

Contextuality as a precondition for entanglement

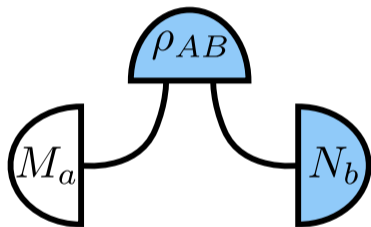
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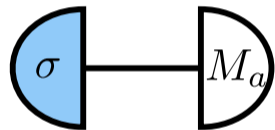


Entanglement



Remote state
preparations

Contextuality



Main idea

1. Alice and Bob share ρ_{AB}
2. Bob measures $\{N_b\}$ and obtains outcome b .
3. Bob announces outcome b .

State of Alice's system:

$$\sigma_A \approx \text{Tr}_B((\mathbf{1}_A \otimes N_b)\rho_{AB})$$

Remote preparations

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Set of all remotely-preparable states:

$$\Lambda_A(\rho_{AB}) = \{\sigma_A \in \mathcal{D}(\mathcal{H}_A) : E_B \geq 0, \sigma_A = \text{Tr}_B((\mathbf{1}_A \otimes E_B)\rho_{AB})\}$$

Contextuality

K - set of preparable states: $K \subset \mathcal{D}(\mathcal{H})$

P&M contextuality

K has preparation & measurement noncontextual model if:

$$\mathrm{Tr}(\rho M_a) = \sum_{\lambda} \mathrm{Tr}(\rho N_{\lambda}) \mathrm{Tr}(\omega_{\lambda} M_a)$$

where

$$\mathrm{Tr}(\rho N_{\lambda}) \geq 0, \quad \forall \rho \in K, \forall \lambda$$

and $\omega_{\lambda} \in \mathcal{D}(\mathcal{H})$.

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$\mathrm{Tr}(\rho N_{\lambda})$ - classical state replacing ρ

$\mathrm{Tr}(\omega_{\lambda} M_a)$ - classical response function replacing M_a

$\Lambda_A(\rho_{AB})$ has P&M noncontextual model



ρ_{AB} is separable



$\Lambda_A((1 - \varepsilon)\rho_{AB} + \varepsilon\tau_{AB})$ has P&M noncontextual model
for almost all separable τ_{AB} and $\varepsilon \in (0, \delta(\tau_{AB}))$

Robust remote preparations are necessary

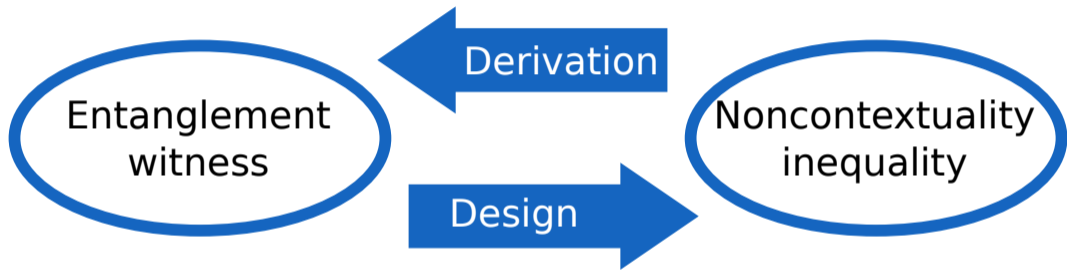
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$\Lambda_A(\rho_{AB})$ has P&M noncontextual model

Counter-example

$$\rho_{AB} = \frac{1}{4}(|0\rangle\langle 0| \otimes |00\rangle\langle 00| + |1\rangle\langle 1| \otimes |01\rangle\langle 01| + |+\rangle\langle +| \otimes |10\rangle\langle 10| + |-\rangle\langle -| \otimes |11\rangle\langle 11|)$$



New entanglement witness

Proposition

Let ρ_{AB} be a separable quantum state.

Let $E_{t,b}$ and $M_{t,b}$ be positive operators, $E_{t,b} \geq 0$ and $M_{t,b} \geq 0$, such that $E_* = \frac{1}{2}(E_{t,0} + E_{t,1})$, $\frac{1}{3} \sum_{t=1}^3 M_{t,b} = \frac{\mathbb{1}_A}{2}$ and $\mathbb{1}_A = M_{t,0} + M_{t,1}$ for all $t \in \{1, 2, 3\}$ and $b \in \{0, 1\}$. Then

$$\sum_{t=1}^3 \sum_{b=0}^1 \text{Tr}[(M_{t,b} \otimes E_{t,b})\rho_{AB}] \leq 5 \text{Tr}[(\mathbb{1}_A \otimes E_*)\rho_{AB}].$$

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For specific choice we get:

$$\text{Tr}((\sigma_x \otimes \sigma_x)\rho_{AB}) + \text{Tr}((\sigma_z \otimes \sigma_z)\rho_{AB}) \leq \frac{4}{3}$$

New contextuality inequality

Proposition

Let K be a set of allowed preparations. Let $\sigma_* \in K$ and let $i \in \{1, 2\}$, let $\sigma_{i+}, \sigma_{i-}, \sigma_{i0} \in \text{cone}(K)$ be subnormalized preparations such that

$$\sigma_{1+} + \sigma_{1-} + \sigma_{10} = \sigma_* = \sigma_{2+} + \sigma_{2-} + \sigma_{20}.$$

Let A_i be observables such that $-\mathbb{1} \leq A_i \leq \mathbb{1}$ for all $i \in \{1, 2\}$. If there is a P&M contextual model for K , then

$$\text{Tr}[(A_1 + A_2)(\sigma_{1+} - \sigma_{1-})] + \text{Tr}[(A_1 - A_2)(\sigma_{2+} - \sigma_{2-})] \leq 2.$$

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<http://www.humboldt-foundation.de>

