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Visual Kinematics
I. Visual Space Metric in Visual Motion

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Visual deformations of moving objects are traditionally attributed to visual mechanisms, such as spatiotemporal summation and visual masking, that transform the distribution of color/brightness in visual space. This paper presents experimental proof that in addition to these distributional transformations, the metric of visual space itself changes in visual motion. Under steady fixation conditions the perceived spatial separation between non-interacting visual images moving with the same physical velocity shrinks along the direction of motion, but not transversely. The ratio of the perceived longitudinal separation-in-motion to the separation-at-rest does not depend on the latter. This ratio decreases with increasing velocity. A simple principle (Mapping Homogeneity) is proposed and experimentally corroborated that conceptually separates geometric transformations in motion from all conceivable transformations of color/brightness distributions in visual space.

1. INTRODUCTION

An object moving in the perceptually frontoparallel plane generally looks different from the same object when stationary. That certain visual deformations in motion should be expected follows from the well-established general principle: in vision, perceptual characteristics assigned to any spatiotemporal position depend on the light distribution within a spatiotemporal neighborhood of this position. The dependence is described in terms of light (luminance) integration–interaction mechanisms in the visual system. Two such (groups of) mechanisms are known to contribute to visual deformations-in-motion.

Visual smear, sometimes crudely described as an apparent elongation of the moving image along the direction of motion, is attributed to spatiotemporal luminance integration. This integration is more complex than a concatenation of

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independent temporal summations occurring within stationary spatial positions along the motion trajectory (Efron, 1970; Efron & Lee, 1971; Smith, 1969). For unidirectional motion the amount of smear is smaller than predicted from independent temporal summations within stationary positions (Burr, 1980), indicating that the summation might occur within spatial areas moving along with the moving image ("spatiotemporal receptive fields" hypothesis: Burr, 1981; Burr, Ross, & Morrone, 1986; see also Adelson & Bergen, 1985; Watson & Ahumada, 1985).

Another mechanism that could be involved in perceived deformations-in-motion is visual masking of a moving stimulus by its preceding and following positions (Day, 1973). This mechanism has been proposed for the Ansbacher effect, crudely described as an apparent contraction of a rotating arc (Ansbacher, 1938, 1944; Stanley, 1964, 1966, 1968), but it can also be applied to the "length contraction" effect in uniform rectilinear motion described by Caelli *et al.* (1978).

Deformations-in-motion due to light integration–interaction mechanisms change the distribution of color/brightness in visual space without affecting its frontoparallel geometry, the metric of spatial relations between visual events in the left–right–above–below plane (Indow & Watanabe, 1988). In this paper the effect of the integration–interaction mechanisms will be referred to as the *distributional* deformations in motion. No evidence has previously been obtained that the frontoparallel metric itself might transform in visual motion. Moreover, it has not previously been clear whether the "geometric" and "distributional" transformations are operationally separable concepts or whether the difference is only terminological. Indeed, if a deformation-in-motion is taken as indicating a change in the frontoparallel metric, is it not always possible to equivalently describe it as a change of the color/brightness distribution in a metrically fixed space (by referring to a known or inventing an ad hoc integration–interaction mechanism)?

In this situation the principle of Occam's razor makes it preferable to assume that *all* deformations-in-motion are distributional (simply because distributional deformations do take place), and the frontoparallel metric of the visual space is in a fixed correspondence with that of the physical space. This assumption is implicit in practically all psychophysical literature dealing with 2-D perception. This is especially clear when visual perception is discussed in terms of linearity/nonlinearity of spatiotemporally invariant operators mapping luminance distribution, $l(x, y, t)$, into brightness distribution, $L(x, y, t)$, in the same coordinates (otherwise the concept of invariance is not defined). The usual practice, for example, is to graph a luminance profile against a spatial or temporal axis, transform the luminance according to a certain rule (in linear analysis by convolving the profile with a point spread or impulse response function), and to refer to the result as a brightness profile (see any analysis of the Mach bands, e.g., Lowry & DePalma, 1961).[1]

[1] For stationary stimuli this approach is justifiable: evidence presented in this paper and in other work referred to in the text suggests that in this case the frontoparallel spatial relations in vision and in the stimulation plane are metrically isomorphic.

This paper presents experimental proof that the frontoparallel metric of perceived spatial relations changes in visual motion in a regular way.[2] A simple principle (Mapping Homogeneity) is proposed and experimentally corroborated that allows one to conceptually separate kinematic transformations of the spatial geometry from distributional changes due to the integration–interaction mechanisms of vision. Based on this principle an experimental paradigm is described (Double-Perturbation paradigm), in which geometric transformations are measured in a pure form, isolated from all conceivable distributional deformations-in-motion.

This paper is the first in a series of three Visual Kinematics papers. Although all three are structured so that they can be read independently, they tell a cumulative story of how the perceptual metric of spatiotemporal relations changes in visual motion (only spatial relations are dealt with in the experiments described, but perceived time enters as an integral component in the theory of spatial transformations in motion). The following brief overview of the second and third papers provides a broader context for the present paper.

Visual Kinematics II (Dzhafarov, 1992a) deals with the relationship between perceived spatial transformations and velocity. The transformations are shown to occur under both steady fixation and free looking conditions: perceived spatial intervals contract along the direction of motion as a function of velocity defined in external (rather than retinal) coordinates. For any given velocity the contraction magnitude increases as a function of factors which increase perceived speed, suggesting that the metric of visual space depends on perceived rather than physical velocity. For example, perceived speed is greater under steady fixation than under free looking, and the contraction magnitude is also greater for steady fixation. In all cases, however, the relationship between physical speed and the contraction magnitude is described by a two-piece power function with some remarkable properties.

Visual Kinematics III (Dzhafarov, 1992b) presents experimental evidence that perceptual deformations-in-motion are generally a complex mixture of geometric and distributional transformations. The logic of the Double-Perturbation paradigm is not only sufficient (as demonstrated in the present paper) but is also necessary for isolating purely geometric transformations. Among other things, this means that geometric analysis is not directly applicable to apparent length contractions and elongations of a moving light segment. The paper also discusses a complex relationship between kinematic transformations and dynamic spatial acuity. In the

[2] The terms "metric" and "geometry" are used to refer to the metrical values of spatial intervals taken along the direction of motion and in the orthogonal direction: the problem studied is whether these metrical values change in motion. This work does not provide a means for reconstructing the dependence of the spatial intervals obliquely oriented with respect to the motion path on their orthogonal projections: thus one cannot speak of the geometry in motion as being Euclidean, Riemanian, etc. Indow and Watanabe (1988) have shown that for stationary stimuli the frontoparallel geometry (or metric) of the visual space is Euclidean, at least to a first approximation. It is likely that this property should not change in motion, but this remains to be demonstrated empirically.

same paper I propose a theoretical language for transformations of spatiotemporal coordinates in visual motion. These transformations are shown to share certain formal properties with the Lorentz and Galileian transformations in physical kinematics (namely, the transformations are linear in coordinates), but differ from them in other fundamental aspects. They do not obey the Galileian relativity principle and do not form a uniparametric group with respect to velocity (Lorentz group).

2. Mapping Homogeneity Principle (MHP)

2.1. *Formulation of the MHP*

Throughout this paper uppercase and lowercase symbols refer to perceptual and physical characteristics, respectively; boldface roman symbols denote vectors; angle brackets denote axes or frames of reference. Let $\langle x, y, t\rangle$ and $\langle X, Y, T\rangle$ be, respectively, physical and perceptual spatiotemporal frames of reference,[3] and let $\mathbf{l}(x, y, t)$ and $\mathbf{L}(X, Y, T)$ denote visual stimulation and the resultant perceptual image (distributions of local physical and perceptual characteristics of light, respectively). The MHP is the proposition

$$\begin{aligned} &\text{if} && \mathbf{l}(x, y, t) \to \mathbf{L}(X, Y, T), \\ &\text{then} && \mathbf{l}(x+\Delta x, y+\Delta y, t+\Delta t) \to \mathbf{L}(X+\Delta X, Y+\Delta Y, T+\Delta T), \end{aligned} \tag{1}$$

where the arrows stand for "is mapped to," or "is perceived as." Thus, if two visual stimuli are shifted replicas of each other, then so are their perceptual images.

Due to retinal inhomogeneities, if a steady fixation is maintained, the MHP may hold only approximately with respect to spatial shifts: thus, tiny spatial details seen in the fovea can disappear in peripheral vision. I will show, however, that deviations from the MHP, if any, are too small to significantly contribute to the results discussed below.

The perceptual shift, $(\Delta X, \Delta Y, \Delta T)$, depends on the physical shift, $(\Delta x, \Delta y, \Delta t)$, but in the general formulation of the MHP it may also depend on certain parameters of the light distribution being shifted, $\mathbf{l}(x, y, t)$.[4] It is reasonable to assume that the absolute spatiotemporal localization of the $\mathbf{l}(x, y, t)$ is not among

[3] A steady fixation was maintained in all experiments reported in this paper. It is not important, therefore, whether $\langle x, y\rangle$ are understood as external or retinal coordinates. It is possible that in a more rigorous analysis $\langle x; y\rangle$ should be complemented by the depth dimension $\langle z\rangle$ with z-values adjusted so that the (x, y)-values form a perceptually (rather than physically) frontoparallel plane (horopter surface; Indow & Watanabe, 1988). It seems, however, that in the relatively small (x, y)-area considered in this paper the two concepts are roughly equivalent.

[4] To avoid confusion: a stimulus "being shifted" does not imply any physical operation of shifting. This should be obvious when "shifting" is in time. Spatial "shifting" has exactly the same abstract meaning: it is a purely mathematical term (operation) meaning that consideration is being switched from one stimulus to another, which is a shifted replica of the former.

those parameters. This means that a replica of $l(x, y, t)$ shifted with respect to the latter by a sum of two physical shifts, $(\Delta x_1, \Delta y_1, \Delta t_1)$ and $(\Delta x_2, \Delta y_2, \Delta t_2)$, is mapped into a replica of $L(X, Y, T)$ shifted by the sum of the corresponding perceptual shifts:

if

$$l(x+\Delta x_1, y+\Delta y_1, t+\Delta t_1) \rightarrow L(X+\Delta X_1, Y+\Delta Y_1, T+\Delta T_1),$$

and

$$l(x+\Delta x_2, y+\Delta y_2, t+\Delta t_2) \rightarrow L(X+\Delta X_2, Y+\Delta Y_2, T+\Delta T_2),$$

then

$$\begin{aligned} &l(x+\Delta x_1+\Delta x_2, y+\Delta y_1+\Delta y_2, t+\Delta t_1+\Delta t_2) \\ &\quad \rightarrow L(X+\Delta X_1+\Delta X_2, Y+\Delta Y_1+\Delta Y_2, T+\Delta T_1+\Delta T_2). \end{aligned} \tag{2}$$

Another way to formulate this property of perceptual mapping is to say that all axes in the two frames of reference, $\langle x, y, t\rangle$ and $\langle X, Y, T\rangle$, are interval scales. In the rest of the paper, the MHP will refer to the general formulation (1) complemented by the assumption of the homogeneity of axes (2). I will assume also that the axes in the two frames of reference are "matched"; i.e., $(\Delta X, \Delta Y, \Delta T)$ depends on $(\Delta x, \Delta y, \Delta t)$ only componentwise. As shown in the Appendix, the MHP implies that perceptual shifts are proportional to physical shifts, with proportionality coefficients depending, in general, on the stimulus being shifted,

$$\begin{aligned} \Delta X &= \phi_{xX}(\mathbf{p})\, \Delta x \\ \Delta Y &= \phi_{yY}(\mathbf{p})\, \Delta y \\ \Delta T &= \phi_{tT}(\mathbf{p})\, \Delta t, \end{aligned} \tag{3}$$

where $\mathbf{p}$ stands for a vector of stimulation parameters, invariant with respect to all possible physical shifts:

$$\mathbf{p} = \mathbf{p}\{l(x, y, t)\} = \mathbf{p}\{l(x+\Delta x, y+\Delta y, t+\Delta t)\}. \tag{3*}$$

2.2. *Mapping of Stationary and Moving Luminance Distributions*

Figure 1 illustrates the MHP for a spatially local light perturbation of a homogeneous field, at rest (panel pairs aA and bB) and in motion (cC and dD), respectively. Denote the stationary light perturbation shown in Fig. 1a by $l_s(x, y)$, and its visual image (Fig. 1A) by $L_s(X, Y)$. According to the MHP, a shifted replica of $l_s(x, y)$ (Fig. 1b) is mapped into a shifted replica of $L_s(X, Y)$ (Fig. 1B):

$$\begin{aligned} &l_s(x+\Delta x, y+\Delta y) \rightarrow L_s(X+\Delta X_s, Y+\Delta Y_s), \\ &(\Delta X_s, \Delta Y_s) = (\phi_{xX}(\mathbf{p})\, \Delta x, \phi_{yY}(\mathbf{p})\, \Delta y). \end{aligned} \tag{4}$$

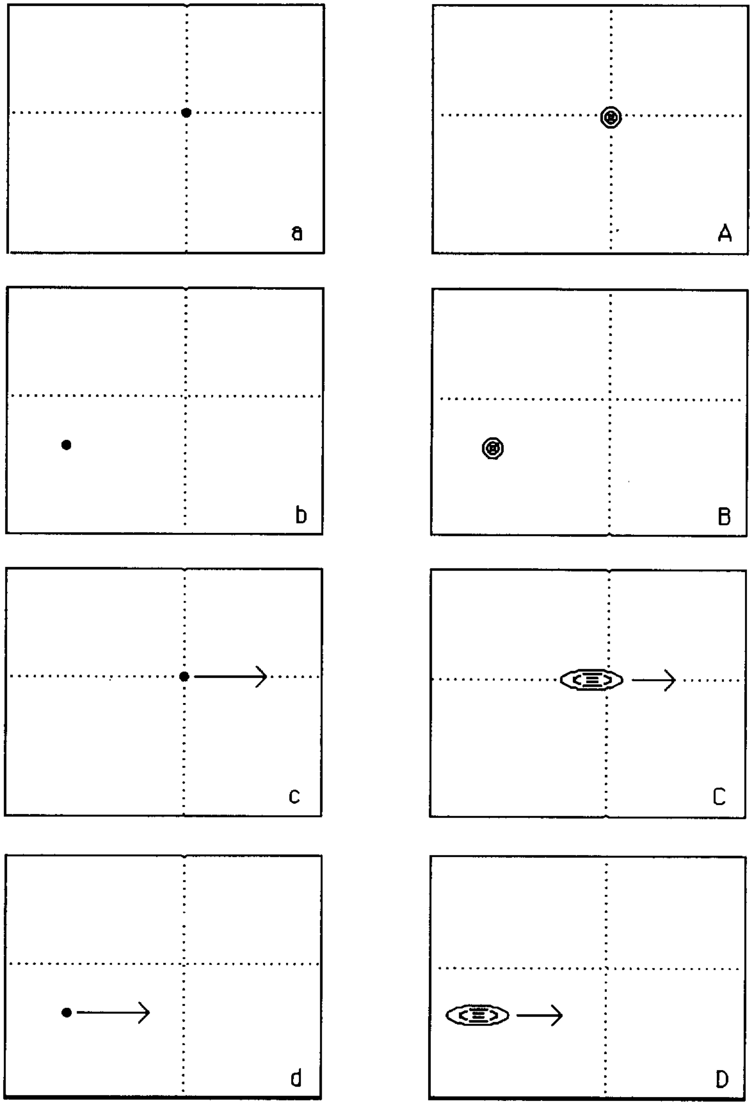

FIG. 1. Schematic illustration of the MHP as applied to stationary and moving perturbations of a homogeneous luminance field. Right panels show perceptual images of the corresponding left panels. See Sections 2.2 and 2.3 for explanations.

When $l_s(x, y)$ is set in motion with velocity v along the $\langle x \rangle$-axis (Fig. 1c), the light distribution is described as $l_s(x - vt, y)$. As shown in the Appendix, it follows from the MHP that this stimulus is perceived as a color/brightness profile, $L_m(X, Y)$, moving along the $\langle X \rangle$-axis with a constant velocity, V (Fig. 1C):

$$l_s(x - vt, y) \to L_m(X - VT, Y). \tag{5}$$

The moving shape is subject to distributional deformations, and, in general, depends on velocity v:

$$L_s(X, Y) \neq L_m(X, Y). \tag{6}$$

According to the MHP, a spatially shifted replica of the moving stimulus (Fig. 1d) maps into a spatially shifted replica of its perceptual image at every moment T of perceptual time (Fig. 1D):

$$\begin{aligned} l_s(x+\Delta x - vt, y+\Delta y) &\to L_m(X+\Delta X_m - VT, Y+\Delta Y_m) \\ (\Delta X_m, \Delta Y_m) &= (\phi_{xX}(\mathbf{p}, v)\,\Delta x, \phi_{yY}(\mathbf{p}, v)\,\Delta y). \end{aligned} \tag{7}$$

The proportionality coefficients can be written in this form, $\phi_{..}(\mathbf{p}, v)$, because, compared to (4), velocity v is the only difference between the two stimuli. Strictly speaking, this means a redefinition of $\mathbf{p}$: it stands now for all stimulation parameters, *except velocity*, that can affect the proportionality coefficients.

2.3. *Fixed Metric versus Motion-Dependent Metric*

The question of whether the frontoparallel spatial metric changes in visual motion now reduces to the question of whether the following equality holds:

$$(\Delta X_m, \Delta Y_m) = (\Delta X_s, \Delta Y_s). \tag{8}$$

A positive answer follows from the common postulate mentioned earlier that the frontoparallel metric of visual space is in a fixed correspondence with that of physical space; i.e., it does not depend on stimulation, being, in particular, the same for stationary and moving stimuli. Put differently, *two identical luminance distributions moving identically except for a spatial shift should undergo identical distributional deformations, and therefore (if the perceptual metric is stimulation-independent) their spatial separation cannot be affected.* Related to the MHP, the postulate means that the perceptual shift, $(\Delta X, \Delta Y, \Delta T)$, depends (componentwise) on the physical shift, $(\Delta x, \Delta y, \Delta t)$, but not on the stimulus being shifted, $l(x, y, t)$. As a result, the proportionality coefficients in (3) are fixed, or, equivalently, the vector of stimulation parameters, $\mathbf{p}$ (on which the coefficients might depend), is empty.

If, on the contrary, (8) is shown empirically not to hold, then the only possible conclusion (under the MHP) can be that the frontoparallel metric as such is different for moving and resting stimuli, whatever distributional deformations are involved. In this paper (8) will be shown to be wrong: ΔX changes systematically as a function of physical velocity v, indicating transformations of the visual space metric.

To avoid misunderstanding, if the frontoparallel geometry changes in visual motion, it must be reflected in all visual deformations, including those of the perceived shape of a single moving stimulus; see (6). However, only when formulated in terms of perceptual shifts and the MHP do the geometric transformations become logically isolated from *all conceivable* distributional transformations: (8)

can be wrong or right irrespective of any distributional transformations underlying (6).

Another, very important, theoretical advantage provided by the MHP is that one avoids the logical difficulty of identifying certain perceptual points as being "the same" across different stimulation displays. Indeed, it would be meaningless to say, when comparing $(\Delta X_s, \Delta Y_s)$ and $(\Delta X_m, \Delta Y_m)$, that the two shifts are measured between "the same" two positions, only in one case the positions are stationary, and in the other, moving. In fact, the shifts are measured between any two spatial points correspondingly located within two similar (due to the MHP) shapes, moving or stationary. Strictly speaking, it is not even necessary that the physical shape of the moving stimulus in Figs. 1c and 1d be identical to that of the stationary stimulus shown in Figs. 1a and 1b.

3. Double-Perturbation (2P) Paradigm

To operationalize the concept of perceptual shift and to make (8) experimentally testable, two spatial positions correspondingly located within two shifted replicas of each other have to be compared simultaneously in perceptual time. This implies that the two light distributions, stationary or moving, have to be presented together, as shown in Figs. 2a and 2b.

Assuming that the two light distributions are mapped into visual images independently, the perceived separation between these images equals the shift between the images of the two distributions presented separately. The shift, $(\Delta X, \Delta Y)$, then can be measured experimentally, by means of either numeric magnitude estimation of the perceived horizontal and vertical separations or adjusting the length of a stationary segment to match these separations. This is the basic logic of the 2P paradigm proposed in this work.

Figure 2B shows the results predicted by the fixed-metric hypothesis, i.e., when (8) holds; Fig. 2C schematizes the factual situation revealed by the experiments reported below. The 2P stimulation actually used in the experiments is shown in Fig. 3: two identical rectangular luminance in/de/crements on a uniform background, stationary or moving with a common velocity along the $\langle x \rangle$-axis. The stimulation parameters are described under Experiments.

The possibility of independent mapping, or lack of interaction, can be formally justified by adopting the following, very intuitive, principle (Mapping Locality): *perturbation of any light distribution (including a homogeneous field) bounded along the $\langle x \rangle$-axis, and/or $\langle y \rangle$-axis, and/or $\langle t \rangle$-axis maps into a correspondingly bounded perturbation of the perceptual image of the initial distribution.* The principle can be easily formalized, but unlike the MHP, the formalization does not lead to any nontrivial mathematical derivations and thus will not be presented here. It is almost obvious that the two stimuli, stationary or moving, can be placed on a uniform background far enough from each other so that the perceived image induced by either stimulus is the same whether the other stimulus is present or

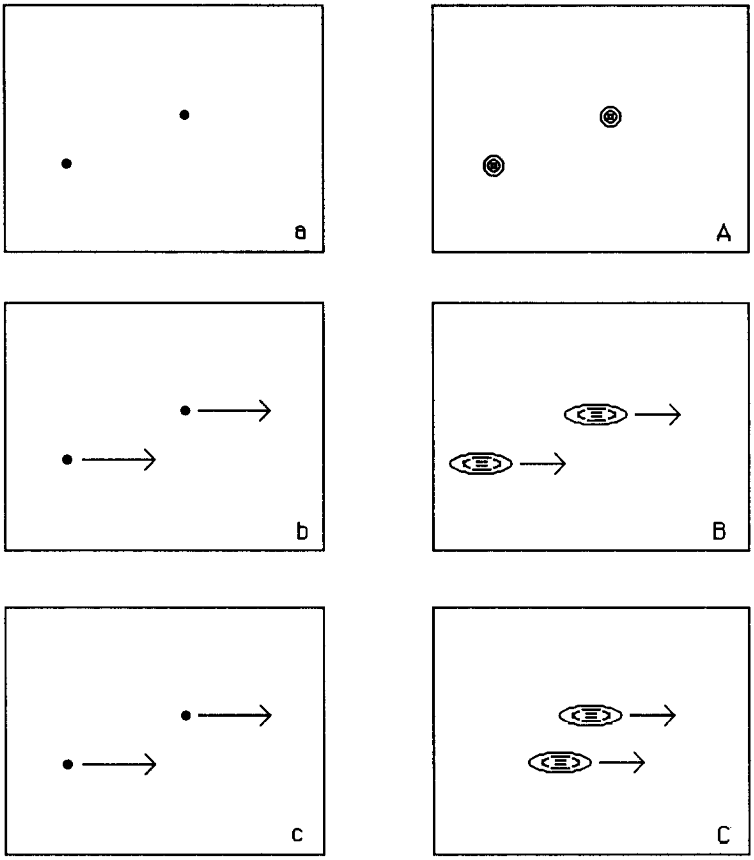

FIG. 2. Schematic illustration of the 2P paradigm, based on the MHP and independent mapping assumption. See Sections 2.2 and 2.3 for explanations.

absent. An empirical question remains, however, whether this situation has been achieved in the experiments reported below.

A priori interactions could occur in the following two ways to produce a violation of (8). First, an interaction could deform the two perceived contours, or shift them spatially with respect to each other, compared to separately presented. Second, an interaction could induce additional cues of spatial separation in the visual surrounding of the two contours, e.g., due to a simultaneous color/brightness contrast. If spatial estimations were based on such cues, then their dependence on velocity could be due to the same integration–interaction mechanisms that are involved in apparent deformations of the contours themselves; see (6).

There is a simple way to find out whether this indeed happens if/when (8) is violated. *If an interaction affects perceived separation, then ΔX-estimates should exhibit a dependence on the light segments' transverse separation; also both ΔX- and ΔY-estimates should depend on the contrast, length, and other parameters of the*

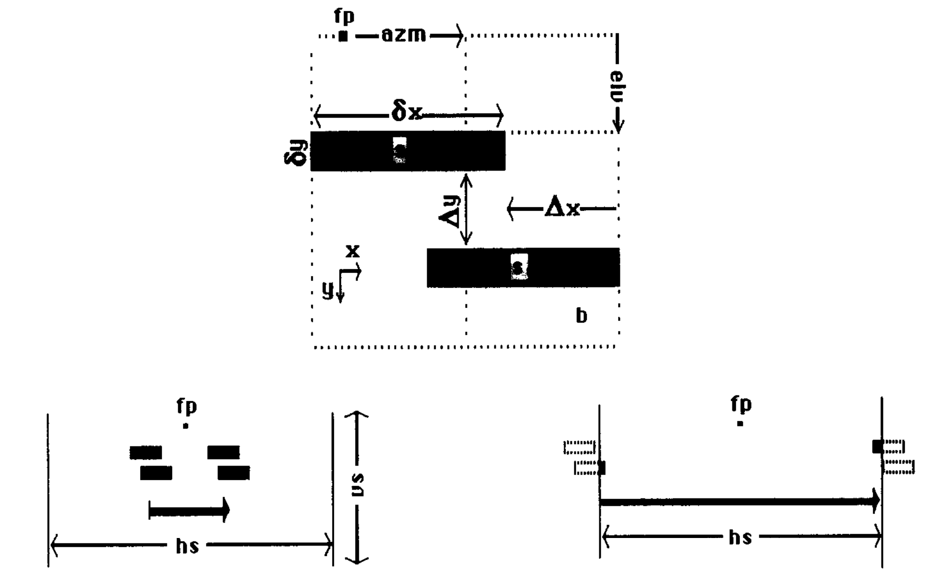

FIG. 3. 2P stimulation used in the experiments reported in this paper; fp, fixation point; azm, azimuth; elv, retinal elevation; $\delta x{:}\delta y$, shape parameters; $s{:}b$, contrast parameters; $\Delta x{:}\Delta y$, shift parameters ($\Delta x > 0$ if the lower segment is shifted to the right from the upper one); hs:vs, screen size parameters. Left and right bottom panels represent the "abrupt" and "gradual" appearance–disappearance modes, respectively. For details see Section 4.1.

segments. Indeed, an "interaction" that does not depend on distance, luminance, and shape parameters, but does depend on motion velocity (all of which will be shown to be the case), might just as well be called a kinematic change in geometry.[5]

Another factor should be taken into account when dealing with the $(\Delta X, \Delta Y)$-estimates as a function of velocity. The derivations presented in the Appendix deal with idealized visual motion, lasting infinitely long. In reality a unidirectional motion of a spatially bounded stimulus should start and end according, say, to one of the presentation modes shown in Fig. 3. As a result motion velocity becomes reciprocally related to presentation time, and it is necessary to make sure that the latter is not responsible for the results obtained. Other factors that should be taken into account as possible (non-kinematic) causes of the changes in $(\Delta X, \Delta Y)$ will be discussed within the context of actual experimental results.

[5] A potential difficulty here is that in (3) the stimulation parameters vector, **p**, determining the spatiotemporal metric might, in principle, contain some of the luminance and shape (but, of course, not inter-segment distance) parameters. If so, the separation of the interactional and geometric effects would be seriously complicated, although still possible through the distance-(in)dependence considerations and quantitative analysis of the luminance and shape factors. In other words, a dependence of the ΔX- and ΔY-estimates on luminance and shape parameters, if found, would constitute a less decisive argument for the existence of interaction than the independence on these parameters does for the lack of interaction. In this regard it is fortunate (for simplicity of analysis) that the ϕ-coefficients of (3) can be shown not to depend on luminance and shape factors.

4. Experiments

This paper presents the results of nine experiments establishing the existence and basic properties of the geometric transformations in visual motion (under steady fixation; experiments with free looking will be described in the following Visual Kinematics papers). In all experiments the dependent variable was an estimate of the perceived spatial separation, ΔX (or ΔY), between two segments constituting a 2P stimulus (Fig. 3).

4.1. *Stimulation*

Refer to Fig. 3, top. A 2P stimulus is characterized by the shape of the light segments ($\delta x:\delta y$); contrast ($s:b$, segment and background luminance, respectively); horizontal level, or elevation (elv) with respect to the fixation point (fp); spatial shift ($\Delta x:\Delta y$); and velocity of motion along the $\langle x \rangle$-axis (v). In addition both moving and stationary 2P stimuli are characterized by presentation time (τ).

Horizontal shift, Δx, is considered to be positive (negative) if the lower light segment is shifted to the right (left) with respect to the upper one. The vertical separation, Δy, is always considered positive. An instantaneous position of a 2P stimulus when in motion, or its permanent position when stationary, is characterized by its horizontal eccentricity, or azimuth (azm), measured from the fp to the vertical midline of the two-segment configuration (middle point line in Fig. 3, top). Azimuth is positive (negative) in the right (left) visual semifield. Luminance was practically zero outside the screen borders; except in one condition of Experiment 3 the background luminance, s, was nonzero, and the borders were clearly seen (dimensions hs:vs).

The moving 2P stimuli could be presented in one of the two appearance–disappearance modes shown in Fig. 3. In the "abrupt" mode (Fig. 3, left bottom) the moving 2P stimulus appeared at a certain azimuth position, uniformly moved rightward to another position, and disappeared. In the "gradual" mode (Fig. 3, right bottom) the 2P stimulus appeared from behind the left screen border, uniformly moved toward the right border, and disappeared behind it.

4.2. *Procedure: Experiments* 1–6

In Experiments 1–5 the task was to give numeric magnitude estimates of ΔX ("in mm"), the apparent horizontal separation between the two segments of moving or stationary 2P stimuli. For moving stimuli the separation was described as that between the leading edges of the two segments. In each experiment there were several "target" 2P stimuli that all had $\Delta x = 1.15°$ but differed in some of the other parameters: vertical separation, Δy (Experiment 2); contrast, $s:b$ (Experiment 3); shape, $\delta x:\delta y$ (Experiment 4); elevation, elv, and sign of the horizontal separation, $\pm\Delta x$ (Experiment 5). The target 2P stimuli could be stationary (presented at different azimuth positions) or moving with velocity 35°/s from azm $= -10.5°$ to azm $= 10.5°$ ("abrupt" mode, Fig. 3, left bottom). Presentation time (τ) for the moving 2P stimulus was, therefore, 0.6 s. Stationary 2P stimuli were presented for 1.5 s, and, in two cruder replications of the experiments, for 0.5 s. Only stationary stimuli were used in Experiment 1. The 2P parameters' values that did not vary within an experiment were chosen from the following list of "standard" values: $\Delta y = 1.0°$; $\delta x : \delta y = 2.3° : 0.5°$; elv $= 0.5°$; $s : b = 30\ \text{cd} \cdot \text{m}^{-2}/3\ \text{cd} \cdot \text{m}^{-2}$; hs : vs $= 50° : 25°$.

In each experiment for every target 2P stimulus, with $\Delta x = 1.15°$, there were 20 "auxiliary" 2P stimuli, identical with the target except for the values of Δx that were evenly spaced between 0.6° and 1.6°. Within an experiment, the target stimuli were presented from 16 to 24 times each (20 on average), and 20 auxiliary stimuli were presented once each, all stimuli in a randomized order. Only for the target stimuli were the numeric estimates recorded: the sole purpose of mixing them with the auxiliary stimuli was to extend the range of numerical estimates and prevent predictability.

An analogous procedure was used in Experiment 6 where the task was to give numeric magnitude estimates of ΔY ("in mm") for stationary and moving 2P stimuli. ΔY was described as the vertical separation between the lower boundary of the upper segment and the upper boundary of the lower segment. The target Δy-value was 1.0°, and 20 auxiliary Δy-values were evenly spaced between 0.5° and 1.5°. All other parameters in this experiment had the "standard" values listed above.

To counterbalance possible long-term changes in estimation, the six experiments of this group were divided into short runs carried out in a random order. A trial was initiated by the observer pushing a designated key. The observer could repeat the same presentation an arbitrary number of times before giving an estimate (usually there were two to five repetitions for moving stimuli and one to two for stationary stimuli).

4.3. *Procedure: Experiment* 7

In Experiment 7 there were only stationary 2P stimuli presented for time (τ) varying from 0.017 to 1 s. Δx varied from 0.33° to 2.7°. Other parameters were constant: $\Delta y = 0.3°$; $\delta x : \delta y = 9° : 0.03°$; $s : b = 30\ \text{cd} \cdot \text{m}^{-2}/0\ \text{cd} \cdot \text{m}^{-2}$; elv $= 0°$; azm $= 0°$ (a fixation point was present before, but not during, presentation); hs : vs $= 17.1° : 11.5°$. The presentation was immediately followed by a high-density quasi-random dot masking field (about nine dots per squared arc deg), with dot dimensions of $0.03° : 0.03°$, and the same contrast as in the 2P stimuli. The task was to numerically estimate ΔX, "in mm". The observer fixated a small dot in the center of the screen. The dot disappeared 1 s after the observer pressed a key initiating a trial, being replaced with a 2P stimulus. After period τ the stimulus was replaced with the high-density quasi-random dot field, which in turn was replaced with the fixation point 1 s later. This cycle was repeated three more times before the observer gave a numeric estimate. Presentation times (τ) and Δx-values were used in a randomized order. There were from 35 to 55 presentations per τ per Δx (45 on average).

4.4. *Procedure: Experiments* 8 *and* 9f[6]

In these experiments the task was to adjust the length of a stationary light segment to match the apparent horizontal separation (ΔX) between the leading edges of the two moving segments. The adjustments were made after a moving 2P stimulus had been presented four times in brief succession. Trials were initiated by the experimenter after a warning signal. All observation and stimulation conditions varying within an experiment were used in a randomized order, with the total of 20 match-estimations per condition. The stationary light segment used for match-estimations was about 1.5° below the upper screen border, above the moving stimulus and fixation point. The matched segment was identical in width and contrast to the segments constituting the 2P stimulus.

The velocity v in Experiment 8 was 32°/s; in Experiment 9f velocity varied from 22.1°/s to 86.4°/s ("gradual" appearance–disappearance mode, Fig. 3, right bottom). The horizontal separaton Δx varied between 0.34° and 2.7° in Experiment 8, and on three levels, 2.5°, 3.2°, 4.1°, in Experiment 9f. Other parameters in these two experiments were held constant: $\Delta y = 1.2°$; $\delta x : \delta y = 12.9° : 0.5°$; $s : b = 30\ \text{cd} \cdot \text{m}^{-2} : 3\ \text{cd} \cdot \text{m}^{-2}$; elv = 1.0°; hs : vs = 36.8° : 10.7°.

4.5. *Experimental Setup*

In Experiments 1–6 the light beams from two slide projectors (S1 and S2), with a mechanical shutter built into S2 (closing/opening time about 2 ms), were optically superimposed by means of a 50/50 beam splitter and front-projected on a flat screen after reflection from a rotating mirror–galvanometer. The latter was driven by a function generator whose onset was synchronized with a programmable impulse generator controlling the shutter. The two images from the slide projectors, S1 and S2, can be considered congruent 2P stimuli, differing in their contrast only, $s_1 : b_1$ and $s_2 : b_2$, so that the resulting contrast $s : b = (s_1 + s_2) : (b_1 + b_2)$. A positive contrast, $s > b$, was created by setting $s_1 = b_1 = b$ (homogeneous field) and $s_2 = s$, $b_2 = 0$ (double-perturbation). The 2P stimulus presentation was provided by opening the shutter in S2, at a certain moment after the observer initiated a trial, and closing it after period τ. A negative contrast, $s < b$, was created by setting $s_1 = s$, $b_1 = b$, and $s_2 = b - s$, $b_2 = 0$, so that when superimposed, the resulting image was a homogeneous field of luminance b. The 2P stimulus presentation in this case was provided by closing the shutter for S2 and opening it after period τ. The resulting image was viewed binocularly from the distance of 1.5 m through a 50° : 25° window. The observer's head was fixed in a chin rest with a forehead support. The experiment was conducted in a completely darkened room. An LED fixed to the screen served as a fixation point.

The setup in Experiments 8 and 9f was a simplified version of the one just described. Because the appearance–disappearance mode was "gradual," the

[6] The experiment is referred to as 9f rather than 9 because the data reported in this paper constitute only one-half of the whole experiment (namely, the ΔX-estimates obtained under steady fixation, hence the suffix f). The experiment will be analyzed in full in the next paper of the Visual Kinematics series.

exposure duration was determined by the horizontal screen size (hs) and velocity (v). A moving stimulus was front-projected on a screen from a slide projector, via a rotating mirror. Two other projectors provided a uniform illumination of the screen and the image of the stationary comparison light segment. The screen was formed by the inner surface of a large cyclinder (2 m in radius), and the mirror was placed in its center. The observer's head was fixed in a chin rest with a forehead support 0.4 m behind and slightly above the mirror position, so that the viewing distance was 2.4 m. Observation was binocular, through a window of 10.7° vertically and of a variable horizontal size (typically 36.8°). Other stimulation/observation conditions were as in Experiments 1–6. An optical–mechanical device allowed the observer to change the visible length of the comparison light segment by moving a lever.

In Experiment 7 the stimuli were presented on a Macintosh SE screen (17.1°:11.5°, 512×342 pixels, 60 Hz vertical refresh), binocularly viewed from a 60-cm distance (chin rest without forehead support). Darkness was not complete: the computer outlines were discernible after several minutes of dark adaptation.

4.6. *Observers*

Three observers participated in Experiments 1–6, two in Experiment 7, five in Experiment 8, and two in Experiment 9f. All had normal or corrected-to-normal acuity. The observers were naive as to the aims and designs of the experiments, except for KVL (in Experiments 8 and 9f) and ED, the author (in Experiments 1–7).

5. Space Contraction in Motion (SCM)

5.1. *SCM: Existence and Properties*

That the SCM effect does occur is demonstrated in Fig. 4, which summarizes the results of Experiments 1–5: numeric magnitude estimates of ΔX, the perceived horizontal separation under various conditions, normalized by the mean ΔX-estimate for a standard 2P stimulus in the fovea. The figure shows that *ΔX is considerably shorter in motion that at rest*: for $v = 35°/\text{s}$ the contraction was by about 30% for observer DL, and by slightly more than that for two other observers. The figure shows also that differences in vertical separation, contrast, shape, elevation, and sign of Δx do not have any noticeable effect on ΔX-estimates, for either moving or resting 2P stimuli.

Recall from Section 4.2 that the ΔX-estimates in Fig. 4 all correspond to one and the same ("target") physical shift value, $\Delta x = 1.15°$. Recall also that the parameters (vertical separation, contrast, shape, elevation, etc.) whose values are not shown on the horizontal axis for a given experiment have "standard" values listed in Section 4.2. Thus, all stimuli in Experiment 1 have standard parameter values (they differ in azimuth which does not belong to the list), and so are the 2P stimuli

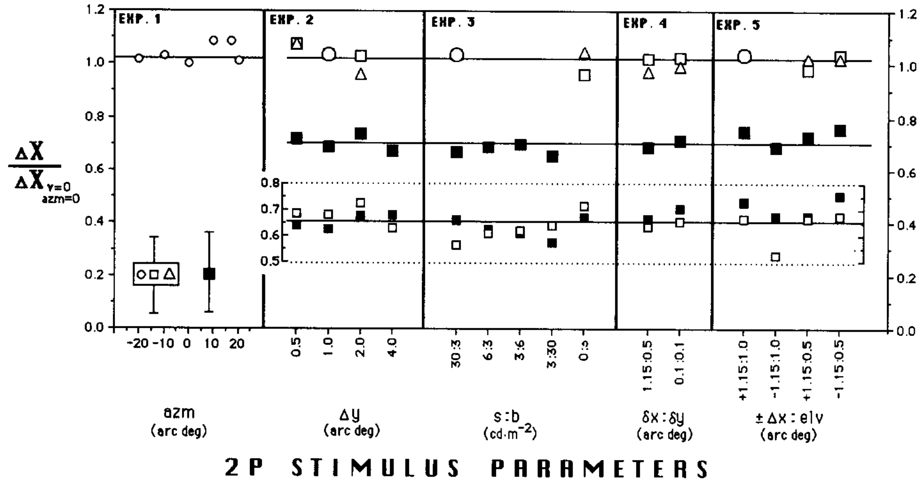

FIG. 4. Experiments 1–5: ΔX (numeric magnitude estimation), in units of mean ΔX for standard stationary 2P stimulus at azm $= 0°$, as a function of 2P stimulation parameters. Symbols ∘ (stationary 2P with standard parameters' values, see Section 4.2), □, △ (stationary 2P at azm $= 0°$ and $17°$, respectively), and ■ (moving 2P, $v = 35°/\mathrm{s}$) represent means of 20 ± 4 estimates by observer DL; ○ is the mean level of all ∘ (see Section 5.1 for explanations). Vertical bars show averaged ± 1 standard deviation per symbol (group of symbols). Symbols ■ and □, shifted down by 0.25, represent motion data for observers MP and ED, respectively (means of 2–3 estimates, normalizing unit was determined from 20 estimates). For stationary stimuli $\tau = 1.5$ s (DL) and 0.5 s (MP, ED); for moving stimuli $\tau = 0.6$ s.

corresponding to $\Delta y = 1°$ in Experiment 2, $s{:}b = 30{:}3$ (cd $\cdot$ m^{-2}) in Experiment 3, and $\pm\Delta x$:elv $= 1.15°{:}0.5°$ in Experiment 5. That is why for the latter three conditions stationary 2P stimuli are represented by the mean value of ΔX computed from Experiment 1.

In Fig. 4, as well as in other figures throughout this paper, vertical bars attached to a symbol (group of symbols) show ± 1 standard deviation averaged over all conditions represented by this symbol (symbols). The averaging of standard deviations leads to little information loss, because the variability shows only weak and nonsystematic dependence on stimulation parameters.

Figure 5 presents the numeric magnitude estimates of ΔY (Experiment 6) normalized by the same (ΔX) value as in Fig. 4. Like ΔX in Experiment 1, the estimates of ΔY for stationary (standard) 2P stimuli do not change with azimuth. In contrast to ΔX, however, the ΔY-estimates do not change in motion either. One concludes that the *SCM effect does not occur transversely, in the direction orthogonal to that of motion.* In a less systematic fashion, this fact has been confirmed with velocities much higher than 35°/s and under a variety of observation conditions. Phenomenologically, the higher the velocity, the more obvious the asymmetry between the longitudinal and transverse directions: changes occur only in the former. (Moreover, even the changes in the perceived profiles of the light segments themselves seem to occur only longitudinally.)

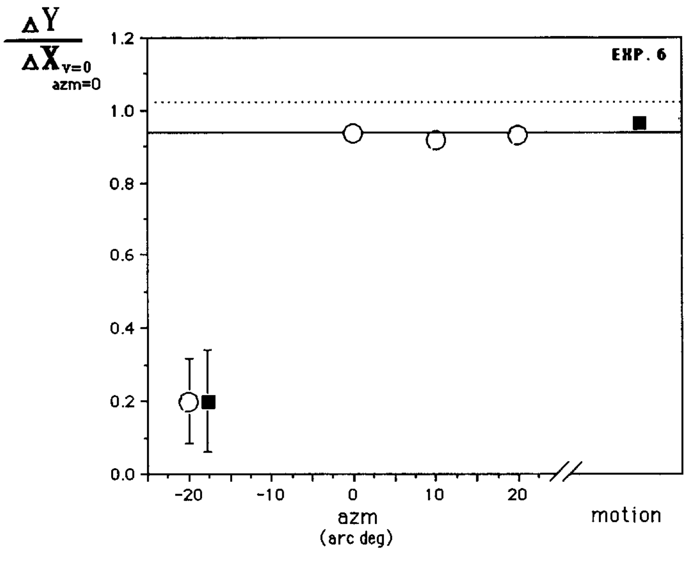

FIG. 5. Experiment 6: ΔY (numeric magnitude estimation), in units of mean ΔX for standard stationary 2P stimulus at azm $= 0°$, as a function of azimuth and motion. Symbols ○ (standard stationary 2P, $\tau = 1.5$ s) and ■ (standard moving 2P, $v = 35°/\text{s}$, $\tau = 0.6$ s) represent means of 20 ± 4 estimates. Observer DL. Vertical bars as in Fig. 4. Point line is the upper horizontal line of Fig. 4 (mean ΔX for stationary 2P).

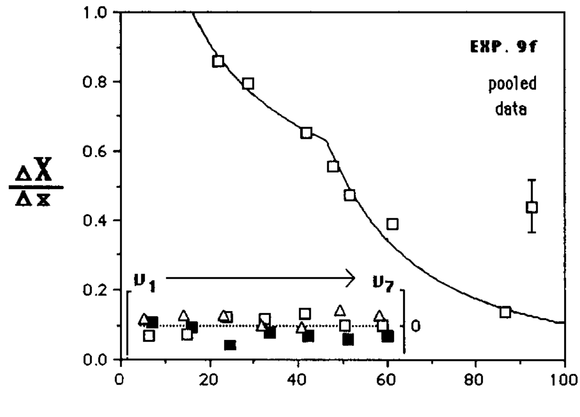

FIG. 6. ΔX (match-estimation), in units of Δx, as a function of angular velocity v and Δx (Experiment 9f). Symbol □ represents 120 ΔX-estimates by two observers (KVL and TSA) for three values of Δx (20 estimates per Δx per observer). The theoretical curve is not discussed in this paper (see footnote 6). Inset: each triad of symbols (■, □, △) corresponds to one of the seven values of v used in the experiment (in increasing order from left to right); vertical deviation of the symbols from the horizontal line equals the difference between the means computed separately for three Δx-values (4.1°, 3.2°, 2.5°, respectively) and their grand mean. Vertical positioning of the inset is arbitrary.

Comparing the mean ΔY-estimate with the mean ΔX for stationary stimuli (point line in Fig. 5), note that their ratio, $\Delta Y/\Delta X$, is about 6% greater than the ratio of the physical separations, $\Delta y/\Delta x = 1°/1.15°$. This overestimation of the vertical separation reflects the well-known vertical–horizontal anisotropy of the fronto-parallel metric (Künnapas, 1955, 1958).

To get a more general picture of the SCM effect, consider the pooled match-estimates of ΔX obtained in Experiment 9f (Fig. 6). The match-estimates are presented in units of the physical shift value, Δx, which is equivalent to normalizing by the (ideal) match-estimation of ΔX at $v = 0$, assuming no systematic error is involved. Ignoring for now the inset (discussed in Section 5.6), the figure shows that *ΔX monotonically decreases as v increases*, reaching as little as 10% of its value at rest as v approaches 90°/s. The data are averaged over two observers, but the individual results (not shown in this paper, see Footnote 6) exhibit exactly the same pattern. My informal observations indicate that for motions below 10–15°/s the SCM effect either does not exist or is too small to be measured by a moderate number of numeric estimations.

5.2. *Inhomogeneity and Interaction Ruled Out*

The purpose of the experiments reported was not to merely demonstrate the existence of the SCM effect. The independence of ΔX and ΔY on various stimulation parameters provides evidence against the possibility that SCM is due to interactions between the light segments or due to violations of the MHP.

As mentioned above, interaction between two light segments, a priori, could deform the two perceived contours unequally, shift them toward each other, or induce in their surrounding space additional color/brightness changes serving as spatial separation cues. On an informal demonstration level, it was apparent with the 2P stimuli used in Experiments 1–6 that the appearance of either segment was the same whether another was switched on or off, and that there were no color/brightness changes that could be utilized as separation cues, either in motion or at rest. On a more analytical level, consider the results of Experiments 2, 3, and 4 (Fig. 4). If ΔX changes in motion are due to a transverse interaction, then ΔX-estimates should depend on the transverse separation (Δy), contrast ($s{:}b$), and shape of the segments ($\delta x{:}\delta y$). A reasonable expectation would be that the SCM effect would diminish as Δy increases and/or $s{:}b$ decreases. If the interaction induces color/brightness changes, then it is reasonable to expect, in addition, that the SCM would be weaker with two small squares (0.1°:0.1°) than with two larger rectangles (1.5°:0.5°) partially overlapping along the $\langle x \rangle$-dimension, or even larger rectangles (2.3°:0.5°, a standard value) used in all experiments other than Experiment 4. No such tendencies can be seen in Fig. 4.

Once the lack of interaction is established, the MHP states that the perceptual images of the two segments should be identical except for a $(\Delta X, \Delta Y)$-shift. Consider now two types of deviation from the MHP that could be responsible for the SCM effect: $\langle y \rangle$-inhomogeneity and $\langle x \rangle$-inhomogeneity. The $\langle y \rangle$-inhomogeneity means that the two moving images can be different in shape and/or velocity because they have different retinal elevations. The $\langle x \rangle$-inhomogeneity means that the difference between the moving images can be due to the fact that one is shifted by Δx with respect to the other. In both cases a difference between the shapes and/or velocities could translate into an underestimated ΔX (if, for example, the upper segment in Fig. 3 appeared contracted more than the lower one). Again, it was phenomenologically obvious that differences between the two moving shapes are negligible. Indeed, a difference that could account for, say, the 70% contraction in Fig. 6 should have been quite apparent.

More formal arguments are provided by Experiments 2 and 5 (Fig. 4). The $\langle y \rangle$-inhomogeneity implies that the ΔX-estimates should change when one or both segments change their retinal elevations. Elevation was varied for the lower segment in Experiment 2 and for both segments in Experiment 5 (Fig. 4). As this does not lead to any differences in ΔX, $\langle y \rangle$-inhomogeneity can be ruled out as a possible cause of SCM. The same can be shown even more simply: when $\Delta x = 0$, the two frontal edges look perfectly $\langle X \rangle$-aligned for all velocities and different elevations.

Experiment 5 also rules out influences of possible $\langle x \rangle$-inhomogeneity: the latter implies that if the two segments exchange their retinal elevations (i.e., Δx changes its sign), then the SCM effect should be reversed. Comparing the positive and negative Δx-values in Fig. 4 for each value of elv, one concludes that this is not the case.

The Experiments 1–6 can also be viewed as demonstrating that parameters of shape, size, and luminance/contrast are not contained in the vector $\mathbf{p}$ of (3) (see footnote 5). Putting it differently, these parameters do not affect the frontoparallel metric of the visual space, or at least their influence is negligibly small compared to that of motion velocity. Experiments 1 (Fig. 4) and 6 (Fig. 5) are direct demonstrations of (2) and (3*) (homogeneity of perceptual axes) for stationary stimuli. Indeed, under steady fixation, these formulae are translated into the assertion that ΔX and ΔY do not depend on retinal location, which is indeed the case between $-20°$ and $+20°$ azimuth. It is important, of course, not to extrapolate this result to very small values of Δx and Δy, comparable to visual acuity thresholds.

5.3. *Motion as a Distance Cue Ruled Out*

It was assumed so far that both moving and stationary 2P stimuli were perceived on á single frontoparallel plane. Superficially, the SCM effect might suggest the possibility that the visual system treats velocity of motion as an egocentric distance cue (the faster the closer) and rescales the apparent size of moving stimuli accordingly. Because no change in the apparent distance has been observed in the experiments (certainly not by the magnitude of the SCM effect), one should dismiss the "size–distance invariance" version of such hypothesis (Kirkpatrick & Ittelson, 1953). It is still possible, however, that visual motion is treated as a "latent" distance cue that does not give rise to a distance change perception, but triggers a size constancy mechanism to recalibrate the perceived size. This is the "misapplied constancy" version of the hypothesis, proposed for some "illusions" of linear extent (Gregory, 1963, 1970). A simple comparison of Experiments 1–5 (Fig. 4) and 6

(Fig. 5) rules out this hypothesis. Indeed, the recalibration required by this hypothesis must necessarily be a homothetic transformation, a uniform shrinkage of the shape toward a central point. In fact, however, the shrinkage occurs in the longitudinal direction, but does not occur transversely.

5.4. *Size Contrast/Assimilation Ruled Out*

Another hypothetical cause of the SCM effect is also borrowed from the area of geometric "illusions." The hypothesis is that SCM is induced by changes in the apparent size of the segments constituting the 2P stimuli. If the segments look elongated one can assume a contrast effect, if they look contracted then the effect can be called assimilation (Coren & Girgus, 1978). This hypothesis is consistent with the fact that SCM does not occur transversely, because the changes in perceived shape of the segments also seem to occur only along the direction in motion. An implication of this hypothesis is, however, that the perceived ΔX can be changed by changing physical size of the two segments. Then ΔX should have been different for three values of δx used in Experiments 1–5: 0.1°, 1.15° (Experiment 4), and 2.3° (standard value used in other experiments). As this is not the case, the hypothesis should be dismissed.

5.5. *Presentation Time Factor Ruled Out*

In Experiments 1–5 with observer DL (Fig. 4), stationary 2P stimuli were presented for a longer time ($\tau = 1.5$ s) than moving stimuli ($\tau = 0.6$ s). The simplest argument excluding the possibility that presentation time affects perceived spatial

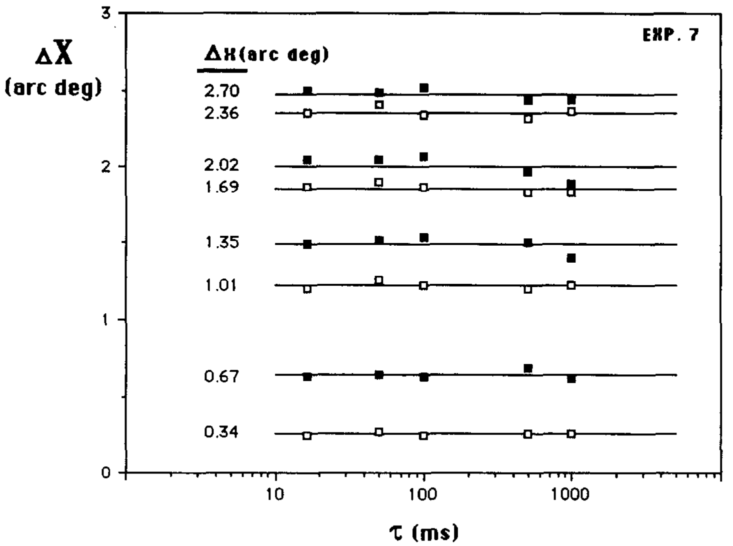

FIG. 7. Experiment 7: ΔX (numeric magnitude estimation) in stationary 2P stimuli as a function of exposure time and Δx. Each symbol represents the mean of 45 ± 10 ΔX-values by observer HL (solid and open symbols are alternated for better readability). Calibration of the vertical axis is explained in the legend to Fig. 8.

intervals follows from the finding that no changes are observed in the transverse direction. It had been found in pilot experiments with observer DL that setting τ at different values (including far below 0.6 s) did not affect estimates, but shorter τ increased the observer's tendency to repeat a presentation more than once (see Experiments). Methodological concerns that led to setting τ at 1.5 s do not seem very compelling in retrospect and will not be discussed here. That the difference in τ cannot be responsible for SCM is clear from the results obtained with two other observers (MP and ED): the estimates shown in the inset of Fig. 4 are normalized by the mean of 20 ΔX-estimates of stationary standard 2P stimuli presented in the fovea for 0.5 s, i.e., for a shorter time than moving stimuli. Although the reliability of the data points in Fig. 4 is lower for these two observers than it is for DL, it is clear that the overall SCM effect is of about the same magnitude, if not greater.

Figure 7 presents the results of Experiment 7, providing direct evidence that ΔX in stationary 2P stimuli does not change with τ ranging from 0.017 to 1 s. A cruder replication of Experiment 7 on the author (two to five estimates per τ per Δx) yielded virtually identical results, with no monotonic trend in ΔX as a function of τ (for any Δx). In Fig. 7 the numeric magnitude estimates are calibrated in angular units to match the vertical format of Fig. 8. In the latter (top curve), the ΔX values averaged over all exposure times are plotted against corresponding values of Δx. The relationship can be reasonably well approximated by a zero-intercept straight line, whose slope is determined by the arbitrary unit of measurement on the scale of ΔX-values.

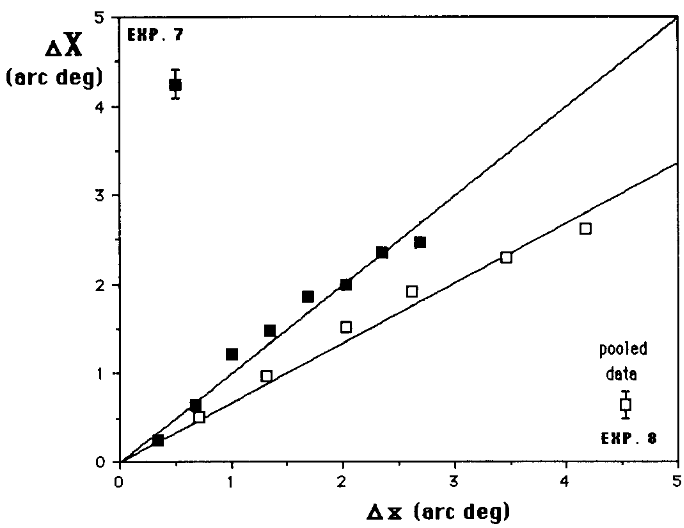

FIG. 8. Solid symbols: ΔX (numeric magnitude estimation) in stationary 2P stimuli of Fig. 7 averaged over five exposure time values and plotted against Δx (about 5×45 estimates per symbol). The values of ΔX are normalized to make the slope of the zero-intercept regression line equal to 1 (see Section 5.5 for justification). Open symbols: ΔX (match-estimation) in moving 2P stimuli, $v = 32°/\text{s}$, as a function of Δx (Experiment 8, pooled for five observers, 5×20 ΔX-estimates per symbol; see Fig. 9 for individual data).

To reflect the degree of contraction as a function of presentation time τ the unit of measurement should be chosen to make the slope equal to 1. Indeed, the contraction is determined with respect to the estimates for long-lasting stationary stimuli. In other words, for these stimuli coefficient ϕ_{xX} in (3) should be set equal to 1. Then, due to the τ-independence, the same unit value should be used for brief stationary stimuli.

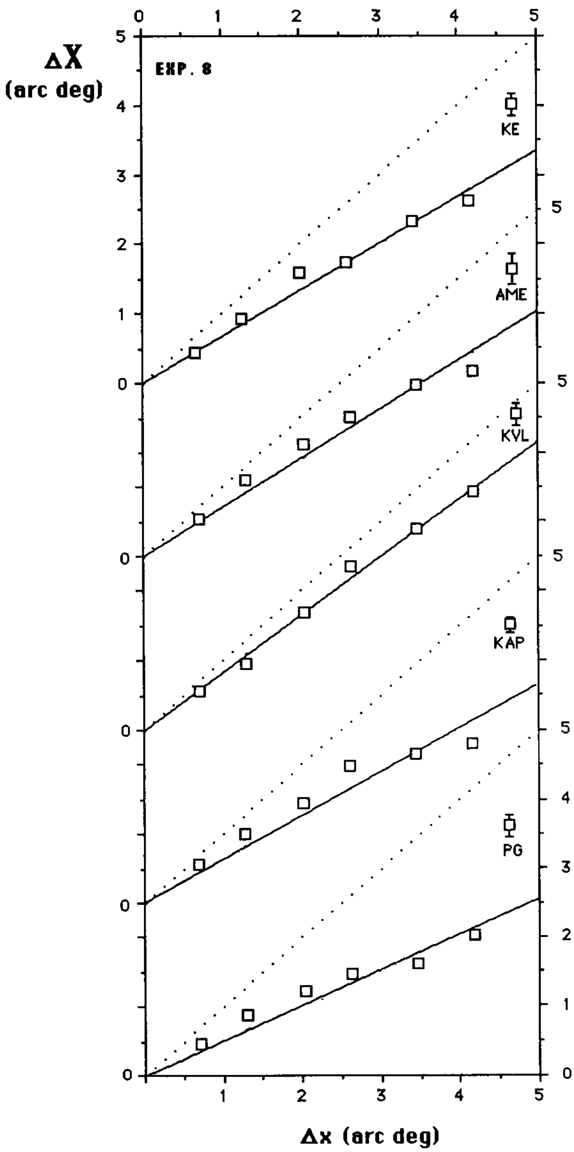

FIG. 9. Same as open symbols in Fig. 8, but shown separately for individual observers. Unit slope is shown by dotted lines. 20 ΔX-estimates per symbol.

For contrast, the data from Experiment 8 pooled across five observers are shown in the same graph (Fig. 8, bottom curve): match-estimates of ΔX in 32°/s motion under steady fixation. These data are also reasonably well described by a zero-intercept straight line—a fact that will be discussed later in more detail. The slope of this line indicates contraction by about 30%, even though at 32°/s a stimulus in Experiment 8 traversed a 36.8° distance in more than 1 s, longer than the longest τ in Experiment 7.

Experimental data reported in the literature on the effect of exposure time on the apparent extent of stationary stimuli (presented at a fixed egocentric distance) show that the effect, if present, is very small (Eisler, 1963; Jaeger & Kraemer, 1980; Newsome, 1965). All these experiments were conducted with single-perturbation (1P) stimuli. In an unpublished experiment I found a slight monotonic decrease ($<2\%$) in the apparent length of a single light segment as τ decreased from 1 to 0.05 s. This magnitude is negligible compared to that of the SCM effect. It is important to realize, however, that were the magnitude of the exposure time effect in a 1P paradigm much greater, it would not contradict the absence of the effect in Experiment 7. One can speculate that in a 1P paradigm the exposure time effect might be a consequence of temporal luminance integration combined with the "irradiation" phenomenon (brighter objects look bigger; see, e.g., Weale, 1975). No such mechanism, however, can influence ΔX in 2P stimuli, by the very logic of the 2P paradigm: *if the MHP holds and the perceptual mappings of the two constituting perturbations do not interact, then they should undergo identical (distributional) deformations as a function of* τ, *leaving their separation intact.* (Additional evidence ruling out exposure time as a possible cause of the SCM effect will be presented in the next paper of the Visual Kinematics series.)

5.6. *Proportionality of Physical and Perceptual Intervals*

That perceived spatial separation between stationary points is proportional to their physical separation is one of the best documented facts in visual psychophysics (Baird & Vernon, 1965; Indow & Watanabe, 1988; Stevens & Galanter, 1957; Stevens & Guirao, 1963; Teghtsoonian, 1965). It is thus not surprising that in the absence of an exposure time effect, all ΔX-estimates for stationary 2P stimuli in Experiment 7 can be approximated by a single zero-intercept straight line (Fig. 8, top curve). It is an important finding, however, that the same is true for moving 2P stimuli (Fig. 8, bottom curve), with a proportionality coefficient (slope) lower than 1. Only pooled data are presented in Fig. 8, but the proportionality holds as a reasonable first-order approximation for the observers considered separately (Fig. 9).[7]

[7] Both Fig. 8 and the individual data for observers AME, KAP, and PG in Fig. 9 exhibit a weak curvilinear trend. The trend does not disappear in log–log coordinates and does not increase as velocity increases (see inset of Fig. 6). Future experiments will have to show whether this trend is attributable to estimation biases and whether it is associated with steady fixation only. The trend could indicate that in final analysis the proposition (2) (at least for steady fixation) will have to be corrected by adding to it nonlinear terms (weak inhomogeneity of spatial axes).

Experiment 9f (Fig. 6), among its other purposes, demonstrates the proportionality between ΔX and Δx for the whole range of velocities used in this experiment. Because ΔX in Fig. 6 is normalized by Δx, the proportionality means that the normalized values are the same for different values of Δx. For each velocity, the three symbols in the inset of Fig. 6 represent mean deviations of $\Delta X/\Delta x$ for three different values of Δx from their grand mean: the horizontal line represents a zero deviation. Obviously, the deviations are small and show no systematic dependence on Δx. The data are averaged over two observers, but, again, the same conclusion can be drawn from the individual results considered separately (not presented in this paper; see footnote 6).

A general conclusion is: *at a given velocity* (v), *the perceived separation is a fixed proportion of the separation at rest, for any value of the latter.* In other words, $\Delta X(v)/\Delta X(0)$ does not depend on $\Delta X(0)$, at least to a first approximation. In the numeric magnitude estimation experiments $\Delta X(0)$ is estimated empirically, in the match-estimation experiments $\Delta X(0)$ is equated with the physical separation, Δx.

6. Conclusion: MHP and SCM

The empirical results discussed in the preceding sections strongly corroborate the MHP. Because the possibility of interaction between the constituting parts of the 2P stimuli has been excluded, the relative contraction, $\Delta X(v)/\Delta X(0)$, can be considered a direct estimator of the coefficient $\phi_{xX}(\mathbf{p})$ in (3). Analogously, $\Delta Y(v)/\Delta X(0)$ is an estimator of the orthogonal coefficient, $\phi_{yY}(\mathbf{p})$. It has been established that the vector of stimulation parameters, $\mathbf{p}$, includes motion velocity, v: if v is directed along the $\langle x \rangle$-axis, then $\phi_{xX}(v)$ is a monotonically decreasing function of v (at least for $v > 10$–$15°/\mathrm{s}$), whereas ϕ_{yY} does not depend on v. Following the logic proposed in the section on the MHP, *one can dismiss the hypothesis that the frontoparallel metric of the visual space is fixed for all visual scenes and that all apparent deformations in motion are distributional.* On the other hand, the frontoparallel metric seems to be stable with respect to changes in other parameters' values, such as shape/size and luminance/contrast. Recall that only dependence on absolute location is precluded by the MHP-based understanding of the metric, but were it strongly dependent on many other stimulation parameters, it would be considerably more difficult to construct operational procedures revealing its structure (see footnote 5).

A comment is necessary here on the fact that the ΔY-separation estimated in Experiment 6 was defined as that between the lower line of the upper segment and the upper line of the lower segment. Strictly speaking, this definition does not follow the MHP-based logic implemented in the procedure of ΔX-estimation: spatial separations are to be measured between spatial positions correspondingly located within two identical shapes. The distance between two upper or two lower boundaries of the segments would be a more rigorous definition of ΔY. The adopted definition, however, seemed to be subjectively more convenient. It is easy to figure out that if the MHP holds and there is no transverse interaction, then the stability of ΔY, defined as it was, also implies stability of the distance between any two correspondingly located points (unless one assumes an exact perceptual cancellation of the geometric transformations by distributional deformations). The definition adopted could only lead to difficulties if ΔY was found to change with velocity, because then additional experiments would be required to dissociate geometric and distributional transformations.

The analysis of the SCM phenomenon presented in this paper leaves unanswered many questions concerning both empirical properties of the phenomenon and its theoretical interpretation. One important question is whether the phenomenon is critically associated with a steady fixation being maintained, or, more generally, whether velocity as the main factor determining SCM should be defined in retinal coordinates. Will, for example, presentation/observation factors affecting perceived speed also affect the size of the SCM effect? These questions will be addressed in the next paper of the Visual Kinematics series, which will also broaden and provide additional evidence for some of the statements arrived at in this study.

APPENDIX: Mathematical Results Associated with the MHP

Proportionality between physical and perceptual shifts. Assume that propositions (1) and (2) are satisfied and $(\Delta X, \Delta Y, \Delta T)$ depends on $(\Delta x, \Delta y, \Delta t)$ only componentwise. Let $\mathbf{p}$ be a vector of stimulation parameters on which $(\Delta X, \Delta Y, \Delta T)$ depends in addition to $(\Delta x, \Delta y, \Delta t)$. Due to (2), $\mathbf{p}$ is invariant with respect to physical shifts; i.e., it satisfies (3*). Let

$$\begin{aligned} \Delta X &= \phi_{xX}(\mathbf{p}, \Delta x) \\ \Delta Y &= \phi_{yY}(\mathbf{p}, \Delta y) \\ \Delta T &= \phi_{tT}(\mathbf{p}, \Delta t). \end{aligned} \tag{A1}$$

Due to (2)

$$\phi_{xX}(\mathbf{p}, \Delta x_1 + \Delta x_2) = \phi_{xX}(\mathbf{p}, \Delta x_1) + \phi_{xX}(\mathbf{p}, \Delta x_2). \tag{A2}$$

This is a variant of the Cauchy functional equation, whose continuous solution is (see, e.g., Aczel, 1966)

$$\phi_{xX}(\mathbf{p}, \Delta x) = \phi_{xX}(\mathbf{p})\, \Delta x. \tag{A3}$$

Identical derivations for ϕ_{yY} and ϕ_{tT} lead to (3). In general, the proportionality coefficients depend on parameters $\mathbf{p}$ of the light distribution. Under the fixed-metric hypothesis the vector $\mathbf{p}$ is empty, and the ϕ-coefficients are constants.

Uniform motion is mapped into uniform motion. Consider a stationary light distribution, $\boldsymbol{l}_s(x, y)$. When set in motion with velocity v along the $\langle x \rangle$-axis, the spatiotemporal light distribution is described as

$$\boldsymbol{l}(x, y, t) = \boldsymbol{l}_s(x - vt, y). \tag{A4}$$

The following identity holds for this distribution,

$$\boldsymbol{l}(x, y, t + \Delta t) = \boldsymbol{l}(x - \Delta x, y, t), \tag{A5}$$

where

$$\Delta x = v\, \Delta t. \tag{A5$'$}$$

The implication can be reversed. If (A5) holds for any $(\Delta x, \Delta t)$ satisfying (A5′), then $\boldsymbol{l}(x, y, t)$ can be represented by (A4), putting

$$\boldsymbol{l}_s(x, y) = \boldsymbol{l}(x, y, 0). \tag{A6}$$

Now, due to the MHP, denoting by $\boldsymbol{L}(X, Y, T)$ the perceptual image of $\boldsymbol{l}(x, y, t)$,

$$\boldsymbol{l}(x, y, t + \Delta t) \to \boldsymbol{L}(X, Y, T + \Delta T) \tag{A7a}$$

$$\boldsymbol{l}(x - v\, \Delta t, y, t) \to \boldsymbol{L}(X - \Delta X, Y, T), \tag{A7b}$$

where

$$\Delta T = \phi_{tT}(\mathbf{p})\, \Delta t \tag{A8a}$$

$$\Delta X = \phi_{xX}(\mathbf{p}) v\, \Delta t. \tag{A8b}$$

The right parts of (A7a) and (A7b) are equivalent because the left parts are equivalent; see (A5) and (A5′). The proportionality coefficients, $\phi_{tT}(\mathbf{p})$ and $\phi_{xX}(\mathbf{p})$, do not depend on Δt. Hence,

$$\Delta X / \Delta T = v \phi_{xX}(\mathbf{p}) / \phi_{tT}(\mathbf{p}) = V = \text{const} \tag{A9}$$

and, denoting

$$\boldsymbol{L}_m(X, Y) = \boldsymbol{L}(X, Y, 0), \tag{A10}$$

we get

$$\boldsymbol{L}(X, Y, T) = \boldsymbol{L}_m(X - VT, Y). \tag{A11}$$

Thus, $\boldsymbol{L}(X, Y, T)$ is a shape, $\boldsymbol{L}_m(X, Y)$, uniformly moving along axis $\langle X \rangle$.

REFERENCES

ACZEL, J. (1966). *Lectures on functional equations and their applications.* New York: Academic Press.

ADELSON, E. H., & BERGEN, J. R. (1985). Spatiotemporal energy models for the perception of motion. *Journal of the Optical Society of America,* **A2**, 284–299.

ANSBACHER, H. L. (1938). Further investigation of the Harold C. Brown shrinkage phenomenon: A new approach to the study of the perception of movement. *Psychological Bulletin,* **35**, 701.

ANSBACHER, H. L. (1944). Distortion in the perception of real movement. *Journal of Experimental Psychology,* **34**, 1–23.

BAIRD, J. C., & VERNON, C. W. (1965). Binary estimation. *Psychonomic Science,* **3**, 469–470.

BURR, D. C. (1980). Motion smear. *Nature (London),* **284**, 164–165.

BURR, D. C. (1981). Temporal summation of moving images by the human visual system. *Proceedings of the Royal Society of London,* **B211**, 321–339.

BURR, D. C., ROSS, J., & MORRONE, M. C. (1986). Seeing objects in motion. *Proceedings of the Royal Society of London,* **B227**, 249–265.

CAELLI, T., HOFFMAN, W., & LINDMAN, H. (1978). Subjective Lorentz transformations and the perception of motion. *Journal of the Optical Society of America,* **68**, 402–411.

COREN, S., & GIRGUS, J. S. (1978). *Seeing is deceiving: The psychology of visual illusions.* Hilldale, NJ: Erlbaum.

DAY, R. H. (1973). Apparent contraction and disappearance of moving objects in the peripheral visual field. *Vision Research,* **13**, 959–975.

DZHAFAROV, E. N. (1992a). Visual kinematics. II. Space contraction in motion and visual velocity. *Journal of Mathematical Psychology,* **36**, 498–523

DZHAFAROV, E. N. (1992b). Visual Kinematics. III. Transformation of spatiotemporal coordinates. *Journal of Mathematical Psychology,* **36**, 524–546.

EFRON, R. (1970). The relationship between the duration of a stimulus and the duration of a perception. *Neuropsychologia,* **8**, 37–55.

EFRON, R., & LEE, D. N. (1971). The visual persistence of a moving stroboscopically illuminated object. *American Journal of Psychology,* **84**, 365–375.

EISLER, H. (1963). Magnitude scales, category scales, and Fechnerian integration. *Psychological Review,* **70**, 243–253.

GREGORY, R. (1963). Distortions of visual space as an inappropriate constancy scaling. *Nature (London),* **199**, 678–680.

GREGORY, R. (1970). *The intelligent eye.* London: Weidenfeld & Nicolson.

INDOW, T., & WATANABE, T. (1988). Alleys on an extensive apparent frontoparallel plane: A second experiment. *Perception,* **17**, 647–666.

JAEGER, T., & KRAEMER, C. (1980). Exposure duration and line length: An analogy to the Broca–Sultzer effect in perception of extent? *Perceptual and Motor Skills,* **51**, 945–946.

KIRKPATRICK, F. P., & ITTELSON, W. H. (1953). The size–distance invariance hypothesis. *Psychological Review,* **60**, 223–231.

KÜNNAPAS, T. M. (1955). An analysis of the vertical–horizontal illusion. *Journal of Experimental Psychological,* **49**, 134–140.

KÜNNAPAS, T. M. (1958). Measurements of subjective length in the vertical–horizontal illusion. *Acta Psychologia,* **14**, 371–374.

LOWRY, E. M., & DEPALMA, J. J. (1961). Sine-wave response of the visual system. I. The Mach phenomenon. *Journal of the Optical Society of America,* **51**, 740–746.

NEWSOME, L. R. (1965). Judgments of size with varying conditions of stimulus duration and illumination. *Perceptual and Motor Skills,* **21**, 447–452.

SMITH, V. C. (1969). Scotopic and photopic functions for visual band movement. *Vision Research,* **9**, 293–304.

STANLEY, G. (1964). A study of some variables influencing the Ansbacher shrinkage effect. *Acta Psychologia,* **22**, 109–118.

STANLEY, G. (1966). Apparent shrinkage of a rotating arc as a function of luminance relations between figure and ground. *Acta Psychologia*, **25**, 357–364.

STANLEY, G. (1968). Apparent length of a rotating arc-line as a function of speed of rotation. *Acta Psychologia*, **28**, 398–403.

STEVENS, S. S., & GALANTER, E. H. (1957). Ratio scales and category scales for a dozen perceptual continua. *Journal of Experimental Psychology*, **54**, 377–411.

STEVENS, S. S., & GUIRAO, M. (1963). Subjective scaling of length and area and the matching of length to loudness and brightness. *Journal of Experimental Psychology*, **66**, 177–186.

TEGHTSOONIAN, M. (1965). The judgment of size. *American Journal of Psychology*, **78**, 392–402.

WATSON, A. B., & AHUMADA, J. A. (1985). Model of human visual motion sensing. *Journal of the Optical Society of America*, **A2**, 322–341.

WEALE, R. A. (1975). Apparent size and contrast. *Vision Research*, **15**, 949–956.

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