Addressing the compatibility loophole in the absence of spacelike separation

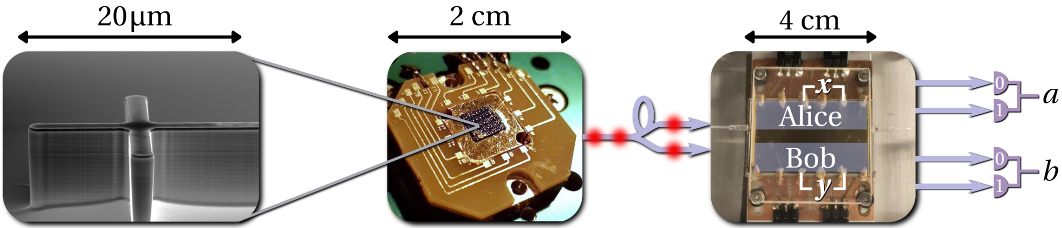
Shane Mansfield

QUANDELA

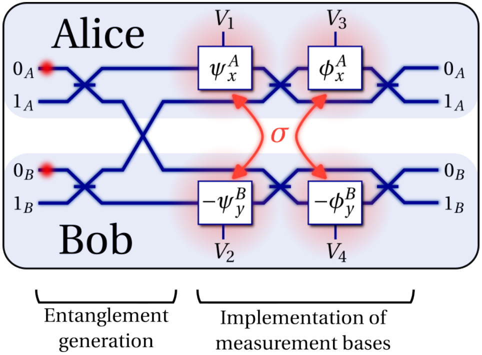
Joint work with
Theory: Boris Bourdoncle, Pierre-Emmanuel Emeriau, Damian Markham
Experiment: Andreas Fyrillas, Alexandre Mainos, Kayleigh Start, Nico Margaria, Martina Morassi, Aristide Lemaitre, Isabelle Sagnes, Petr Stepanov, Thi Huong Au, Sebastien Boissier, Niccolo Somaschi, Nicolas Maring, Nadia Belabas

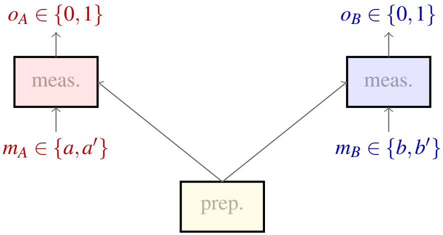
QCQMB 22, 18 Dec 2022

A Photonic Experiment at Quandela



Chip, Abstract Description, Empirical Data





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Formalising Empirical Data – Abramsky–Brandenburger

A **measurement scenario** $\mathbf{X} = \langle X, \Sigma, O \rangle$:

- X – a finite set of measurements
- Σ – an abstract simplicial complex on X
  faces are called the *measurement contexts*
- $O = (O_x)_{x \in X}$ – for each $x \in X$ a finite
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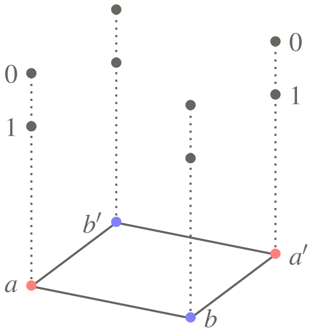

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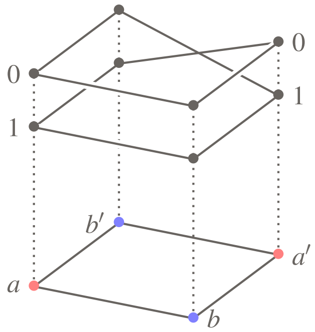

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An **empirical model** $e = \{e_\sigma\}_{e \in \Sigma}$ on $\mathbf{X}$:

- Each e_σ is a prob. distribution over joint outcomes for σ
- Marginals are well-defined; i.e. $\forall \sigma, \tau \in \Sigma$.

$$e_\sigma|_{\sigma \cap \tau} = e_\tau|_{\sigma \cap \tau}$$

(*generalised no-signalling property*)

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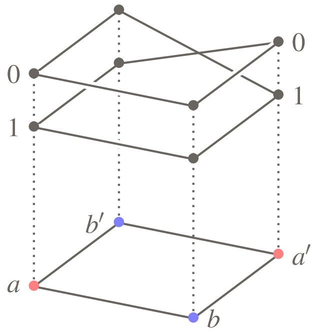
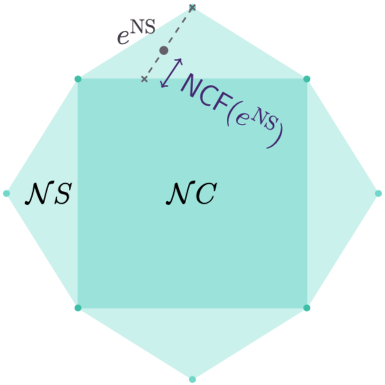
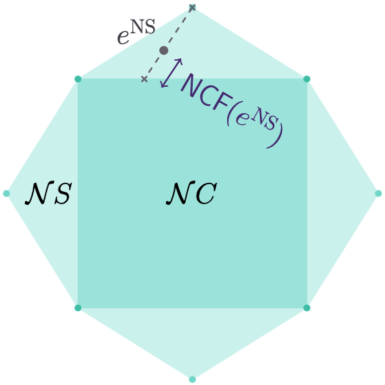


Quantifying Contextuality



- Empirical models for our 'CHSH' scenario live in an 8 dimensional Euclidean space
- Non-contextual polytope: convex hull of the deterministic models
- If e is outside NC, quantify its degree of contextuality by 'how far' outside it is?

Quantifying Contextuality

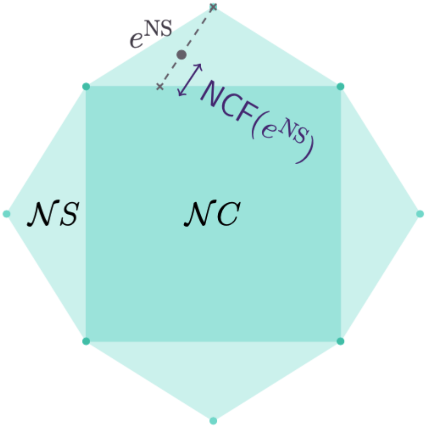


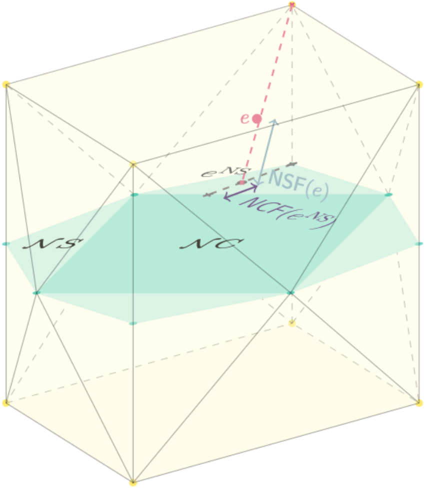
$$e = \mathsf{NCF}(e)\, e^{NC} + \mathsf{CF}(e)\, e^{SC}$$

- NCF is optimised for such decompositions
- CF Corresponds to the normalised violation of an optimal Bell inequality
- Master inequality for witnessing contextuality

$$\mathsf{CF}(e) > 0$$

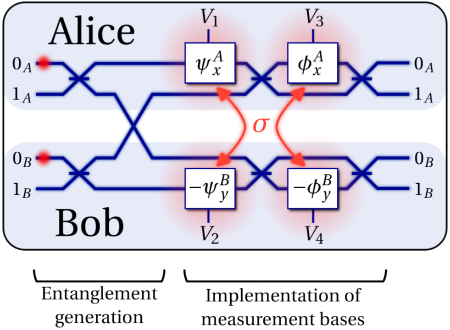
Ideal Data versus Lab Data





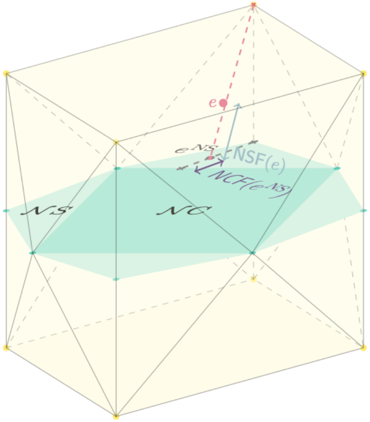
Signalling

$$e_\sigma|_{\sigma\cap\tau} \neq e_\tau|_{\sigma\cap\tau}$$



- Signalling arises due to cross-talk
- Can also arise due to finite statistics
- **First:** introduce a **signalling fraction** to empirically quantify signalling
- **Next:** the Abramsky–Brandenburger framework neatly abstracted away from hidden variable models, CbD allows for measures in the presence of signalling, but to address the issue in certification protocols we will need to get our hands dirty with HVMs again…

Quantifying Signalling

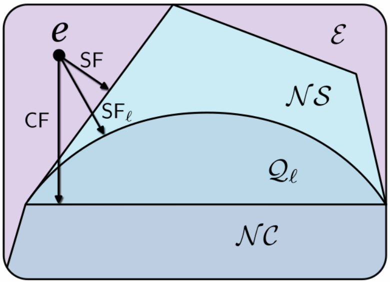


$$e = \mathsf{NSF}(e)\, e^{NS} + \mathsf{SF}(e)\, e^{SC}$$

- NSF is optimal for such a decomposition
- SF is an irreducible amount of signalling

(Measures above from works with Rui and Samson)

Quantifying "Quantum" Signalling



How far we are from a quantum-NS model?

$$e = \mathsf{NSF}_l(e)\, e^{Q_l} + \mathsf{SF}_l(e)\, e'$$

- e^{Q_l} in the lth level of the NPA hierarchy;
  i.e. approximately in the (bipartite) quantum set
- NSF_l is optimal for such a decomposition

Will be useful later to rule out quantum adversaries in randomness certification

Hidden Variable Models

- Postulate a (finite) set Λ of hidden variables
- The hidden variable should determine empirical behaviour
  i.e. to each $\lambda \in \Lambda$ corresponds a notional empirical model h^λ
- But we might only have probabilistic knowledge of the HV via a distribution $\mu \in \mathscr{D}(\Lambda)$
- An empirical model e is realised by a hidden variable model $\langle \Lambda, (h^\lambda)_{\lambda \in \Lambda}, \mu \rangle$ iff

$$e = \sum_{\Lambda} \mu(\lambda) h^\lambda$$

- IDEA: explain empirical behaviour via h^λs with *nicer* properties
  (e.g. determinism, independence,...)

Noncontextual Hidden Variables

Noncontextuality can be expressed as the conjunction of two HV assumptions:

1. Determinism

$$\forall \lambda \in \Lambda, \sigma \in \Sigma. \quad \exists o \in O_\sigma. \quad h^\lambda = \delta_o$$

2. Parameter Independence

$$\forall \lambda \in \Lambda, \sigma, \tau \in \Sigma. \quad h^\lambda_\sigma|_{\sigma\cap\tau} = h^\lambda_\tau|_{\sigma\cap\tau}$$

- An empirical model e is noncontextual if it can be realised by a noncontextual HV model
- This matches the definition from earlier
- To see this, note that together the above assumptions imply that

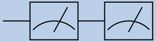
$$\forall \lambda \in \Lambda. \quad \exists g \in \prod_{x\in X} O_x. \quad h^\lambda = \mathbf{g}$$

Questioning the Validity of the Assumptions

Determinism

$$\forall \lambda \in \Lambda, \sigma \in \Sigma. \quad \exists o \in O_\sigma. \quad h^\lambda = \delta_o$$

- Valid to the extent that measurements are empirically *sharp*
- A measurement is sharp if upon repeating it we always obtain the same outcome
- Or if for each outcome there exists a preparation from which it deterministically results

ρ 

Parameter Independence

$$\forall \lambda \in \Lambda. \forall \sigma, \tau \in \Sigma. \qquad h^\lambda_\sigma|_{\sigma\cap\tau} = h^\lambda_\tau|_{\sigma\cap\tau}$$

- Valid to the extent that the empirical model is *no-signalling/no-disturbing*
- Recall e is NS iff

$$\forall \sigma, \tau \in \Sigma. \qquad e_\sigma|_{\sigma\cap\tau} = e_\tau|_{\sigma\cap\tau}$$

Weakening the HV Assumptions

Parameter Independence

A HV model is $(1-\sigma)$ parameter-independent if for all λ, there exists a decomposition of h^λ of the form

$$h^\lambda = c^\lambda_{\mathrm{NS}} h^\lambda_{\mathrm{NS}} + (1 - c^\lambda_{\mathrm{NS}}) h'^\lambda$$

with h^λ_{NS} parameter-independent, and $c^\lambda_{\mathrm{NS}} \geq 1 - \sigma$.

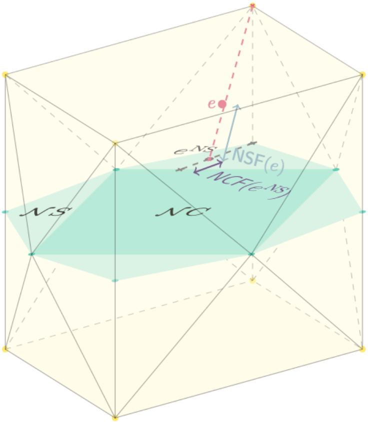
Determinism

A parameter-independent HV model is $(1-\eta)$ deterministic if for all λ, there exists a decomposition of h^λ of the form

$$h^\lambda = c^\lambda_{\mathrm{OD}} h^\lambda_{\mathrm{OD}} + (1 - c^\lambda_{\mathrm{OD}}) h''^\lambda$$

with h^λ_{OD} outcome deterministic and no-signalling, and $c^\lambda_{\mathrm{OD}} \geq 1 - \eta$.

Contextuality in Non-Ideal Data



- Allow HV models some ‘signalling’ and ‘unsharpness’
- Master inequality for escaping HV explanations:

$$\mathsf{CF}(e) > \sigma + \varepsilon - \sigma\varepsilon$$

- Constraints: $(1 - \sigma)$ Parameter Independence, $(1 - \eta)$ Determinism

Proof Sketch

Decomposing the HV model as in the diagram, it’s possible to extract a weight of $(1 - \sigma)(1 - \varepsilon)$ on vertices of NC, which lower bounds NCF, and results in the above bound on CF.

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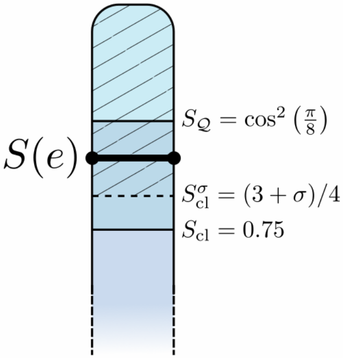

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Empirical Data from Quandela

- $\mathsf{CF} \approx 0.34$
- Tsirelson bound: $\mathsf{CF} \approx 0.41$
- Observed signalling: $\mathsf{SF}, \mathsf{SF}_{l=3} < 0.005$
- Estimated unsharpness: $\eta^{emp} < 0.00001$

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Randomness Certification

- Adapted the Miller–Shi protocol to certify generation of private unpredictable randomness
- (Un)sharpness becomes irrelevant
- Need only a gap between observed CF and the maximal score for admissible HVMs
- For idealised non-signalling data need $\mathsf{CF} > 0$
- Modified protocol can certify randomness whenever $\mathsf{CF} > \sigma$

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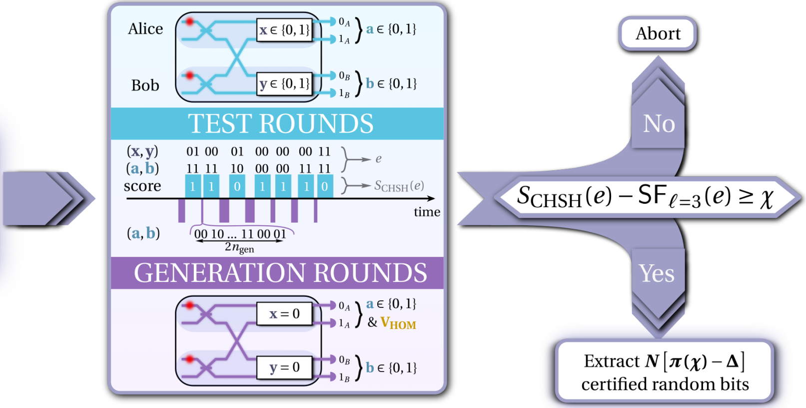
- Secure against quantum side-information
- Guaranteed up to an amount of information leakage parametrised by σ, which also affects the generation bitrate
- Assuming $\sigma = \mathsf{SF}_{l=3}$, current bitrate is 21.2bit/s

Protocol

Input parameters:

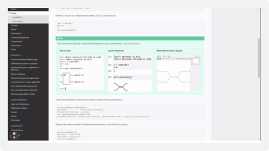
- N: number of rounds
- q: test probability
- χ: threshold



Try it for yourself

QUANDELA Cloud

Making the future of computing brighter



Design circuits and run jobs in Python

Use Open-source framework Perceval to design photonic circuits, develop and test algorithms and launch jobs on powerful simulators or on real quantum processing units.

Quandela's cloud-based platform gives you access to photonic quantum computing, enabling you to develop and deploy algorithms that optimise solutions.

Get Started for Free

qpu:achernar
Achernar QPU
In Deployment

sim:clifford
Web simulator
Available

qpu:ascella
QPU Ascella
Available

Quandela Cloud beta 0.1

Bonus Slides

Factorisable Hidden Variables

Noncontextuality can also be expressed the following HV assumption:

- Factorisability

$$\forall \lambda \in \Lambda, \sigma \in \Sigma. \quad h^{\lambda}_{\sigma} = \prod_{x \in \sigma} h^{\lambda}_{\sigma}|_{\{x\}}$$

- Factorisability implies parameter independence
- (Fine–Abramsky–Brandenburger Theorem) The following are equivalent:
  - Realisability by a factorisable HV
  - Realisability by a deterministic parameter independent HV
  - $\mathsf{CF} = 0$
- Factorisability is what's meant by 'locality' in discussions of Bell's Theorem
- Perhaps Bell's theorem is most interesting for the violation of locality
- But we're more interested by the applications accessible through violations of determinism

Classical Correlations

	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
(a,b)	$1/2$	0	0	$1/2$
(a,b')	$1/2$	0	0	$1/2$
(a',b)	$1/2$	0	0	$1/2$
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'Classical' premises:

- Observable properties have definite values
- Measurements reveal these
- Without disturbing the state

Thus classical data should arise as (a convex combination of) global value assignments:

$$
\begin{aligned}
(a,a',b,b') &\mapsto (0,0,0,0),\\
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\end{aligned}
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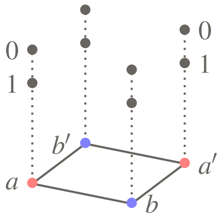

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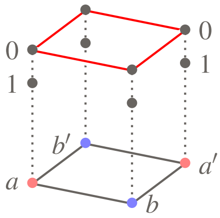

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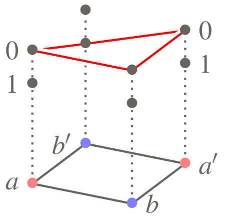

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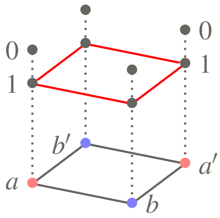

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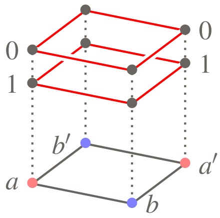

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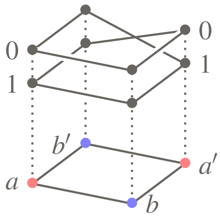
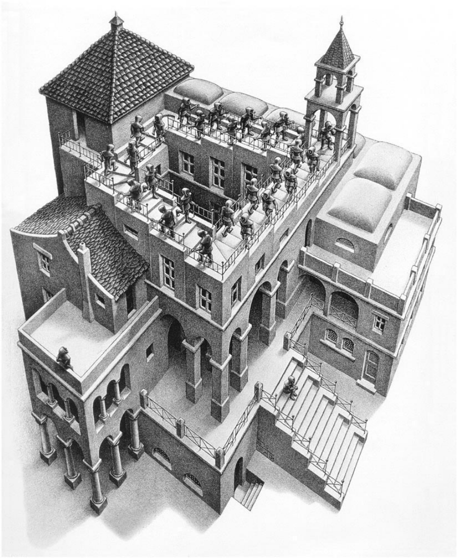
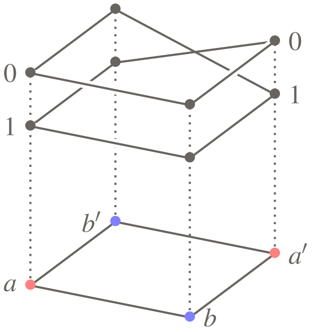


Some empirical correlations *cannot* be obtained as a convex combination of global assignments?!

Contextuality Analogy & Definition





$$\nexists d \in \mathscr{D}\Big(\prod_{x \in X} O_x\Big). \quad \forall \sigma \in \Sigma. \quad d|_\sigma = e_\sigma$$