1 + \eta\, \tilde{b}_0 b_0 b_0'\right] \times$$

$$\frac{1}{2}\left[1 + \eta\, \tilde{b}_1 b_1 b_1'\right]$$

Then:

$$\sqrt{|\langle I\rangle|} + \sqrt{|\langle J\rangle|} = 2\sqrt{\mu_1\mu_2\eta}$$



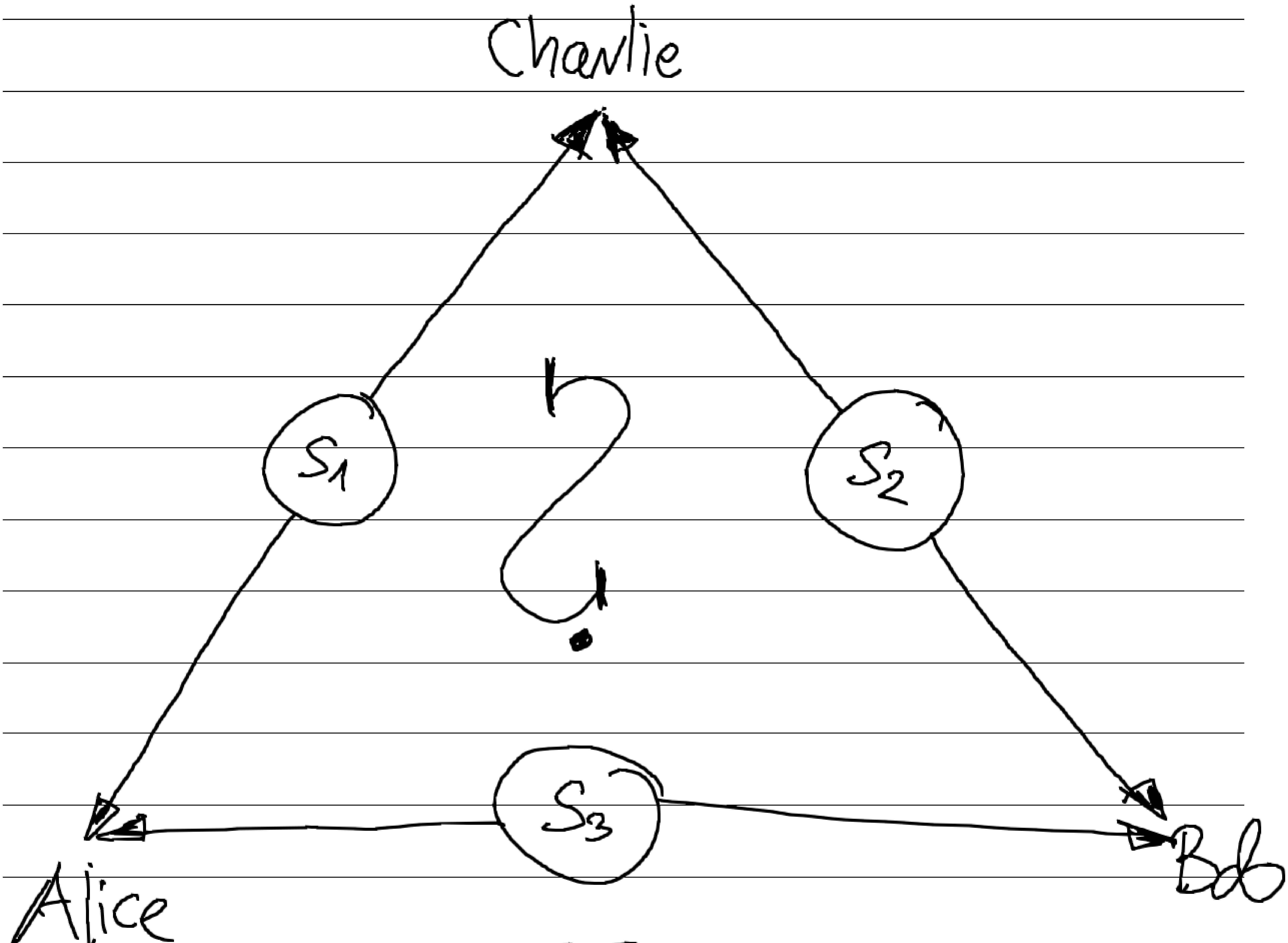
BL ≡ bilocality

BQ ≡ biquantum

BNS ≡ binosignalling

C

Gisin triangle



N-locality $\supset$ Q. networks