Synchronous observation of Bell nonlocality and state-dependent contextuality

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Joint work with Peng Xue[1], Lei Xiao[1], Gabriel Ruffolo[2], André Mazzari[2], Tassius Temistocles[3], and Marcelo Terra Cunha[2].

[1]Beijing Computational Science Research Center

[2]State University of Campinas

[3]Federal Institute of Alagoas

KS contextuality scenario

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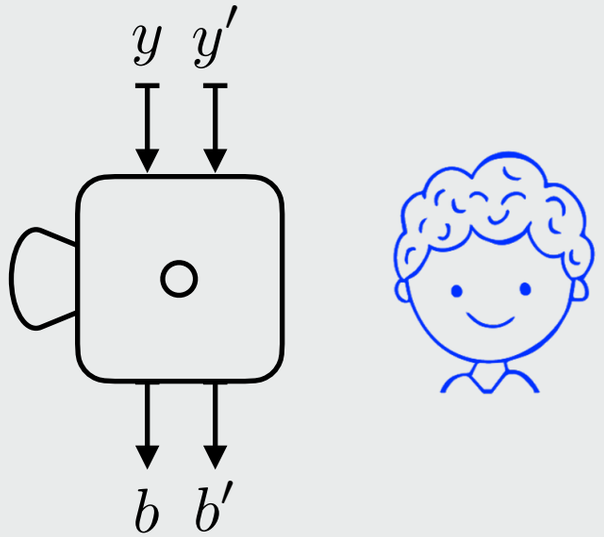

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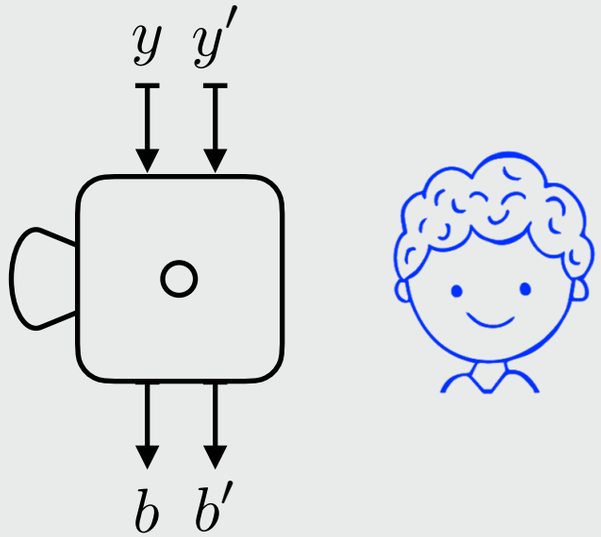

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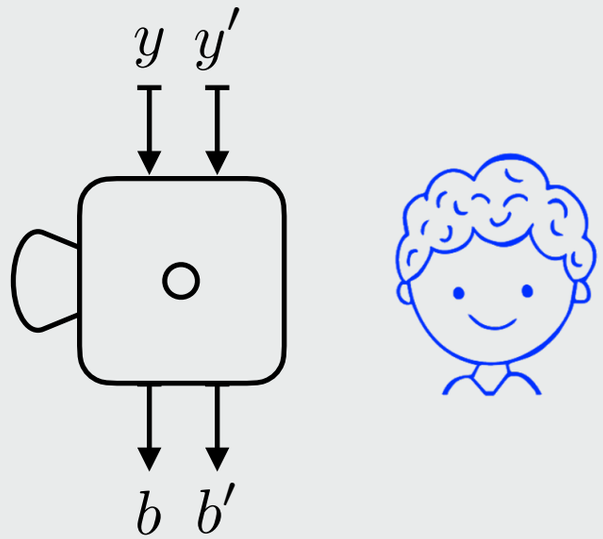
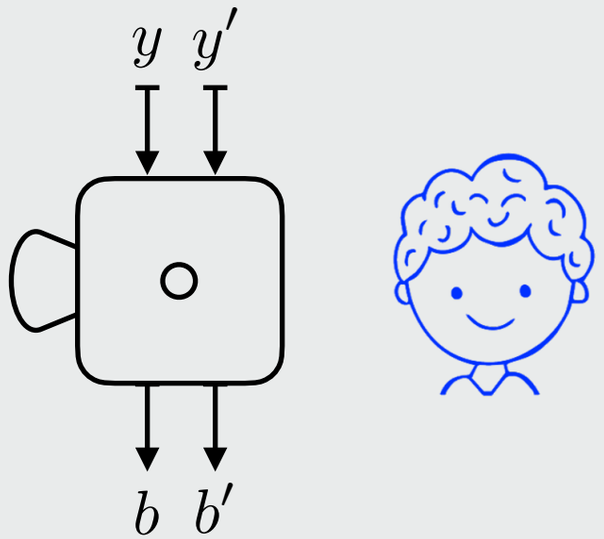


- Behavior of the boxes:

$$p(b, b'|y, y')$$

KS contextuality scenario



- Behavior of the boxes:

$$p(b, b'|y, y')$$

- Noncontextual behaviors:

$$p(b, b'|y, y') = \int p(b|y, \lambda)p(b'|y', \lambda)p(\lambda)d\lambda$$

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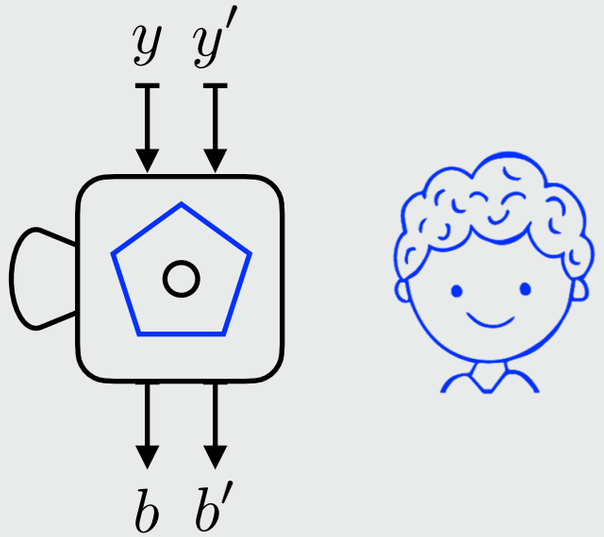

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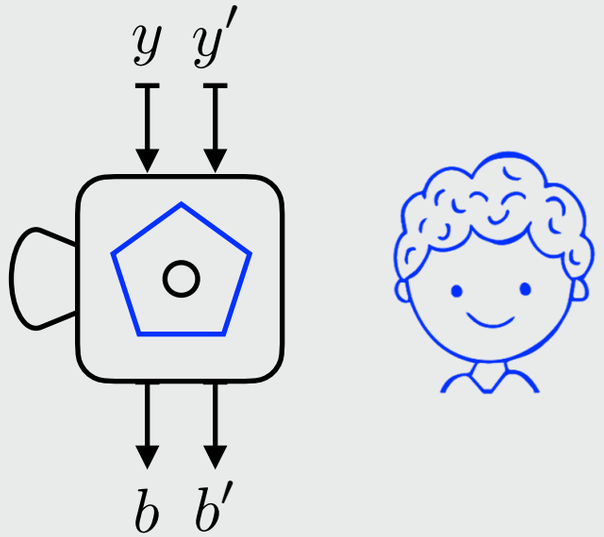

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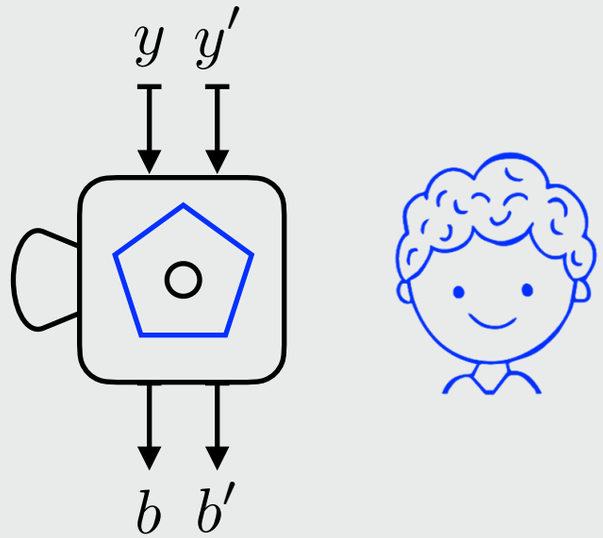

- KCBS inequality [Klyachko etal. (2008)]:

$$\langle B_0 B_1 \rangle + \langle B_1 B_2 \rangle + \langle B_2 B_3 \rangle + \langle B_3 B_4 \rangle - \langle B_0 B_4 \rangle \leq 3$$

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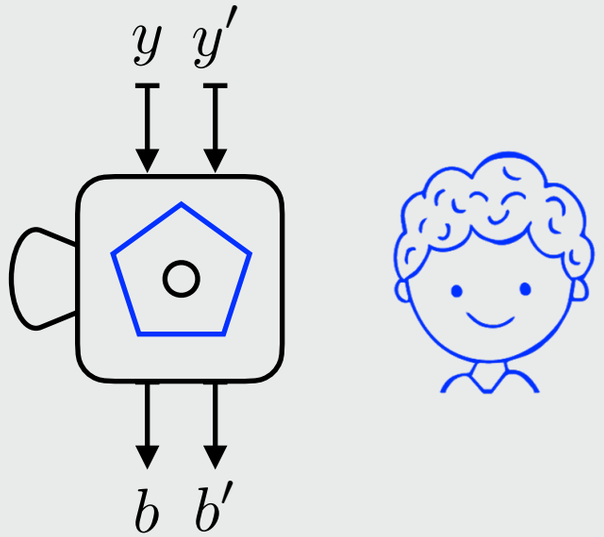

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$$\langle B_y B_{y'} \rangle = p(b = b'|y, y') - p(b \neq b'|y, y')$$

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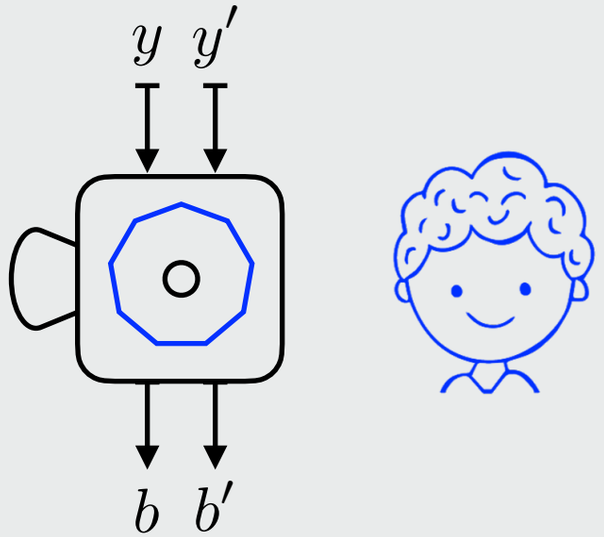

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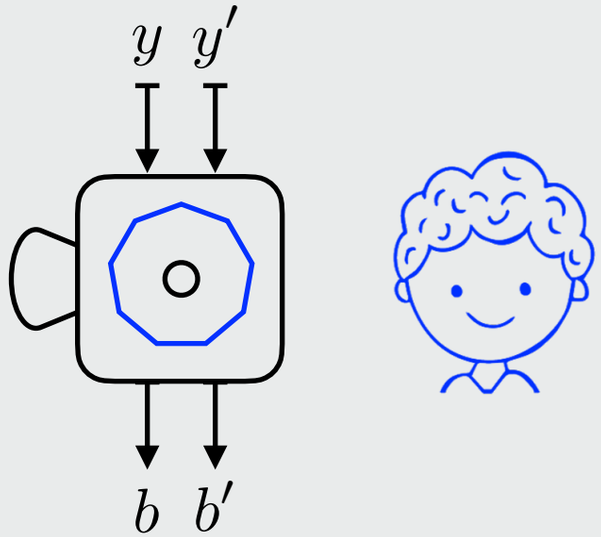

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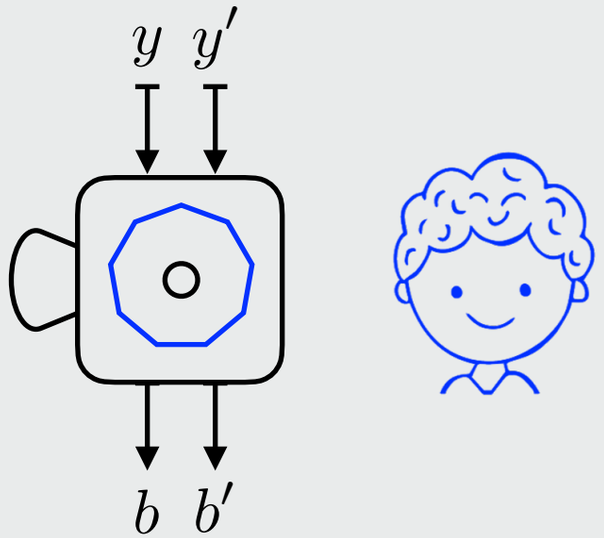

- n-cycle inequality [Araújo etal (2013)]:

$$\sum_{i=0}^{n-2} \langle B_i B_{i+1} \rangle - \langle B_0 B_{n-1} \rangle \leq n-2$$

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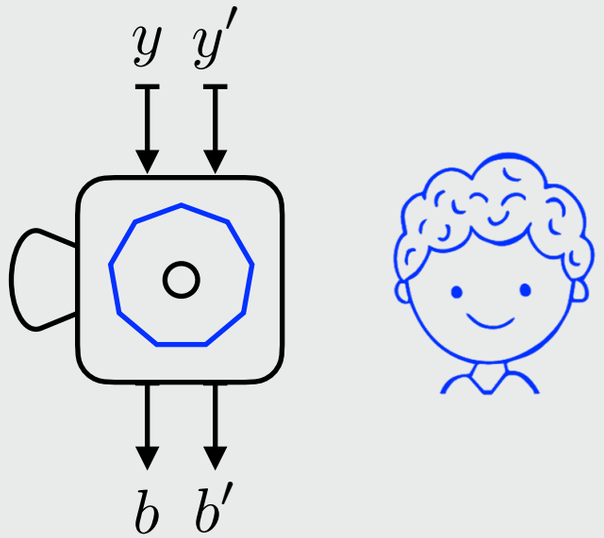

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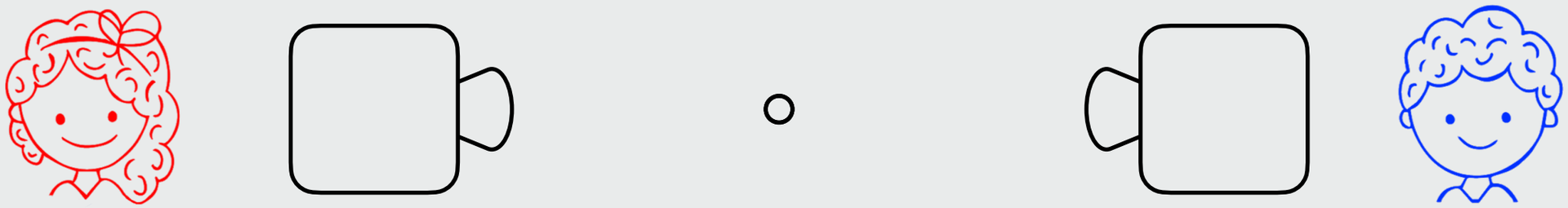
Bell scenario



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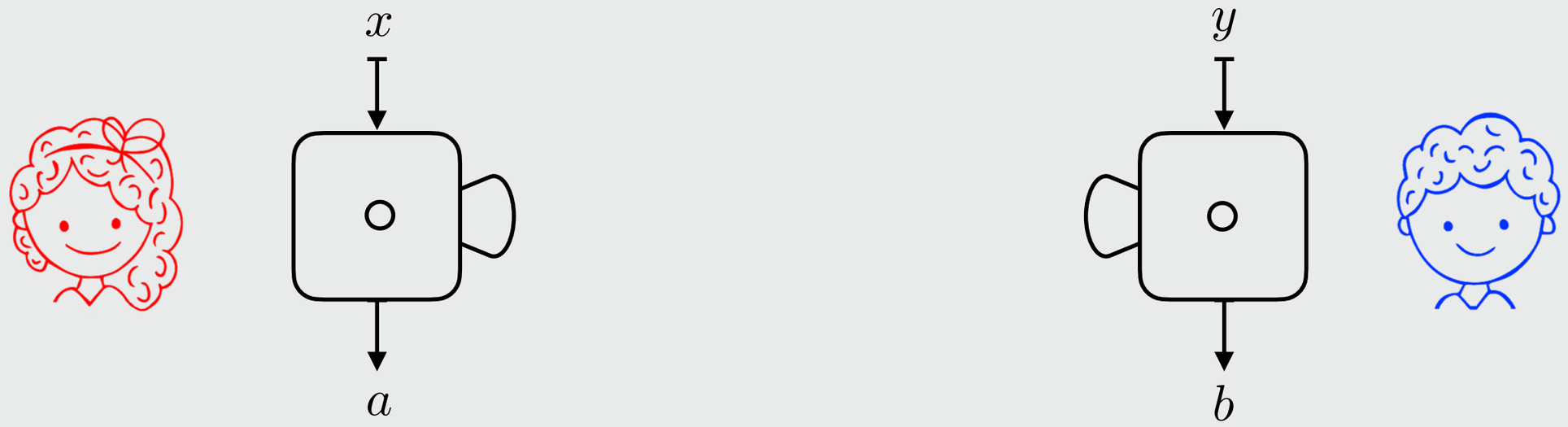

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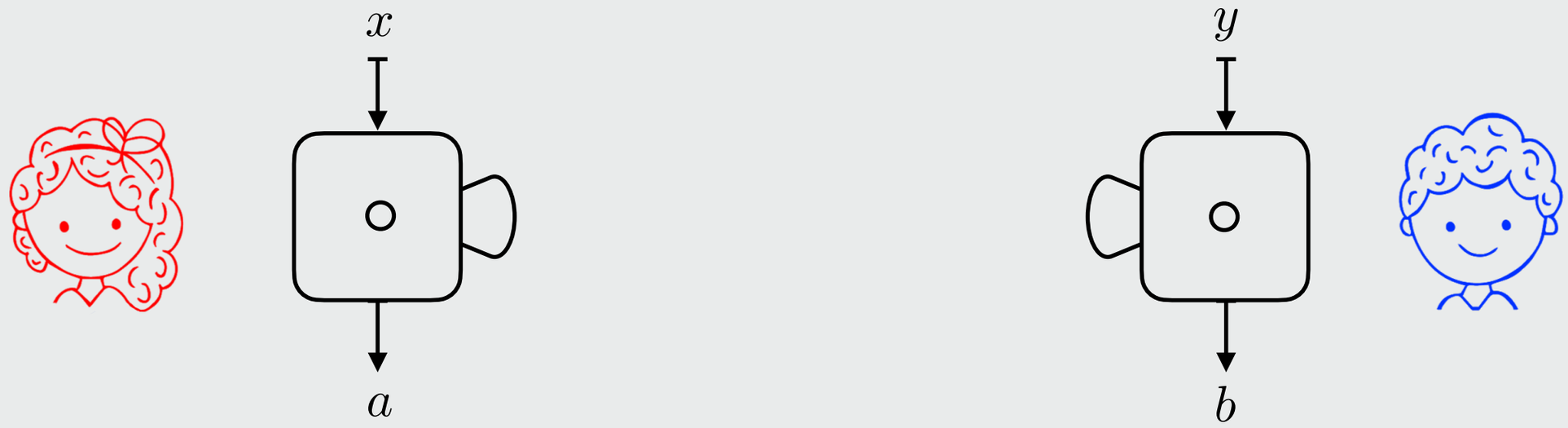

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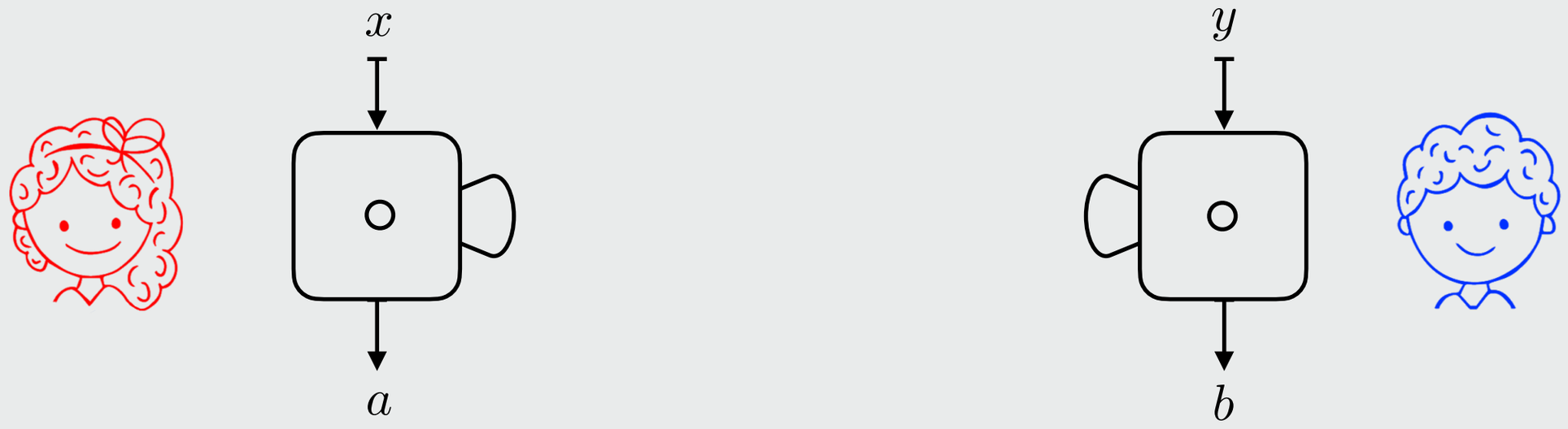

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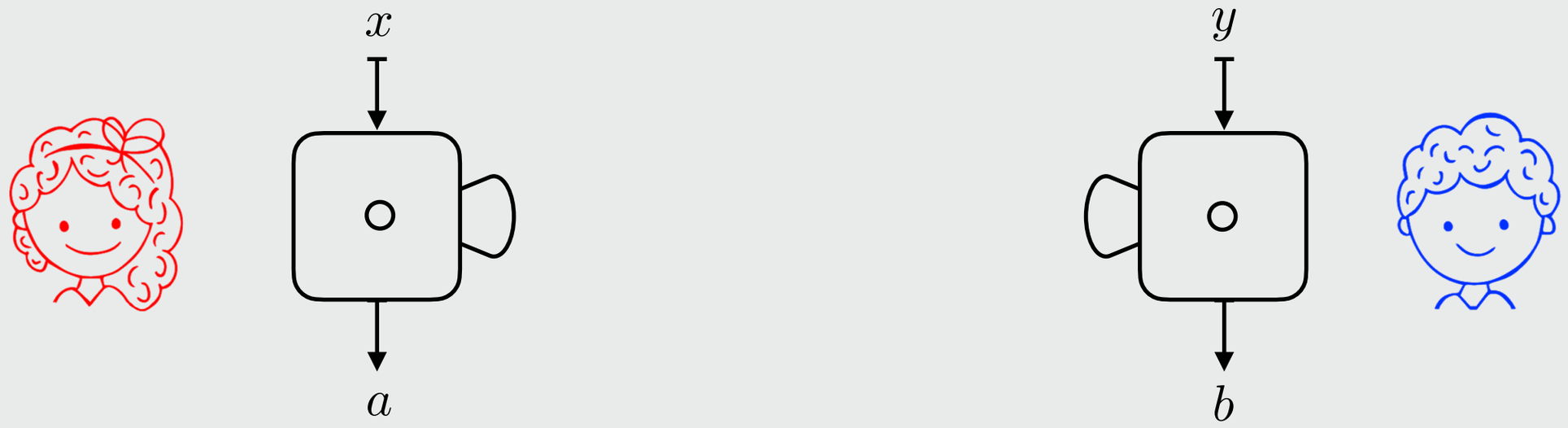

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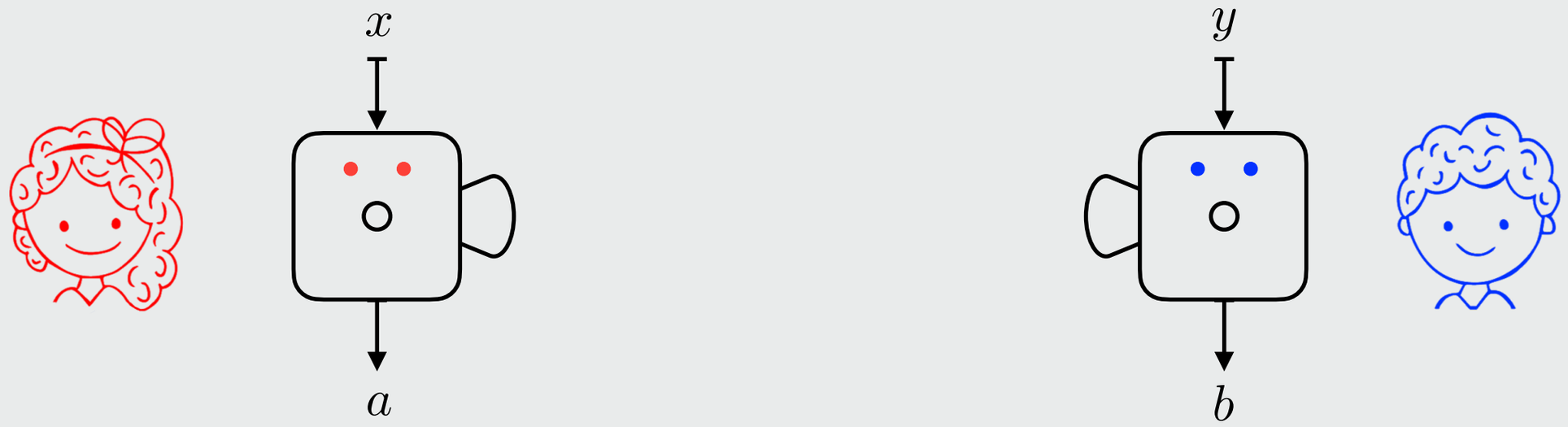
Bell scenario



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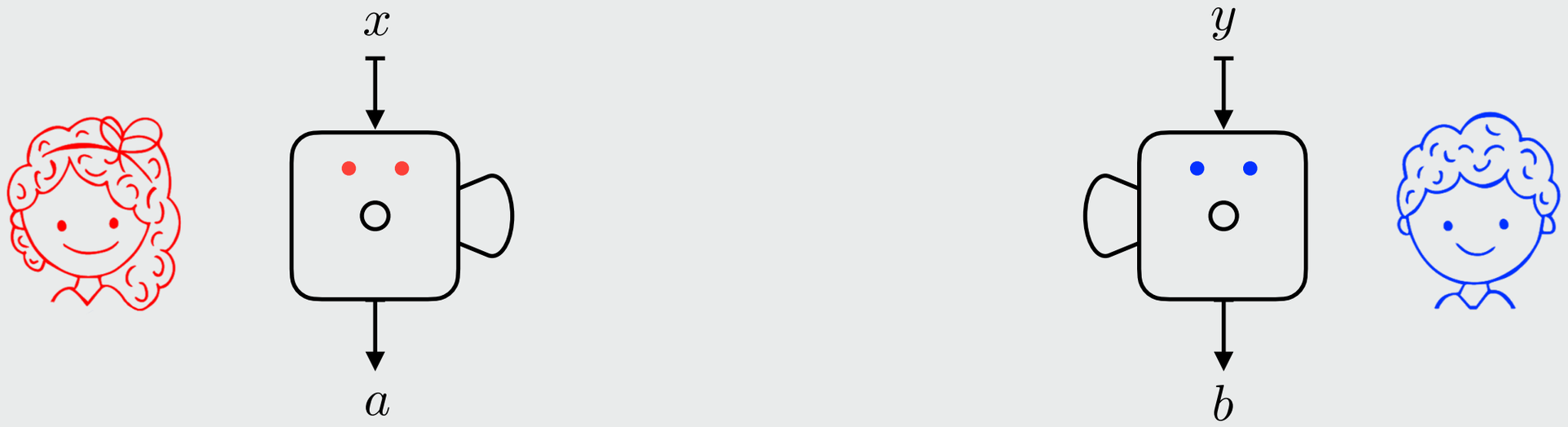

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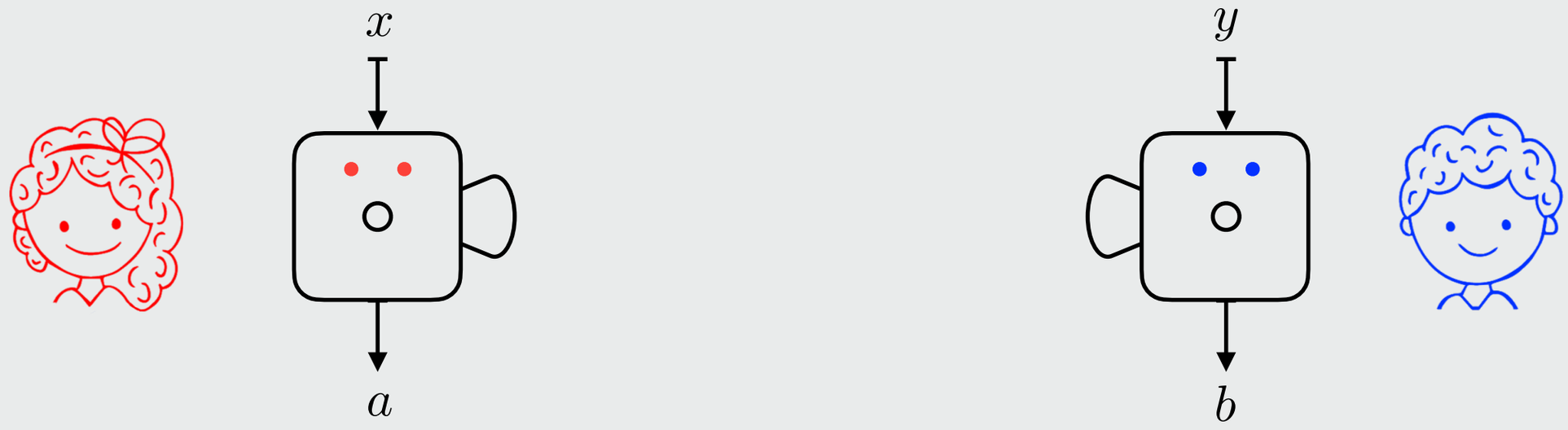

- CHSH inequality [Clauser etal (1969)]:

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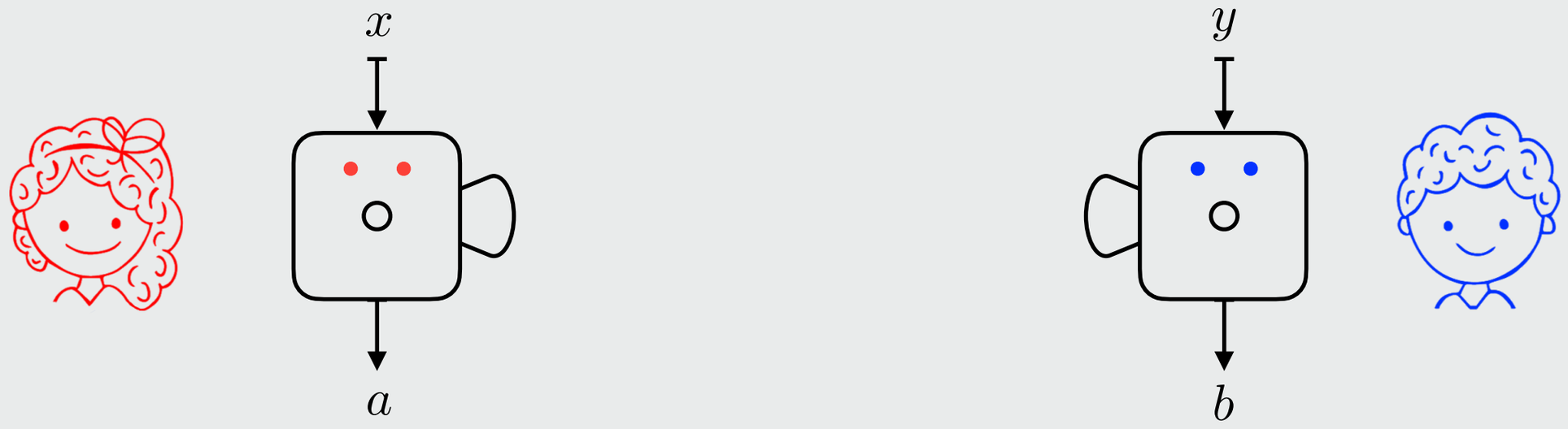

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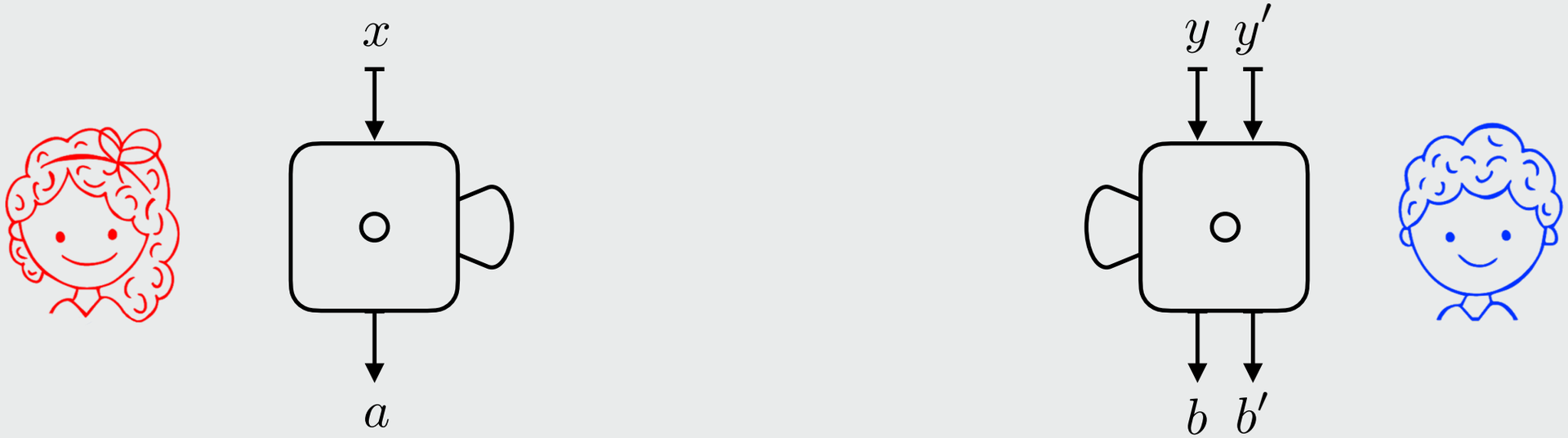
Extended Bell scenario



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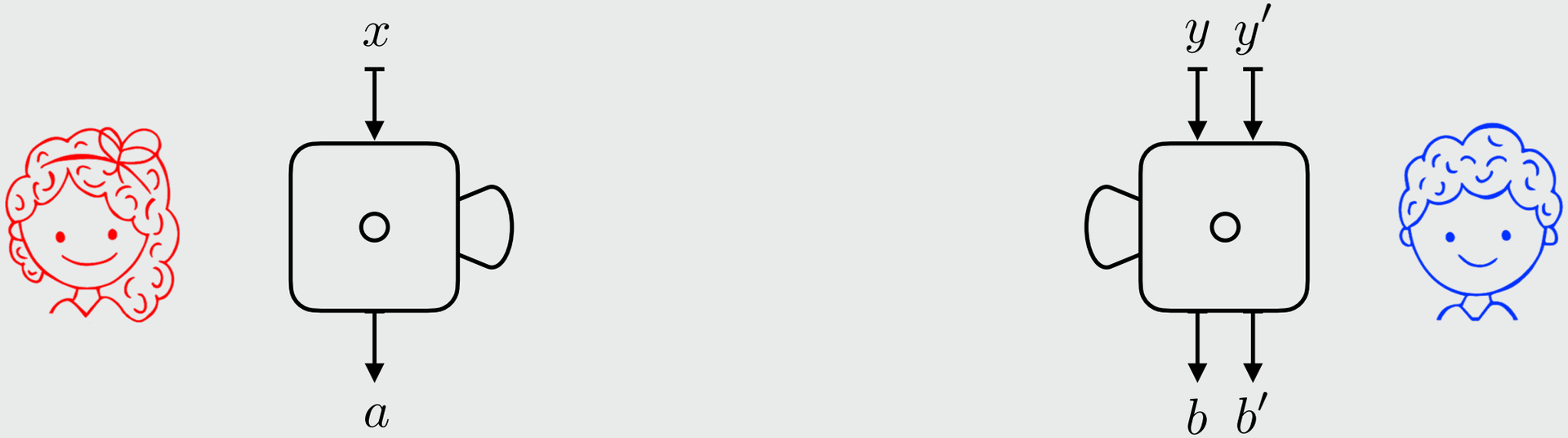

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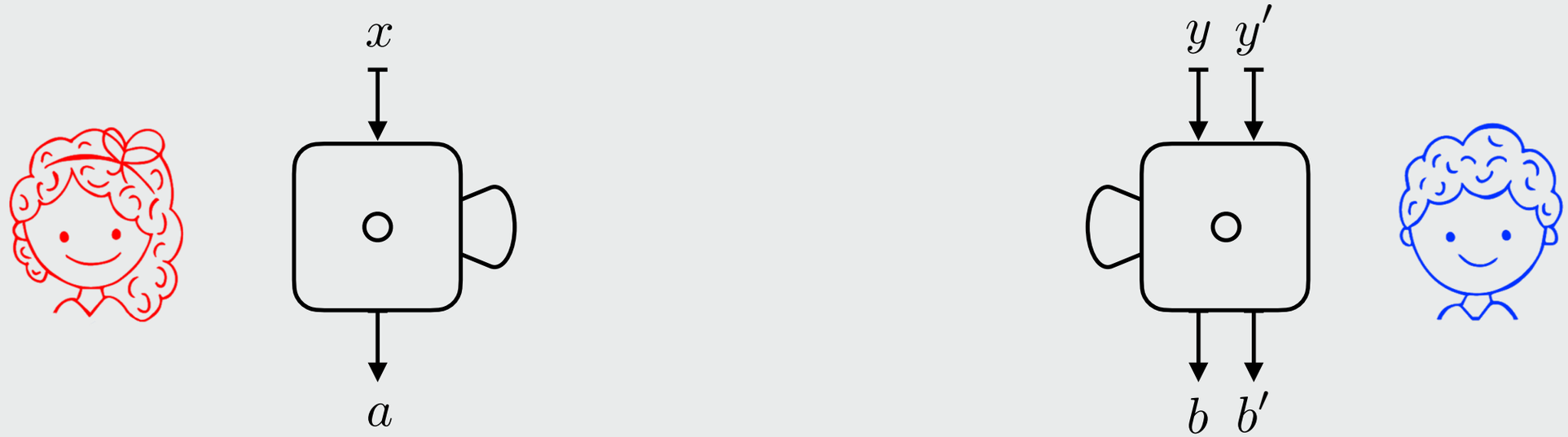

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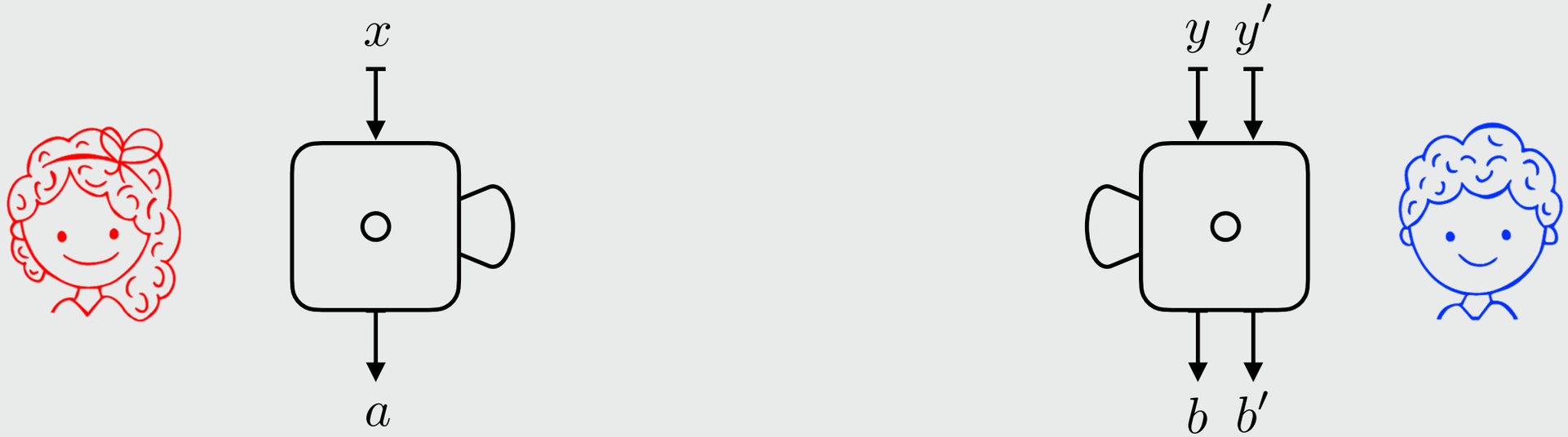

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- Marginal behaviors:

$$p(a, b|x, y) = \sum_{b'} p(a, b, b'|x, y, y') \qquad p(b, b'|y, y') = \sum_{a} p(a, b, b'|x, y, y')$$

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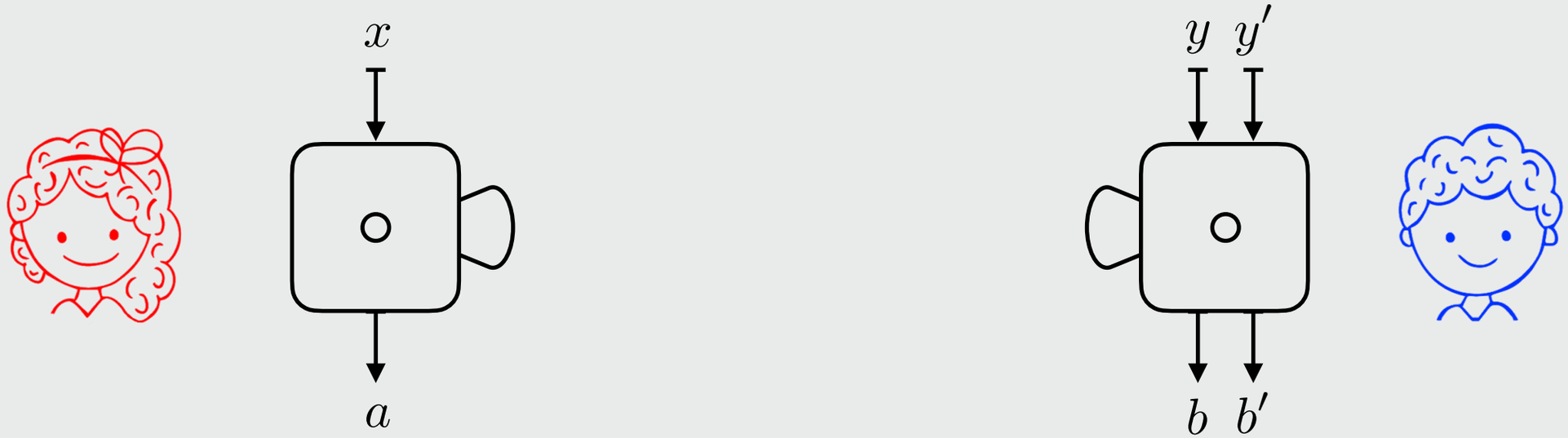

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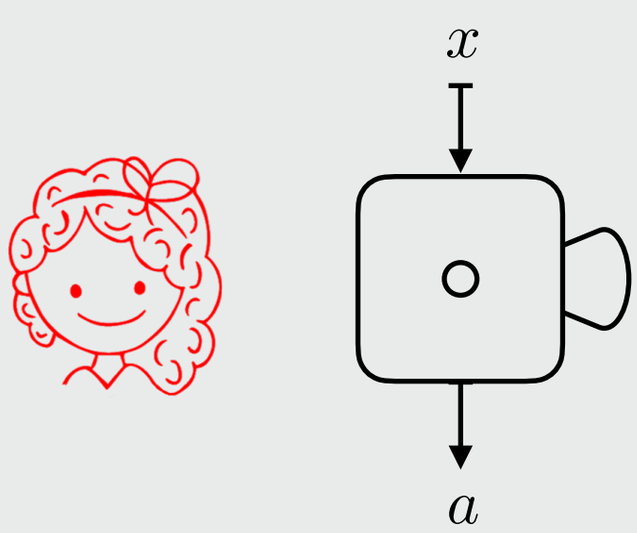
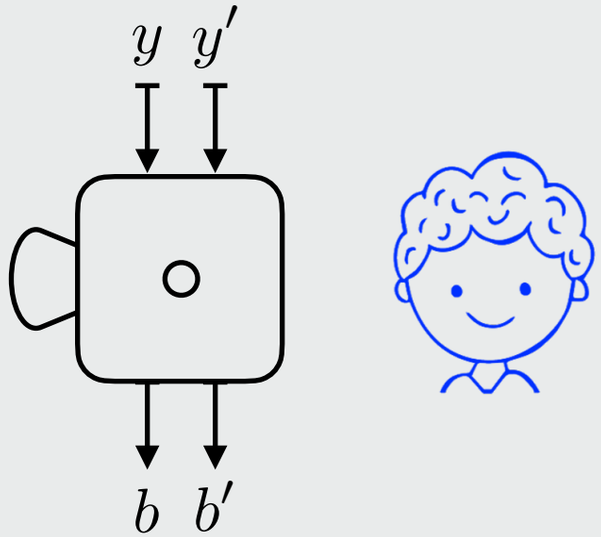
Locality

Extended Bell scenario





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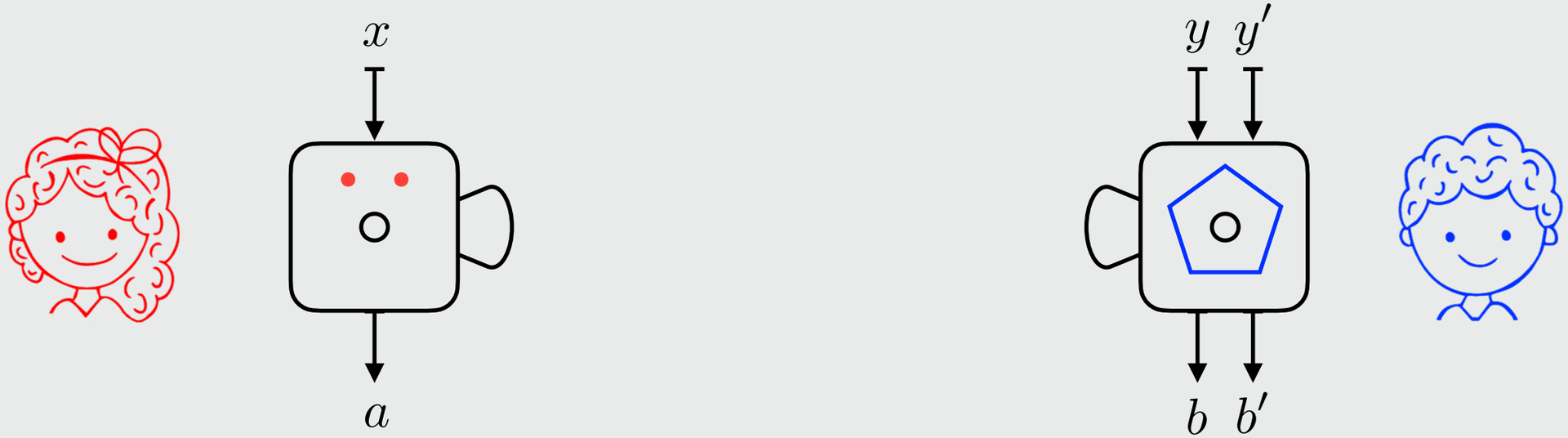
Locality

Noncontextuality

Monogamy between CHSH and KCBS



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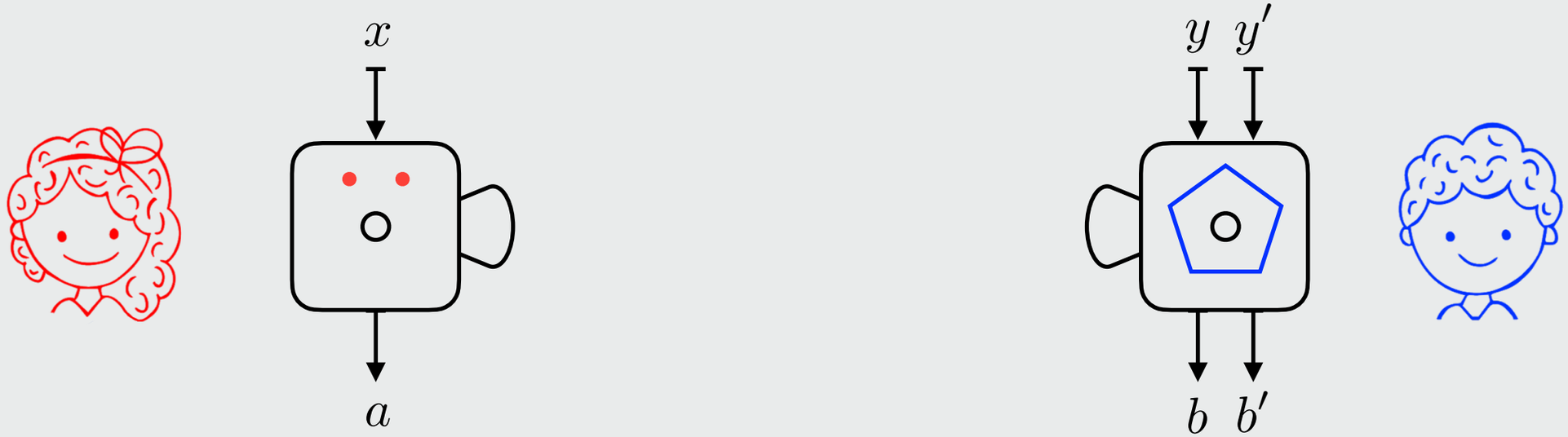

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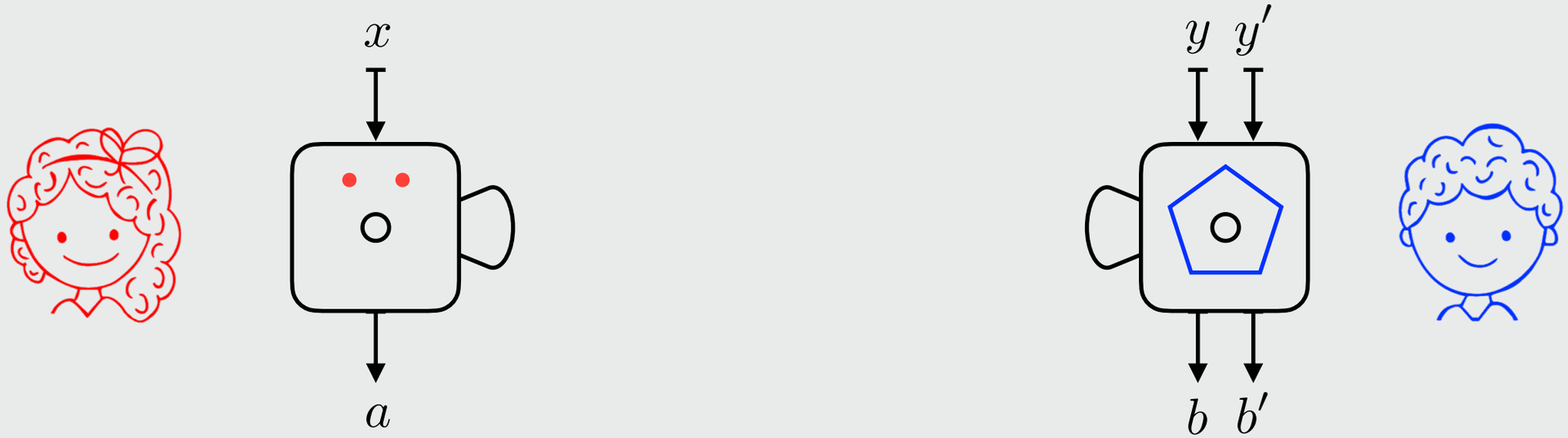

- There is no quantum behavior $p(a, b, b'|x, y, y')$ such that [Kurzynski etal (2014)]:

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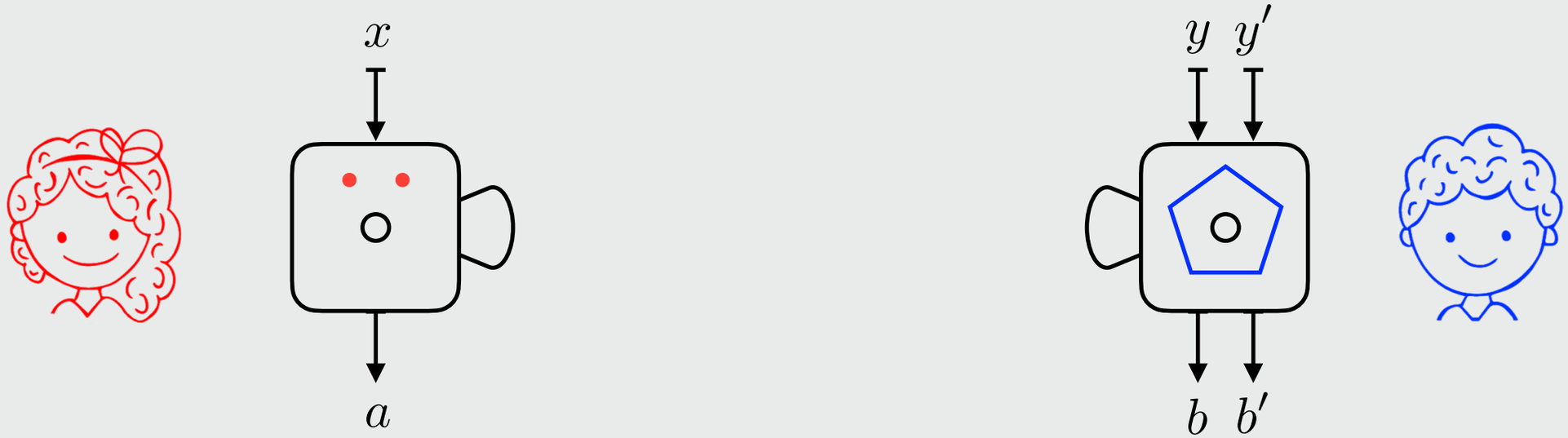

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  - $p(a, b|x, y)$ violate the CHSH inequality; and

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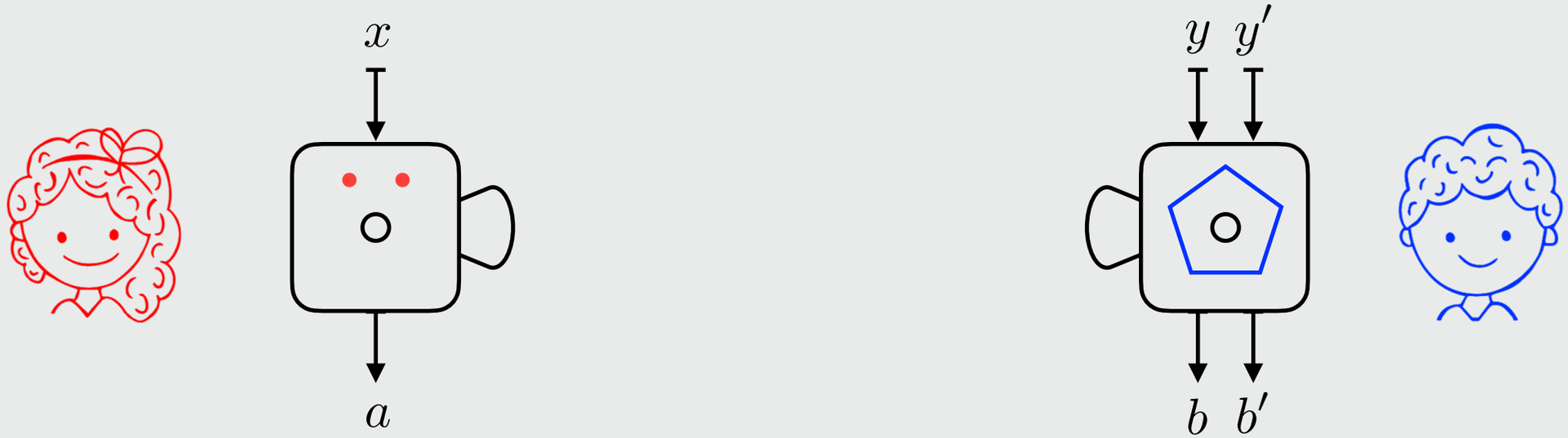

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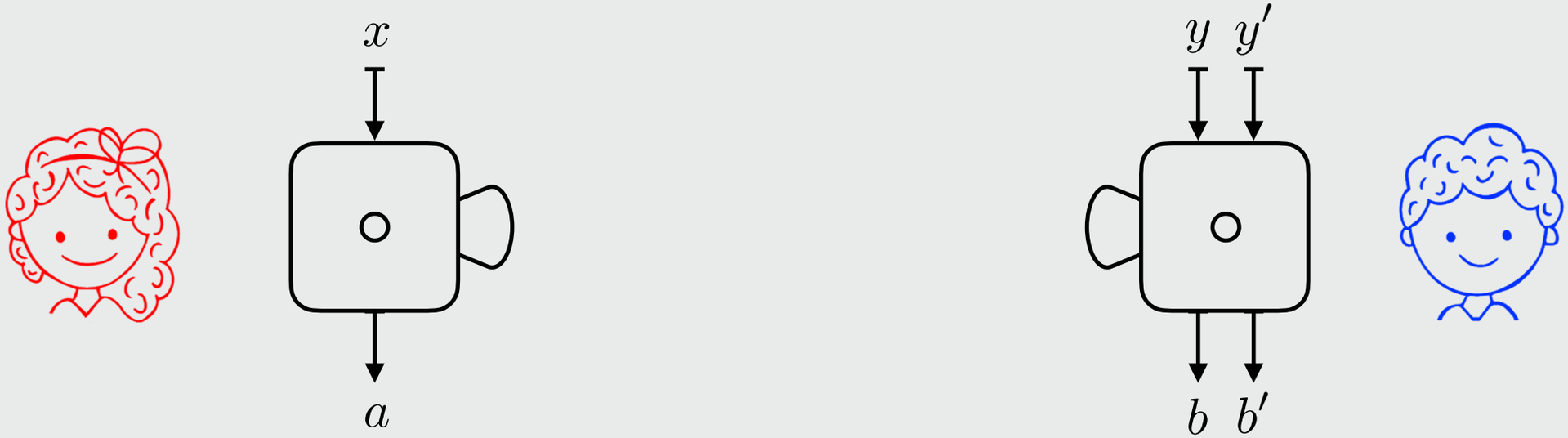
Locality in extended Bell scenarios



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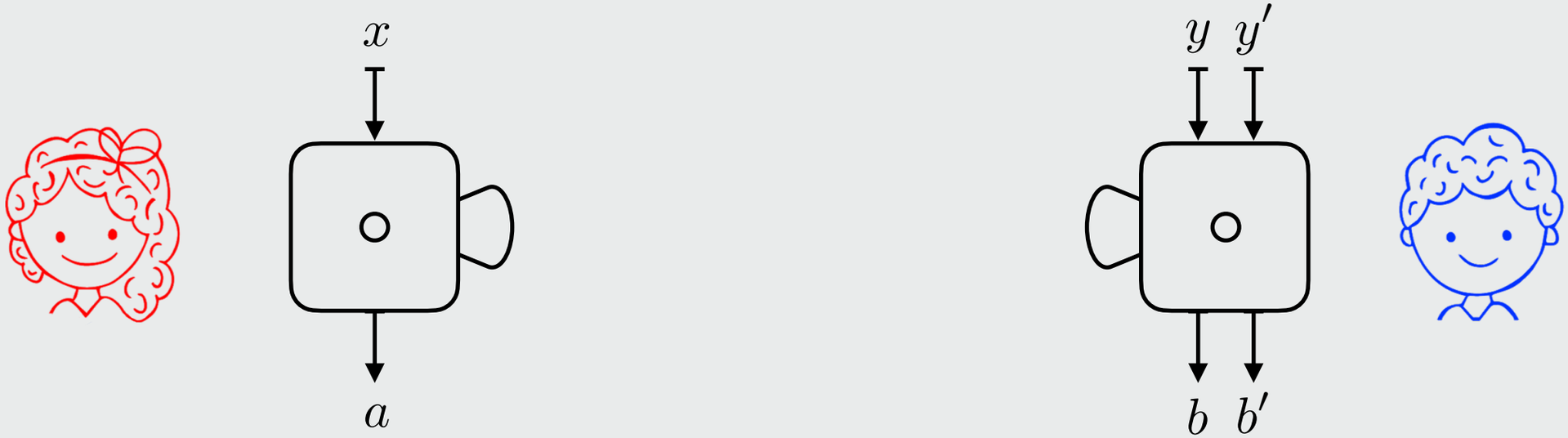

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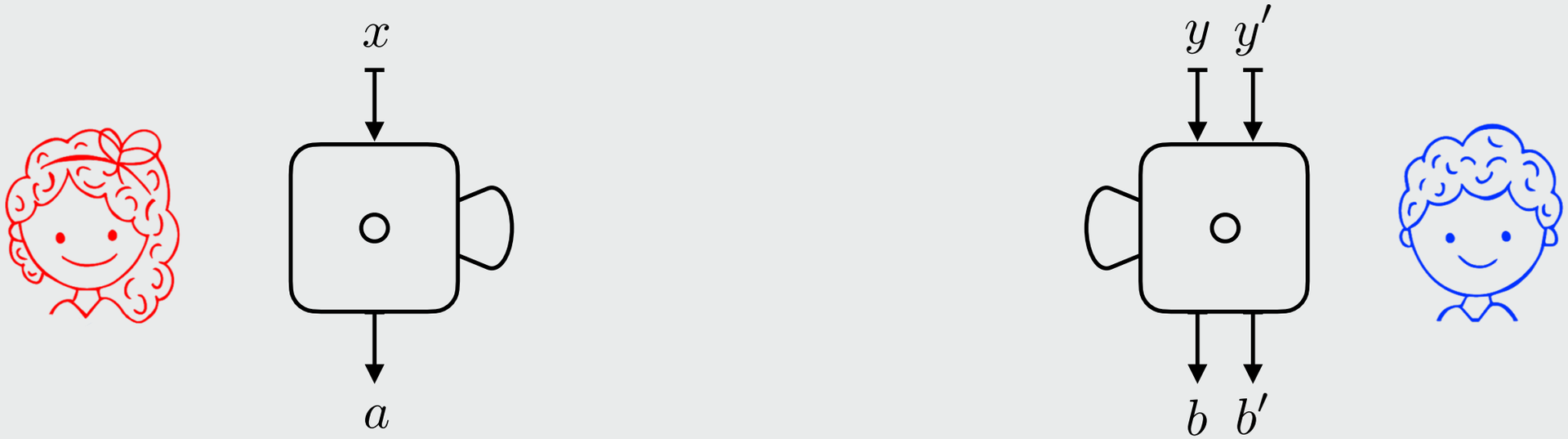

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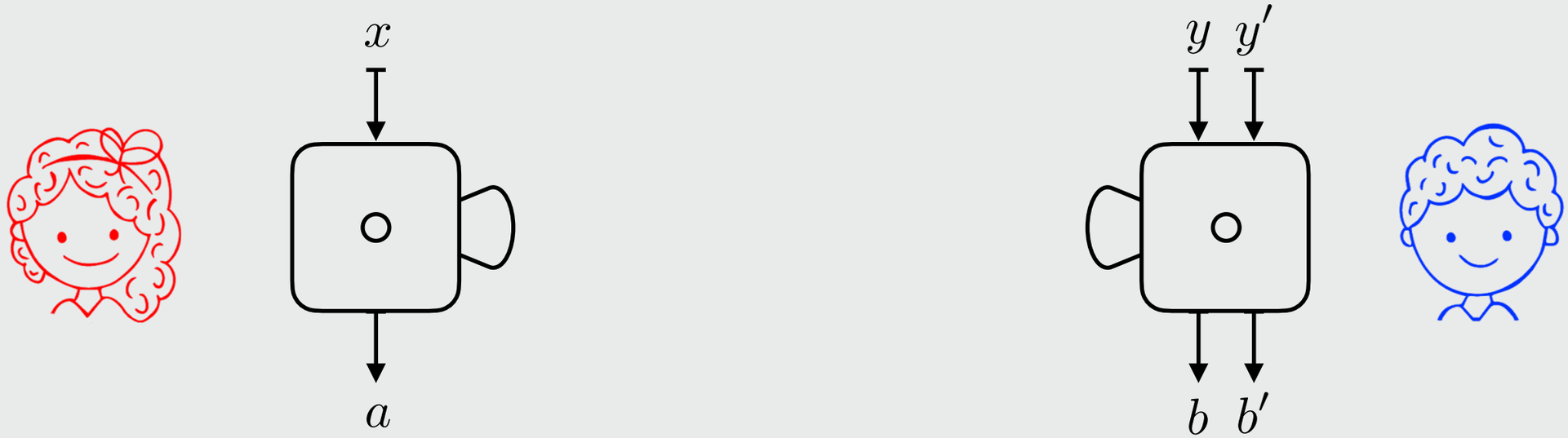

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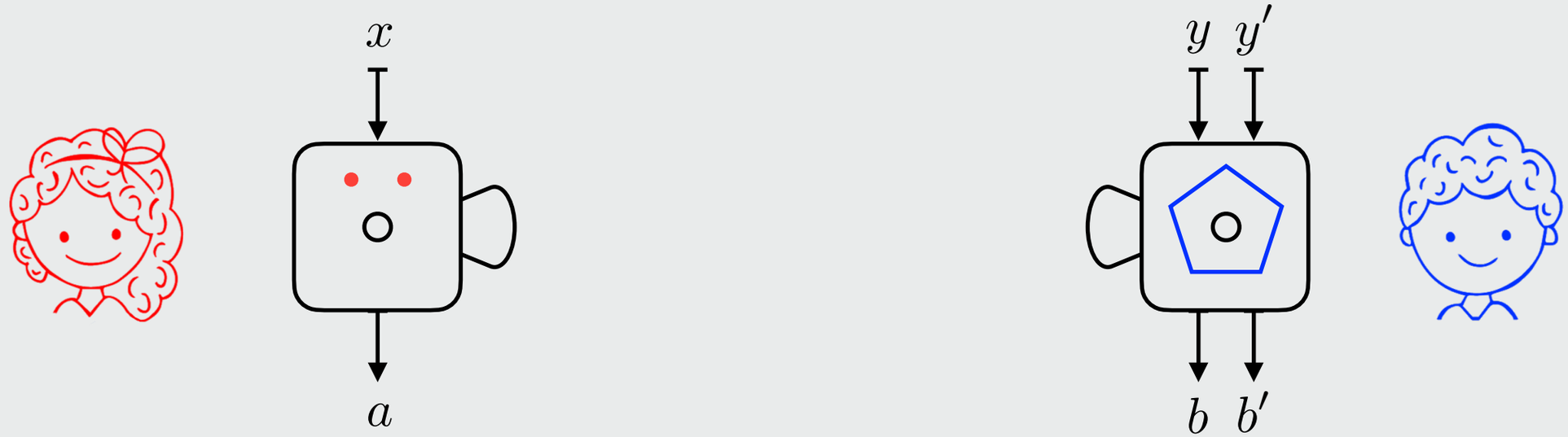
- Locality [Temistocles etal (2019)]:

$$p(a, b, b'|x, y, y') = \int p(a|x, \lambda)p(b, b'|y, y', \lambda)p(\lambda)d\lambda$$

CHSH inequalities



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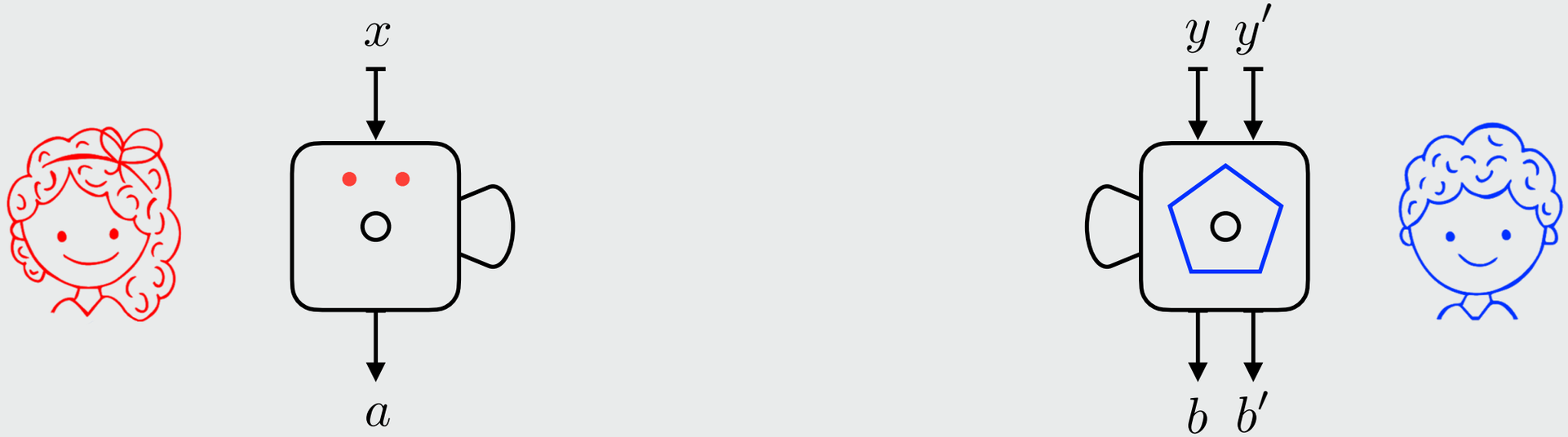

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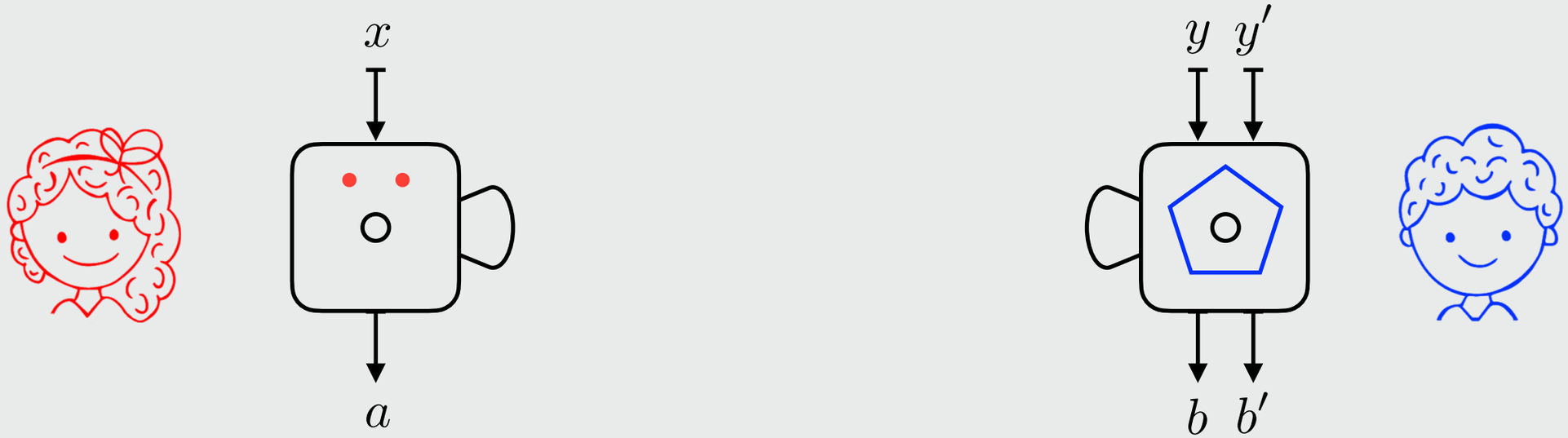

- CHSH_1 inequality (alternative version 1):

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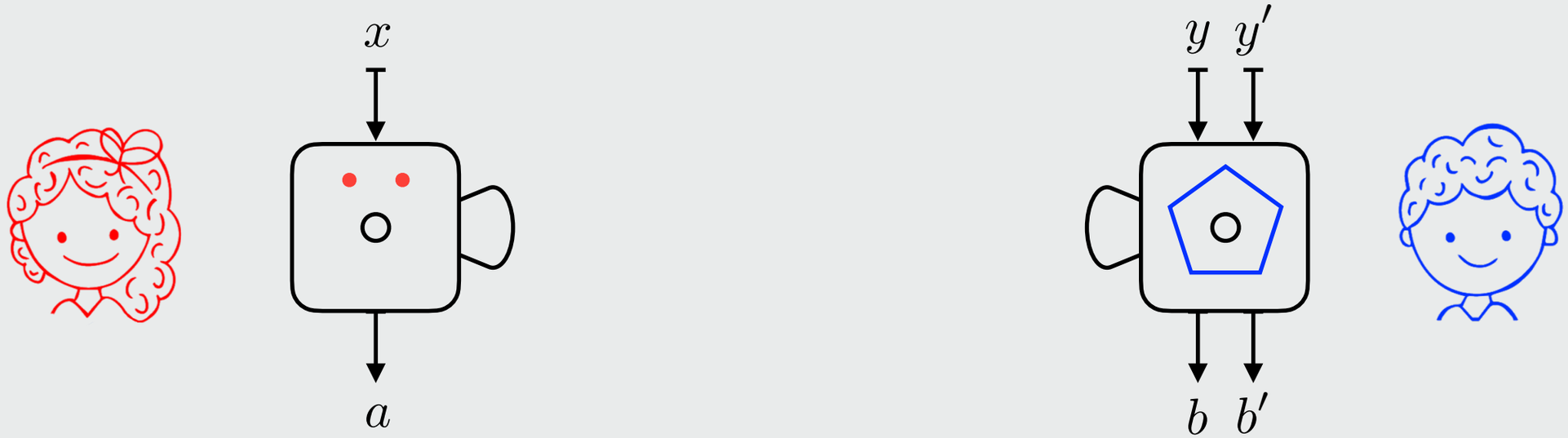
CHSH inequalities



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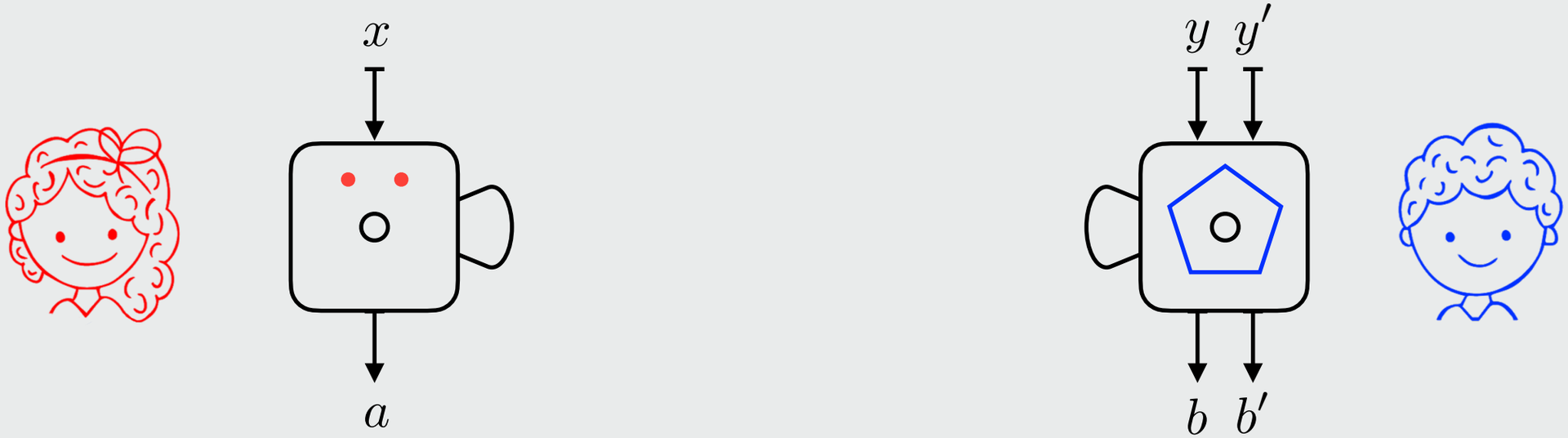
- CHSH_2 inequality (alternative version 2):

$$\langle A_0 B_0 B_1 \rangle + \langle A_0 B_2 B_3 \rangle + \langle A_1 B_0 B_1 \rangle - \langle A_1 B_2 B_3 \rangle \leq 2$$

Nonmonogamy between CHSH_1 and KCBS



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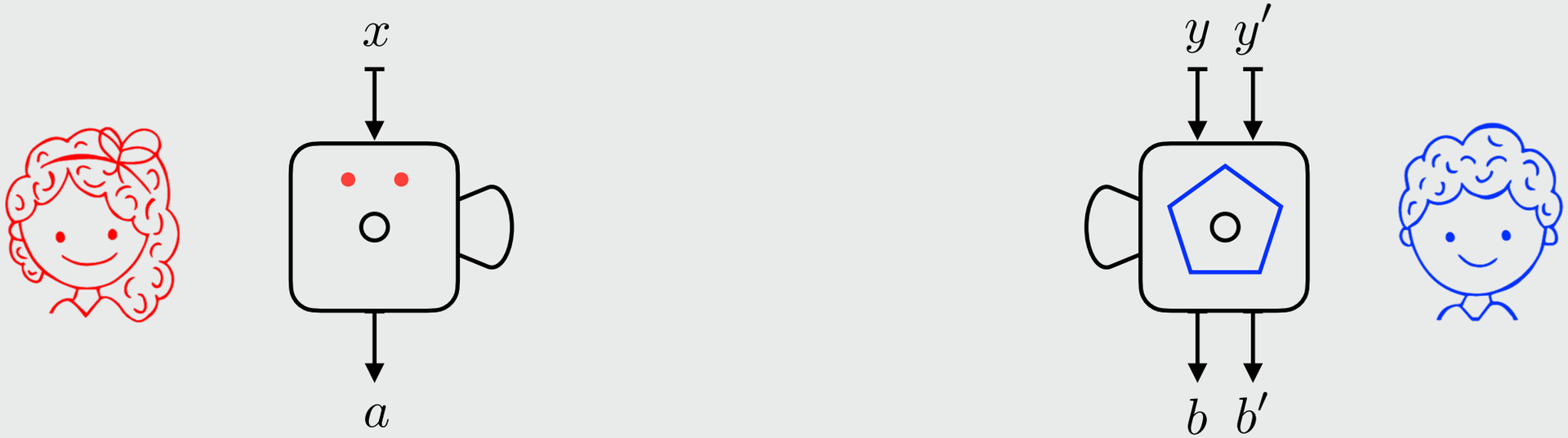

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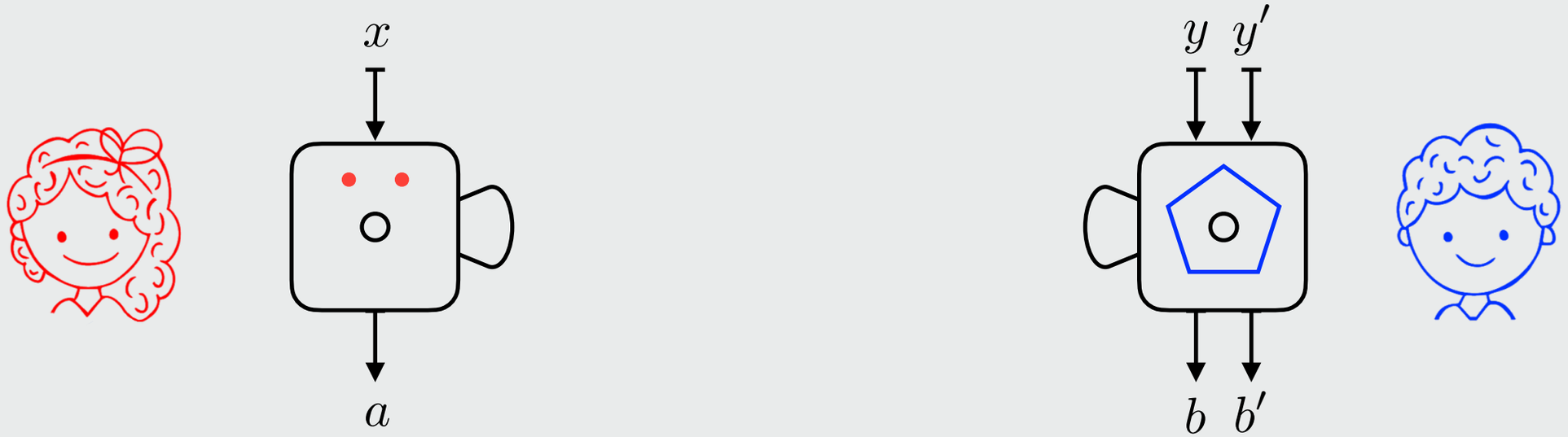

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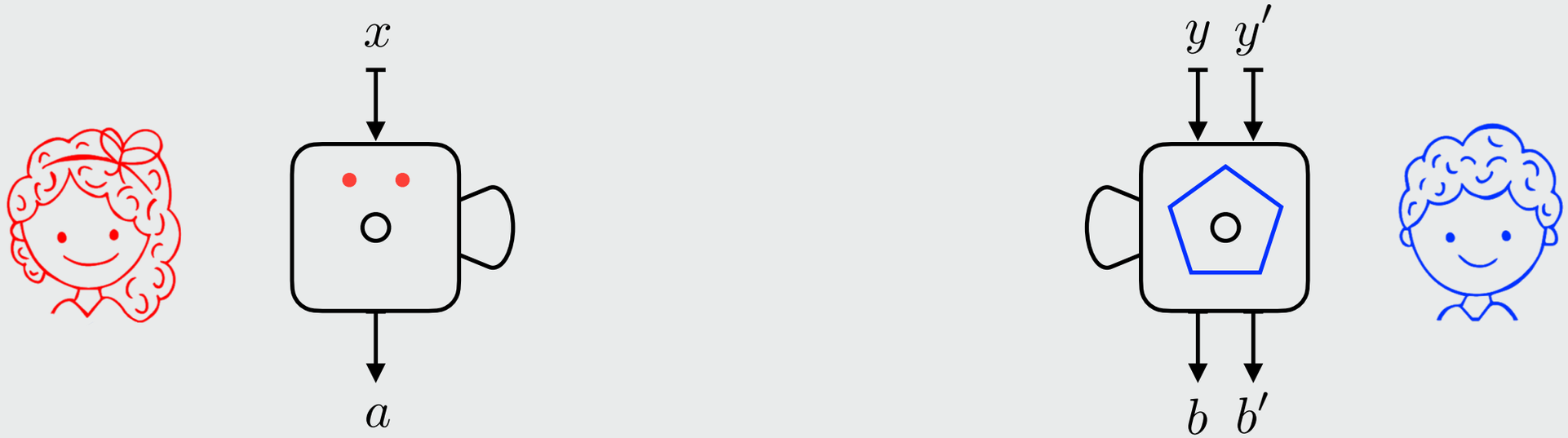

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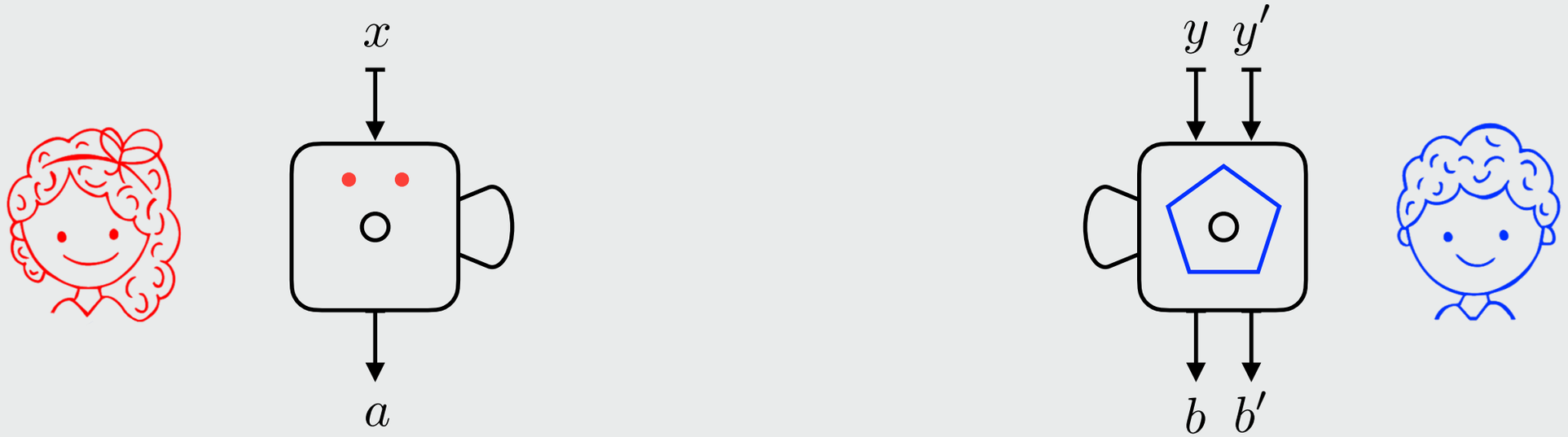

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Sketch of the methods

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- We fixed the measurements of Alice and Bob:

Sketch of the methods

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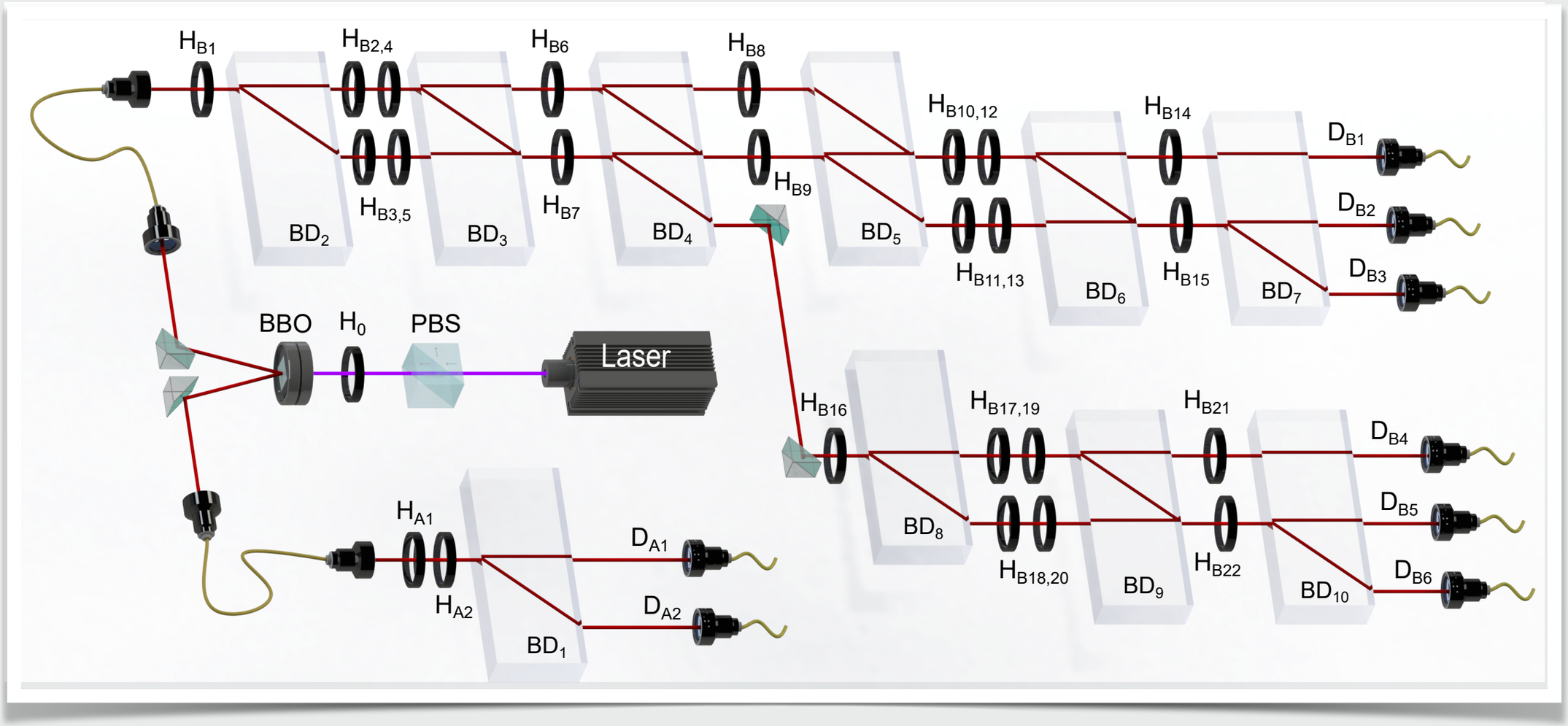
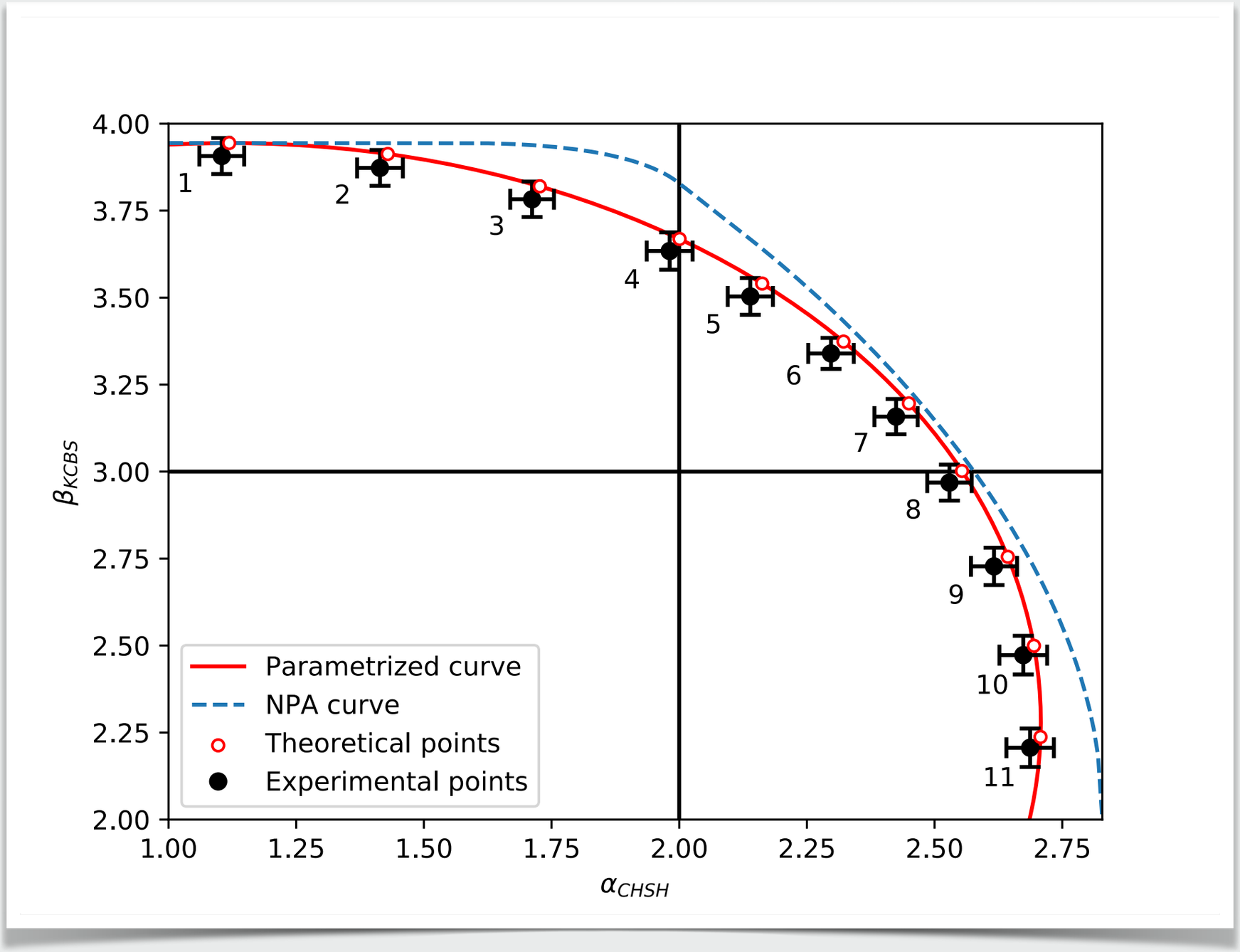
- Then, we found a one-parameter family of pure states such that, for a specific range of the parameter, both CHSH_1 and KCBS are simultaneously violated.

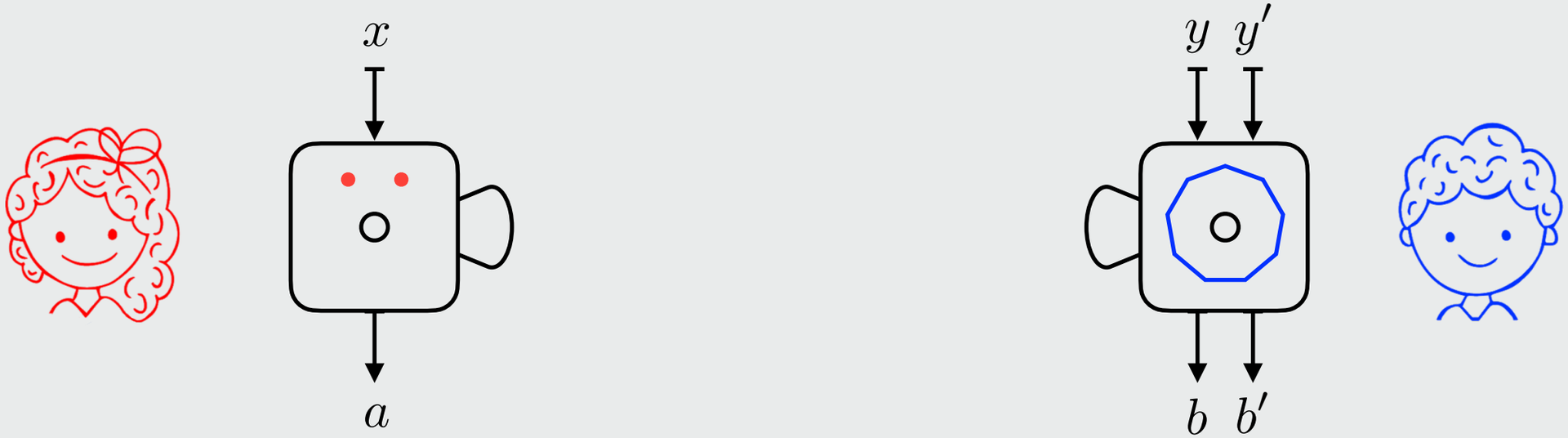
Photonic implementation



Results


4.00
3.75
3.50
3.25
3.00
2.75
2.50
2.25
2.00
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
β_{KCBS}
α_{CHSH}
1
2
3
4
5
6
7
8
9
10
11
Parametrized curve
NPA curve
Theoretical points
Experimental points

Generalization to n-cycles



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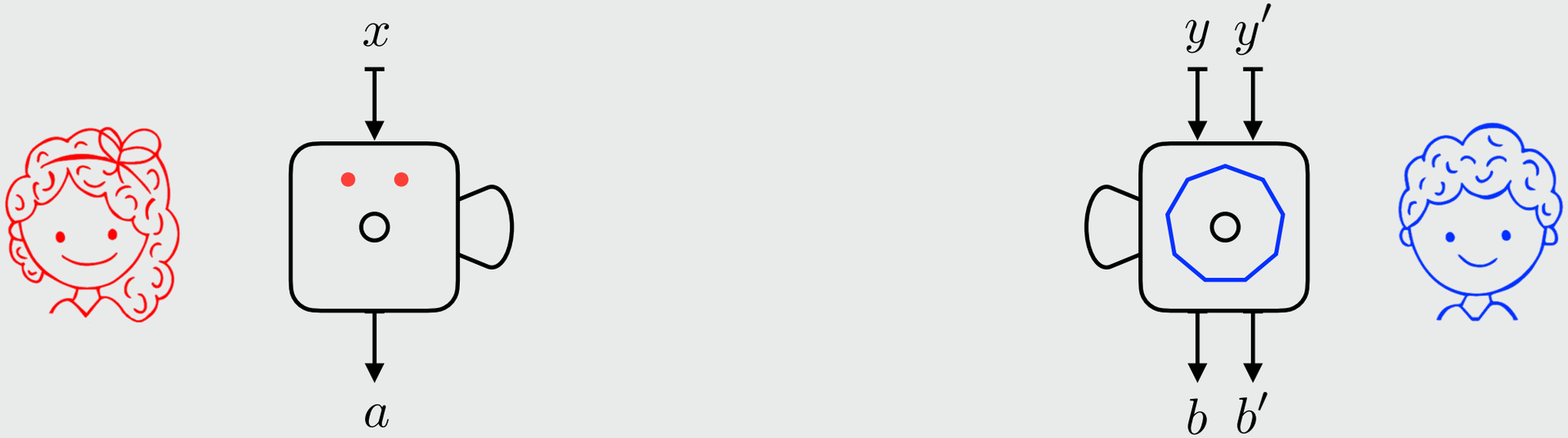

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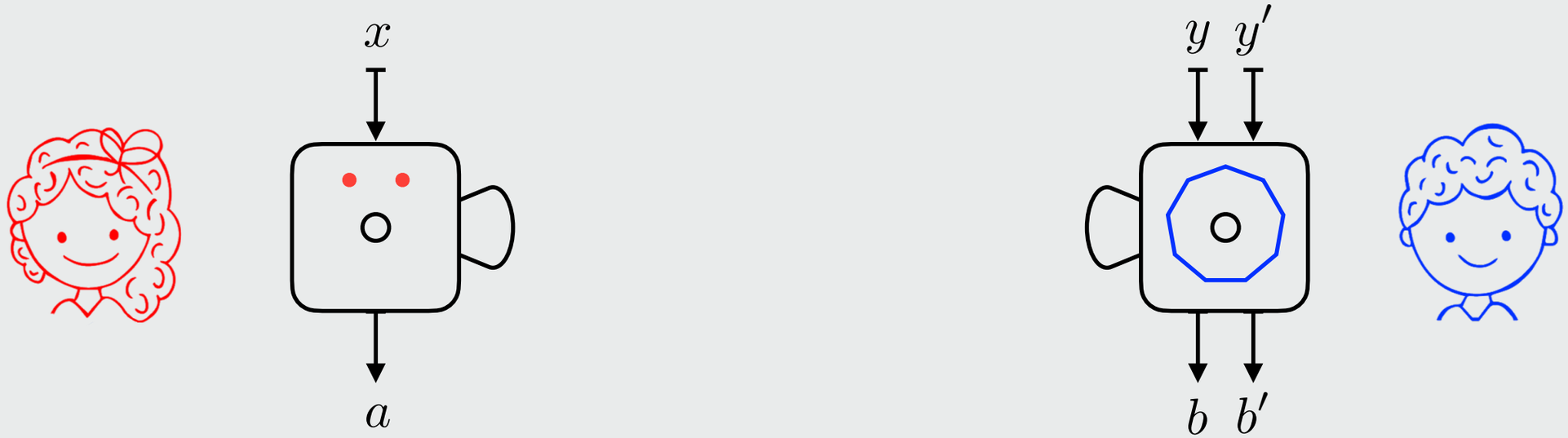

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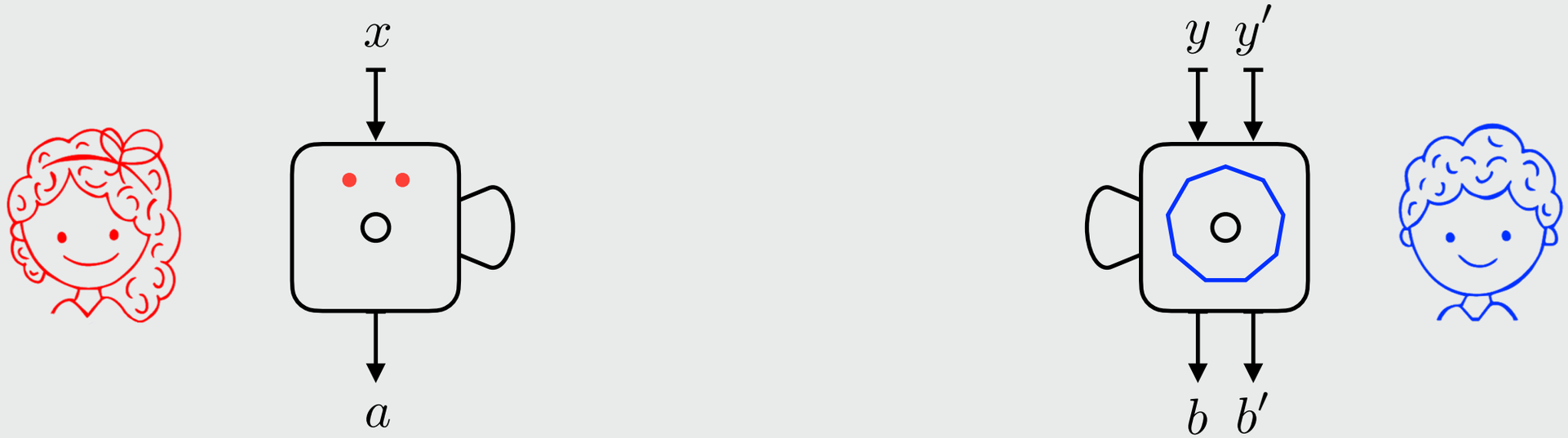

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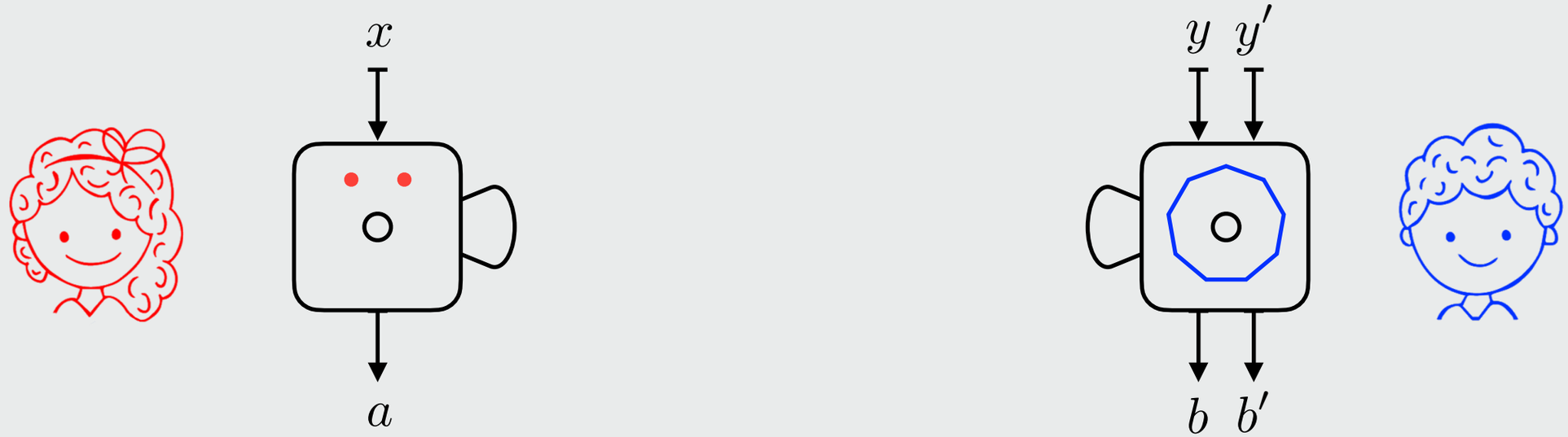

- There are quantum behaviors $p(a, b, b'|x, y, y')$ such that:
  - $p(a, b, b'|x, y, y')$ violate the CHSH_2 inequality; and
  - $p(b, b'|y, y')$ violate the n-cycle inequality (for n<10000).

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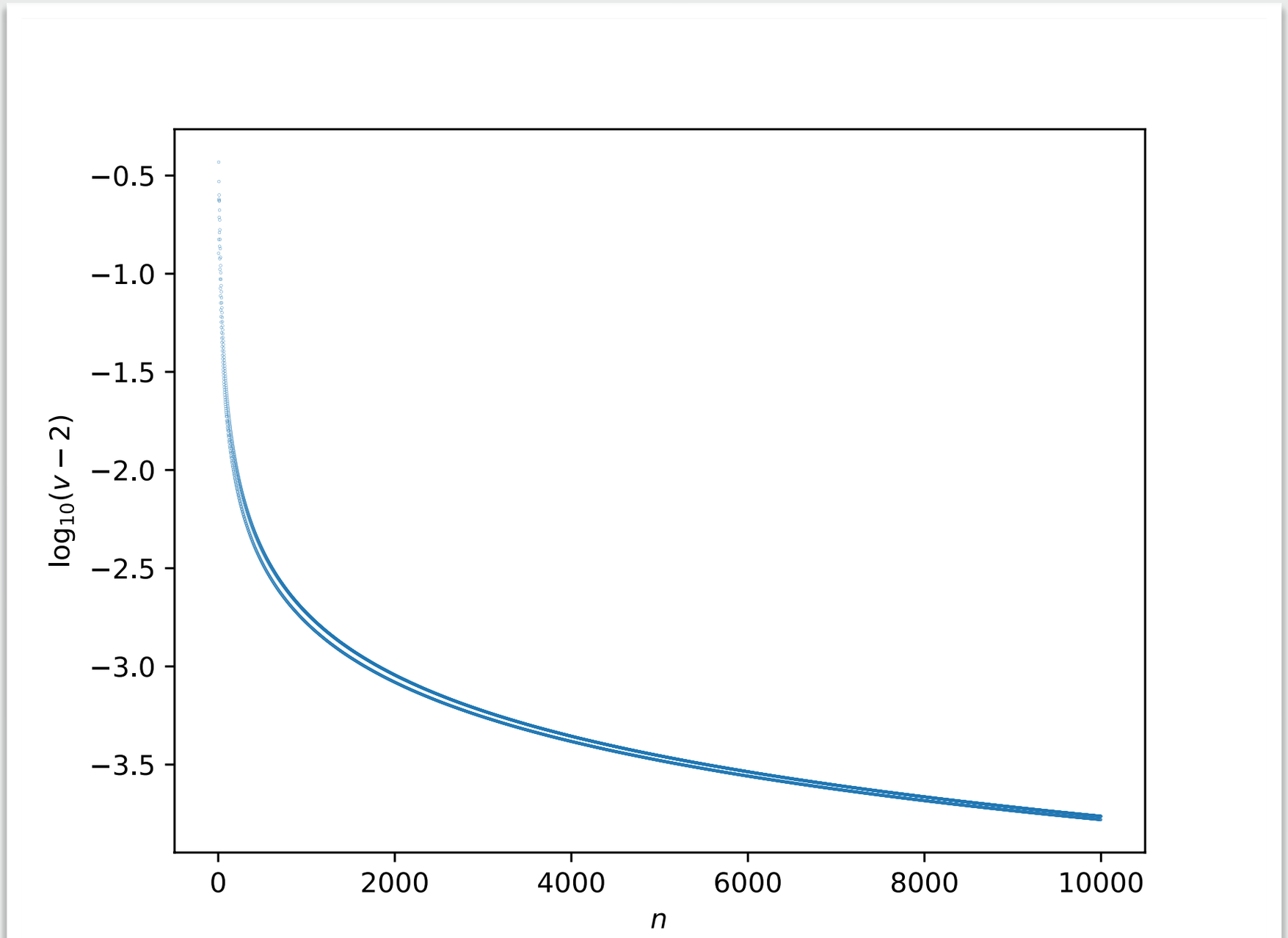
- Then, we numerically optimized the violation of the CHSH_2 inequality, over quantum states, imposing that the n-cycle inequality is violated by, at least, 0.001.

Results



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- This has been experimented and observed in a photonic implementation.

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- By using a standard definition of locality, quantum systems cannot concomitantly violate the CHSH and the KCBS inequalities.

- By using a more general definition of locality, quantum systems can concomitantly violate the CHSH_1 and the KCBS inequalities.

- This has been experimented and observed in a photonic implementation.

- More generally, quantum systems can concomitantly violate the CHSH_2 and the n-cycle inequalities (up to $n<10000$).

Thank you for your attention!

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