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Visual Kinematics
III. Transformation of Spatiotemporal Coordinates in Motion

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The perceived spatial distances between visual objects moving with the same velocity shrink along the direction of motion. The distance-in-motion to distance-at-rest ratio decreases as a function of physical velocity and, for a given physical velocity, as a function of parameters increasing perceived velocity. This effect (Space Contraction in Motion) implies that spatiotemporal coordinates and motion vectors are assigned to visual objects in an interdependent way, reflecting the fundamental structure of visual space–time. This structure can be described by non-Galileian transformations of spatiotemporal coordinates of visual objects in visual motion. The transformations are linear in coordinates, but do not obey the Galileian relativity principle (there is a privileged, absolutely resting system of coordinates), and they do not form a uniparametric (Lorentz) group with respect to velocity. Nor do they necessarily imply that perceived time intervals, and/or perceived simultaneity, must change in motion too. Visual deformations-in-motion are a complex mixture of geometric transformations and changes in color/brightness distribution due to visual integration–interaction mechanisms. Changes in spatial resolution of moving acuity targets (including moving verniers) cannot be derived from the geometric transformations only; one has to specify in addition detection rules and the logical order of geometric and distributional changes.

1. INTRODUCTION

In this paper I present a theory (or theoretical language) that places the Space Contraction in Motion (SCM) effect, described in the preceding papers of the Visual Kinematics series (Dzhafarov, 1992a, b), within a general context of spatiotemporal coordinate transformations in motion. The results established in the preceding papers, both theoretical and empirical, that are essential for the issues discussed in this paper will be briefly recapitulated.

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1.1. *Basic Concepts and Facts*

The analysis of SCM is based on the Mapping Homogeneity Principle (MHP), according to which if a luminance distribution $\mathbf{l}(x, y, t)$ is perceptually mapped into a color/brightness distribution $\mathbf{L}(X, Y, T)$, then a shifted replica of $\mathbf{l}(x, y, t)$, along $\langle x \rangle$, $\langle y \rangle$, or $\langle t \rangle$ axes, is mapped into a correspondingly shifted replica of $\mathbf{L}(X, Y, T)$.[1] The shifts ΔX, ΔY, and ΔT are proportional to the corresponding physical shifts Δx, Δy, and Δt, respectively. The coefficients of proportionality, ϕ_{xX}, ϕ_{yY}, and ϕ_{tT}, generally depend on parameters of the stimulus being shifted, $\mathbf{l}(x, y, t)$.

If $\mathbf{l}(x, y, t) = \mathbf{l}_s(x - vt, y)$, i.e., it is a luminance profile $\mathbf{l}_s(x, y)$ moving along the $\langle x \rangle$-axis with velocity v, then it is perceptually mapped into $\mathbf{L}_S(X - VT, Y)$, a color/brightness profile $\mathbf{L}_S(X, Y)$ moving along the $\langle X \rangle$-axis with velocity V:

$$\begin{aligned} V/v &= \phi_{xX}(v, V)/\phi_{tT}(v, V); \\ V &= V(v, \mathbf{p}). \end{aligned} \tag{1}$$

The vector $\mathbf{p}$ stands for all presentation/observation parameters other than v on which the perceived velocity V might depend (such as steady fixation versus free looking; see Visual Kinematics II). Since $V(v, \mathbf{p})$ is a strictly increasing function of v, *for a fixed vector* $\mathbf{p}$ the proportionality coefficients ϕ_{xX} and ϕ_{tT} are functions of one argument only, which can be chosen to be either v or V. The reason for presenting ϕ_{xX} and ϕ_{tT} as functions of both physical and perceived velocity is as follows.

It has been shown in Visual Kinematics I and II that ϕ_{xX} does not depend on luminance/contrast, shape/size, and other parameters of moving stimuli that do not noticeably affect the perceived velocity of motion. However, ϕ_{xX} decreases as a function of physical velocity v and, for a given v, as a function of other observation/stimulation parameters that increase the perceived velocity V (the SCM phenomenom). Generalizing, a decrease in ϕ_{xX} is always associated with an increase in V, whether or not it is due to an increase in v (The Perceived Velocity Hypothesis, PVH). The following formula provides a fair approximation for the dependence of ϕ_{xX} on v:

$$\phi_{xX}(v, \mathbf{p}) = \begin{cases} 1 & \text{if } \; v \leqslant v_0 \\ (v_0/v)^\alpha & \text{if } \; v_0 < v < v_1 \\ (v_1/v)^\beta \, (v_0/v_1)^\alpha & \text{if } \; v \geqslant v_1, \end{cases} \tag{2}$$

[1] In this paper I follow the notation agreements adopted in Visual Kinematics I and II: uppercase and lowercase symbols will refer to perceptual and physical parameters and coordinates, respectively; boldface roman symbols denote vectors of parameters; angle brackets denote axes or frames of reference. Physical spatial coordinates and velocity are measured in external rather than retinal coordinates. The $\langle x \rangle$-axis in the physical plane and the $\langle X \rangle$-axis in the perceptual plane will always be assumed to be collinear with the direction of physical motion and perceived motion, respectively.

where $v_0 \approx 10°/s$, $v_1 \approx 45°/s$.[2] The constants α and β (but not v_0 and v_1) depend on observer and observation/presentation conditions, **p**, that affect the perceived velocity V: an increase in V corresponds to increases in both α and β (the value of β/α was shown to be relatively stable across different conditions and observers, $\beta/\alpha \approx 7$).

It follows from this formula that ϕ_{xX} can be rewritten as a function of V. For example, assuming that v^α is a fixed strictly increasing transformation F of V, we have

$$\phi_{xX}(V, V_0, V_1) = \begin{cases} 1 & \text{if} \quad V \leqslant V_0 \\ F(V_0)/F(V) & \text{if} \quad V_0 < V < V_1 \\ [F(V_0)/F(V)]^{\Phi} F(V_0)/F(V_1) & \text{if} \quad V \geqslant V_1, \end{cases} \tag{3}$$

where V_0 and V_1 are the "reference" perceived speeds corresponding to v_0 and v_1, and Φ in (3) replaces β/α in (2). One can see that in both representations, (2) and (3), ϕ_{xX} (*considered across all possible sets of observation/presentation conditions*, **p**) is a function of both V and v, allthough only one of these arguments explicitly enters in each expression.[3]

No systematic data are available on metrical transformations of time intervals in motion. Assuming, however, that the MHP holds for temporal as well as spatial shifts, ϕ_{tT} should also be representable as a function of V and v. Indeed, from (1) $\phi_{tT} = \phi_{xX}(v, V)\, v/V$. Finally, the coefficient ϕ_{yY} can be set identically equal to unity because SCM does not occur in the direction orthogonal to that of motion.

[2] The value of $v_0 \approx 10°/s$ is obtained by extrapolation of the middle part of Eq. (2). Informal observations did not reveal any noticeable SCM below 10–15°/s, but it is possible that putting $\phi_{xX} = 1$ in this region is just an approximation for a very slowly decreasing branch. In any case, there should be a transition point v_0 in the proximity of 10–15°/s, separating the branches with distinctly different rates of decrease.

[3] As a tentative speculation, one might relate the transition velocities v_0 and v_1 in (2) and (3) to processing limits of the parvocellular (P) and magnocellular (M) pathways in primates (Lennie *et al.*, 1990). Both M and P pathways contribute to both spatial and movement characteristics of moving stimuli, but the relative role of the M pathways has been shown to be substantially greater at 20°/s than at 1°/s (Merigan, Byrne, & Maunsell, 1990). Perhaps $v_0 \approx 10°/s$ is a transition point from predominantly P (or M − P) processing to predominantly M processing. There is psychophysical evidence that the perception of fine spatial details (primarily processed by the P pathways; Merigan & Eskin, 1986) is effective for motion below 10–15°/s, but not above (Burr, 1980; Fahle & Poggio, 1981). The M pathways (via V_1 layer 4B and indirectly via V2 and V3) project to area MT, which is believed to play a major role in motion perception (Movshon, Adelson, Gizzi, & Newsome, 1986) and in smooth-pursuit eye velocity control (Newsome, Wurtz, Dursteler, & Mikami, 1985). The second transition velocity in (2) and (3), $v_1 \approx 45°/s$, is in the range of the maximum smooth-pursuit velocities in response to an unexpected motion (Westheimer, 1954; Hallett, 1986); it is also in the range of v at which the Weber fraction $\Delta v/v$ begins to increase from a 5–7% plateau (Sekuler, Anstis, Braddick, Brant, Movshon, & Orban, 1990). One might speculate, therefore, that velocities exceeding 45°/s are beyond the optimal range of area MT (in Sekuler *et al.*, 1990, the optimal range is related to the representation of different velocities in the population of velocity-tuned MT cells).

1.2. *The Space–Time–Velocity Triad in Vision*

In a color/brightness distribution $L(X, Y, T) = L_S(X - VT, Y)$, the perceived velocity V equals the ratio of spatial and temporal intervals traversed by a point taken on the moving shape $L_S(X, Y)$. Equation 1 is essentially a restatement of this simple proposition, except that the intervals ΔX and ΔT (defining ϕ_{xX} and ϕ_{tT}) represent perceptual shifts between two otherwise identical images, rather than intervals traversed by a single point. The diagrams in Fig. 1 help to illustrate this point and clarify the operational meaning of the coefficients ϕ_{xX} and ϕ_{tT}.

The configuration shown in the top left diagram is an example of the double-perturbation (2P) stimulation. In general, the 2P stimulation consists of two identical perturbations of a uniform luminance field shifted with respect to each other along both $\langle x \rangle$ and $\langle y \rangle$ dimensions and moving with a common velocity v along the $\langle x \rangle$-axis. If $v \neq 0$ the Δx-shift translates into a Δt-shift measured between the moments when a fixed $\langle x \rangle$-position is crossed by two points correspondingly located within the two perturbations: $\Delta x/\Delta t = v$. The parameters of the two perturbations and their separation can be chosen so that the perceptual mapping of either perturbation is independent of that of the other (in Visual Kinematics I it was shown how this lack of interference can be verified experimentally). Due to the MHP the two distributions are mapped into identical, except for a spatiotemporal shift, visual objects. Then the $\langle X \rangle$, $\langle Y \rangle$, and $\langle T \rangle$ distances between any two

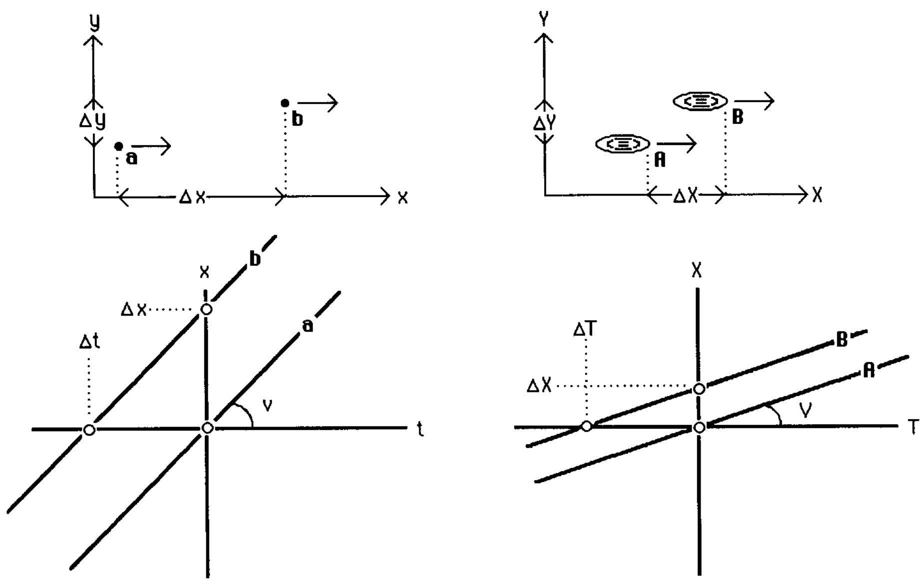

FIG. 1. (Top) $\langle x, y \rangle$ and $\langle X, Y \rangle$: a uniformly moving 2P stimulus with spatial separation $(\Delta x, \Delta y)$ is mapped into two uniformly moving identical shapes with spatial separation $(\Delta X, \Delta Y) = (\phi_{xX}\Delta x, \Delta y)$. (Bottom) $\langle x, t \rangle$ and $\langle X, T \rangle$: ΔX is a spatial shift between A and B taken at a given time moment (open circles with $T = 0$); ΔT is a time shift between A and B taken in a given $\langle X \rangle$-position (open circles with $X = 0$); analogously for Δx and Δt; $V = \Delta X/\Delta T = \phi_{xX}\Delta x/\phi_{tT}\Delta t = v\phi_{xX}/\phi_{tT}$ (Eq. (1)).

points identically located within the two objects should represent ΔX, ΔY, and ΔT in the MHP formulation. These distances can be estimated numerically or by a matching technique (Visual Kinematics I). An operational definition of ΔT, therefore, is the estimated interval between the moments when the two images first cross a transversely oriented spatial marker (provided the latter can be shown to not affect the perceived motion).

It follows that by estimating the transformations of ΔX and ΔT for different values of v (with a fixed Δx or a fixed Δt) one can obtain a valid psychophysical function for perceived velocity ($\Delta X/\Delta T = V$). Given the controversial character of perceived velocity measurements by means of "direct scalling" techniques (see Visual Kinematics II), the reconstruction of the psychophysical function through measurements of spatial and temporal intervals in motion is a desirable prospect. Indeed, perceived linear extent is approximately proportional to physical length both in motion and at rest (Visual Kinematics I and II), and an approximate proportionality seems to hold between perceived and physical durations, at least for stationary stimuli (Eisler, 1976; Allan, 1979). It remains to be demonstrated experimentally that the same is true for time intervals in motion, as required by the MHP.

Note that given the proportionality between ΔX and Δx for any v, a lack of such proportionality between ΔT and Δt would indicate that a uniform stimulus motion is mapped into a perceptual motion with varying speed; it would not indicate that a constant visual velocity does not obey the "distance–time ratio law." If a characteristic of a perceptually uniform motion does not obey this "law," it simply should not be called velocity , allthough it may be related to the latter by a one-to-one function. Numerous attempts have been made to test the "distance–time ratio law" empirically (e.g., Mashour, 1964; Rachlin, 1966). From the viewpoint adopted in this paper, what is really tested in such work is whether certain measurement procedures do measure perceived velocity or (if motion is non-uniform) whether observers average instantaneous speeds arithmetically. The problem goes beyond just correcting the scales for nonsensory factors (Poulton, 1979; Gescheider, 1988). Even if a true sensory scale is obtained, the "ratio law" is the only criterion for deciding that the measured quantity is velocity. Think of a physical device labeled "rapidness" and found to measure a quantity proportional to the squared distance–time ratio. A proper conclusion would be that "rapidness" means something different from, although functionally related to, velocity (in this case it could be kinetic energy). Aglom and Cohen-Raz (1984, 1987) state explicitly the validating function of the "ratio law" for a perceived velocity scale.

Another aspect of the "ratio law," quite apparent when presented in the form of (1), is that neither the numerator nor the denominator of the ratio can be replaced with estimates of length and time intervals in stationary stimuli. The assumption that such replacement is possible is equivalent to the fixed-metric hypothesis ($\phi_{xX} = \phi_{tT} = 1$) which was shown to be wrong in the previous Visual Kinematics papers. Note in Eq. (1) that if the coefficients ϕ_{xX} and ϕ_{tT} did not depend on v, the perceived velocity V would necessarily be proportional to v.

2. Visual Kinematics

2.1. *Frames of Reference*

The theory will be presented on a semi-formal level. It is based on the following idealized picture of visual perception. A visual image is viewed as a distribution of "point-events" uniquely localized in the perceptual (frontoparallel) space and time, with motion vectors assigned to them. The spatiotemporal coordinates of a point-event with a motion vector V are denoted by (X_V, Y_V, T_V). These are the coordinates of the point-event in the stationary frame of reference $\langle X, Y, T\rangle$, which will be referred to as PFR_0 (Perceptual Frame of Reference, stationary). Consider now another frame of reference, moving with respect to the former along its $\langle X\rangle$-axis with velocity V: PFR_V. In this moving frame of reference the point-event under consideration is stationary, and its coordinates can be denoted as (X_0, Y_0, T_0). Note that the subscripts for the frame of reference and the coordinates are symmetrically opposite, as it should be since they refer to motion velocity in both cases.

Visual kinematics is (described by) a system of transformations from (X_0, Y_0, T_0) to (X_V, Y_V, T_V). Formally, this definition is identical with that of physical kinematics. However, unlike in physical kinematics, *the two frames of reference in visual perception are not interchangeable*. It is meaningless to say that the observer can perceptually switch from one to the other. If the observer experiences a real or illusory self-motion with velocity V, the very fact that this motion is perceived indicates that the frame of reference does not change. One can perceive oneself as moving along with a moving visual object, and this, obviously, is not equivalent to stabilizing the latter in a moving frame of reference. It is also obvious that a physical motion of the observer that is not perceived as such does not lead to a change of the perceptual frame of reference either. A conclusion is that the observer is bound to a single frame of reference, which therefore constitutes an "absolute rest" position. Strictly speaking, it is the only frame of reference in visual perception, $\mathrm{PFR}_0 \equiv \langle X, Y, T\rangle$, whereas the moving frame of reference, PFR_V, is merely a mathematical abstraction. To make this abstraction a useful analytic tool, it should be constructed consistent with the MHP and operationally consistent with the MHP-based 2P paradigm.

2.2. *MHP–Consistency of Visual Kinematics*

Somewhat loosely defined, the MHP-consistency means that a system of moving visual objects "frozen" in the PFR_V has a spatiotemporal metric isomorphic to that of the system of moving stimuli "frozen" in the imaginary physical frame of reference moving along with the stimuli. A rigorous definition involves the following requirement imposed on the coordinate transformation formulae from (X_0, Y_0, T_0) to (X_V, Y_V, T_V) and from (X_0, Y_0, T_0) to (x, y, t).

Refer to Fig. 1 once again. Let two points, **A** and **B**, be chosen correspondingly located in the two shifted visual objects. Ignoring for the moment the $\langle Y\rangle$-axis, the points are represented by two parallel motion lines shifted with respect to each other by $(\Delta X, \Delta T)$ (bottom right diagram). The coordinate transformation formulae applied to each point of the two motion lines will map them into two lines in the PFR_V parallel to the time axis (because the PRF_V, by definition, stabilizes the moving points in place). Consider now these two horizontal lines in the PFR_V *as if* they were representing two points, **a** and **b**, correspondingly located within two constituting perturbations of a moving 2P stimulus, but viewed from a physical frame of reference moving with the same velocity v. Then switching to a stationary physical frame of reference $\langle x, t \rangle$ would map the two horizontal lines into two parallel lines with the slope of v, shifted with respect to each other by $(\Delta x, \Delta t)$ (as in the bottom left diagram of Fig. 1).

The MHP-consistency now can be formulated as the requirement that the resulting Δx and Δt equal the factual shift values in the 2P stimulus whose visual image was used to define the points **A** and **B**. Put differently, the transformation formulae from (X_0, Y_0, T_0) to (X_V, Y_V, T_V) should be set so that the values of $\Delta X/\Delta x$ and $\Delta T/\Delta t$ equal the empirically found coefficients $\phi_{xX}(V, v)$ and $\phi_{tT}(V, v)$, respectively. Generalizing this statement to include $\Delta Y/\Delta y$, and recalling that $\phi_{yY} = 1$, one can put $Y_V = Y_0$ in the coordinate transformations and thereby reduce them to $(X_0, T_0) \to (X_V, T_V)$.

To make the MHP-consistency unambiguous, the PFR_V and the physical frame of reference $\langle x, t \rangle$ should be considered isomorphic, except that the PFR_V is moving with respect to $\langle x, t \rangle$ with velocity v. Formally this means that the transformations from (X_0, T_0) to (x, t) are Galileian:

$$\begin{bmatrix} x \\ t \end{bmatrix} = \mathbf{m} \times \begin{bmatrix} X_0 \\ T_0 \end{bmatrix}; \qquad \mathbf{m} = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix}. \tag{4}$$

Note that the MHP-consistency does not imply that one can find a physical point **a** "corresponding" to a perceptual point **A**, so that **A** is a visual image of **a**. Such a statement would be meaningless, for any point of a visual image is determined by the luminance distribution within a certain spatiotemporal area. When considering the mapping from $\langle X, Y, T \rangle$ to $\langle x, y, t \rangle$ (via the auxiliary PFR_V), corresponding points are always taken within two parts of a 2P stimulus or two parts of its image. Due to the logic of the 2P paradigm it is irrelevant what two points are taken in either case.

2.3. *Transformation Formulae from PFR_0 to PFR_V*

Since all spatial and temporal axes considered are interval scales, to be invariant with respect to shifts of the origins the transformations $(X_0, T_0) \to (X_V, T_V)$ should be linear in coordinates. Making the origins of PFR_0 and PFR_V coincide and observing that

$$(0_0, T_0) \to (VT_V, T_V) \tag{5}$$

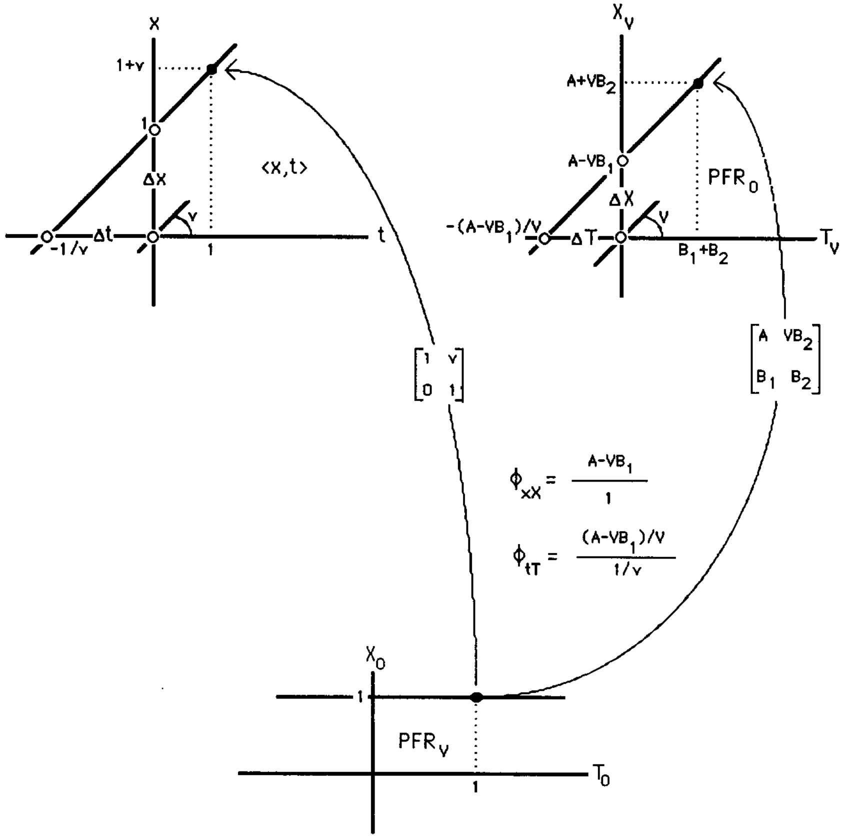

FIG. 2. Solid circles: coordinates (X_V, T_V) in PFR_0 (perceptual frame of reference) and (x_v, t_v) in $\langle x, t\rangle$ (physical frame of reference) corresponding to unit coordinates in PFR_V (imaginary frame of reference "stabilizing" moving percepts); the coordinates (X_V, T_V) and (x, t) are obtained by applying the two transformation matrices to $(X_0 = 1, T_0 = 1)$. Open circles are as in Fig. 1.

(by the definition of uniform motion), we have

$$\begin{bmatrix} X_V \\ T_V \end{bmatrix} = \mathbf{M} \times \begin{bmatrix} X_0 \\ T_0 \end{bmatrix}, \qquad \mathbf{M} = \begin{bmatrix} A & VB_2 \\ B_1 & B_2 \end{bmatrix}. \tag{6}$$

Here $\mathbf{M} = \mathbf{M}(V, v)$, and for a fixed set of presentation/observation parameters $\mathbf{M}$ can be considered a function of V alone (see Section 1.1).[4] This is the most general form of linear kinematics possible.

Figure 2 shows how the coefficients $\phi_{xX}(V, v)$ and $\phi_{tT}(V, v)$ should be expressed

[4] Considered as functions of V the coefficients A and B_2 are even-symmetric, and B_1 is odd-symmetric: $A(-V) = A(V)$; $B_1(-V) = -B_1(V)$; $B_2(-V) = B_2(V)$. A proof is obtained from symmetry considerations by switching V to $-V$ and (X_0, T_0) to $(-X_0, T_0)$. Denoting the dimensionality of A by $[[A]]$, the dimensionality of V by $[[V]]$, and so on, and referring to a dimensionless value as $[[1]]$, we have: $[[A]] = [[B_2]] = [[1]]$, $[[B_1]] = [[T]]/[[X]] = [[V]]^{-1}$.

through the functions A, B_1, and B_2 to provide the MHP-consistency of the matrix $\mathbf{M}$:

$$\begin{aligned} \phi_{xX}(V, v) &= (A - VB_1) \\ \phi_{tT}(V, v) &= (A - VB_1)\, v/V, \end{aligned} \tag{7}$$

which obviously agrees with (1).

2.4. *Identification Problem*

To identify visual kinematics means to find the values of the transformation coefficients, $\mathbf{M}(V, v)$, for any given values of v and other stimulation/observation parameters on which V depends. A complete identification is not possible on the basis of measuring the coefficients ϕ_{xX} and ϕ_{tT} only. Equation (7) provides information concerning only the values of $A - VB_1$ and V; this, of course, does not allow one to unconfound the values of A and B_1 and to find B_2. Visual kinematics can be identified, however, if in addition to ϕ_{xX} and ϕ_{tT} measurements one can find operational procedures for measuring, or plausible theoretical grounds for deriving, the coefficients B_1 and B_2.

This problem is far beyond the scope of this paper: its solution should involve non-rigid motion (temporal changes embedded in moving luminance profiles), which might turn out to be methodologically more complex a task than the SCM measurements. An interpretation for B_1 and B_2 can be derived from considering the $\mathbf{M}$-transformations of the following two pairs of point-events in the PFR_V: $\mathbf{O}(0, 0)$, $\mathbf{I}(X_0 = 1, T_0 = 0)$, i.e., two simultaneous point-events in the PFR_V separated by a unit space interval; and $\mathbf{O}(0, 0)$ and $\mathbf{J}(X_0 = 0, T_0 = 1)$, i.e., two isotopic point-events in the PFR_V separated by a unit time interval. In the PFR_0 the first pair maps into $\mathbf{O}(0, 0)$ and $\mathbf{I}(X_V = A, T_V = B_1)$, so the two point-events are now separated by a time interval B_1 ("simultaneity transformation coefficients"). The second pair maps into $\mathbf{O}(0, 0)$ and $\mathbf{J}(X_V = VB_2, T_V = B_2)$ in the PFR_0, which means that the two point-events are now separated by a time interval B_2 ("time transformation coefficient"; note that the point-events are no more isotopic).

The coefficient B_2 should not be confused with ϕ_{tT} (Eq. (7)), which might also be referred to as a "time transformation coefficient." In general, one should be cautious in labeling transformations along one axis without specifying intervals along the other. A unit time interval isotopically measured in the PFR_V transforms into B_2 in the PFR_0; a unit time interval isotopically measured in the PFR_0 transforms into $B_2(A - VB_1)/A$ in the PFR_V, which is derived by applying $\mathbf{M}^{-1}$ to $\mathbf{J}'(X_V = 0, T_V = 1)$; ϕ_{tT} in Eq. (7) relates two intervals isotopically measured both in PFR_0 and in the physical frame of reference $\langle x, t \rangle$. All these coefficients can be referred to as "time transformation" coefficients, but they have different meanings and quantitative values.

2.5. *Comparison with Physical Kinematics*

It is of great theoretical importance that the three types of kinematic transformations, of space, of time, and of simultaneity, are logically independent. Thus one cannot exclude a priori that $B_1 = 0$ ("absolute simultaneity"), and/or $B_2 = 1$ ("absolute time"), even though $\phi_{xX} = A - VB_1$ changes with V (SCM).

The reciprocity of the time transformation coefficient B_2 and the space transformation coefficient $(A - VB_1)$, which holds for the two forms of kinematics considered in mechanics (Galileian and Lorentzian), follows from the Galileian relativity principle

$$\mathbf{M}^{-1}(V, v) = \mathbf{M}(-V, -v). \tag{8}$$

This principle says that the transformation formulae from PFR_0 to PFR_V should be identical with those from PFR_V to PFR_0, except that the direction of motion is reversed. Stated in a better known form: all uniformly moving frames of reference are equivalent. Equation 8 yields

$$\mathbf{M} = \begin{bmatrix} A & VA \\ (A^2 - 1)/VA & A \end{bmatrix}. \tag{9}$$

Relating (6) and (7) to (9), observe that $\phi_{xX} = A - VB_1 = A^{-1}$ and $B_2 = A$ in this kinematics. The Galileian relativity principle, however, has no operational meaning in the visual kinematics. As discussed above, an observer cannot change the frame of reference with respect to the same visual percept: in this sense there is only one, "absolutely resting," frame of reference in visual perception. Only empirical analysis can show whether the form (9) still holds in visual kinematics as a mathematical coincidence.

If, by another coincidence, $\mathbf{M}$ formed a group with V as the group parameter (Hoffman, 1966, 1978; Dodwell, 1983), then the kinematics should be formally Lorentzian. Indeed, the uniparametric group assumption is equivalent to the requirements that (a) the Galileian relativity holds; and (b) for any two velocities, V and W, there is a velocity U, such that $\mathbf{M}(V) \times \mathbf{M}(W) = \mathbf{M}(U)$. A straightforward algebraic derivation leads to

$$[A^2(V) - 1]/[A(V)V]^2 = [A^2(W) - 1]/[A(W)W]^2,$$

which means that $[A^2 - 1]/[AV]^2$ is a constant independent of V. Because $A = \phi_{xX}^{-1}$, and $\phi_{xX} \leqslant 1$ (SCM), this constant is positive and can be denoted by Ω^2. Because A is dimensionless (see footnote 4), $[[\Omega]] = [[V]]^{-1}$. The resulting kinematics is represented by

$$\mathbf{M} = \begin{bmatrix} (1 - \Omega^2 V^2)^{-1/2} & V(1 - \Omega^2 V^2)^{-1/2} \\ \Omega^2 V(1 - \Omega^2 V^2)^{-1/2} & (1 - \Omega^2 V^2)^{-1/2} \end{bmatrix}. \tag{10}$$

As a special case, if Ω is zero, the kinematics is Galileian ($\mathbf{m}$ in Eq. (4) with V replacing v), but this possibility is ruled out for visual kinematics by the SCM effect.

Caelli, Hoffman, and Lindman (1978) proposed that (10) could be appropriate for visual kinematics if V is assumed to be proportional to v and Ω^{-1} is interpreted as the "maximum perceivable velocity." (The first assumption, $V \propto v$, is not made

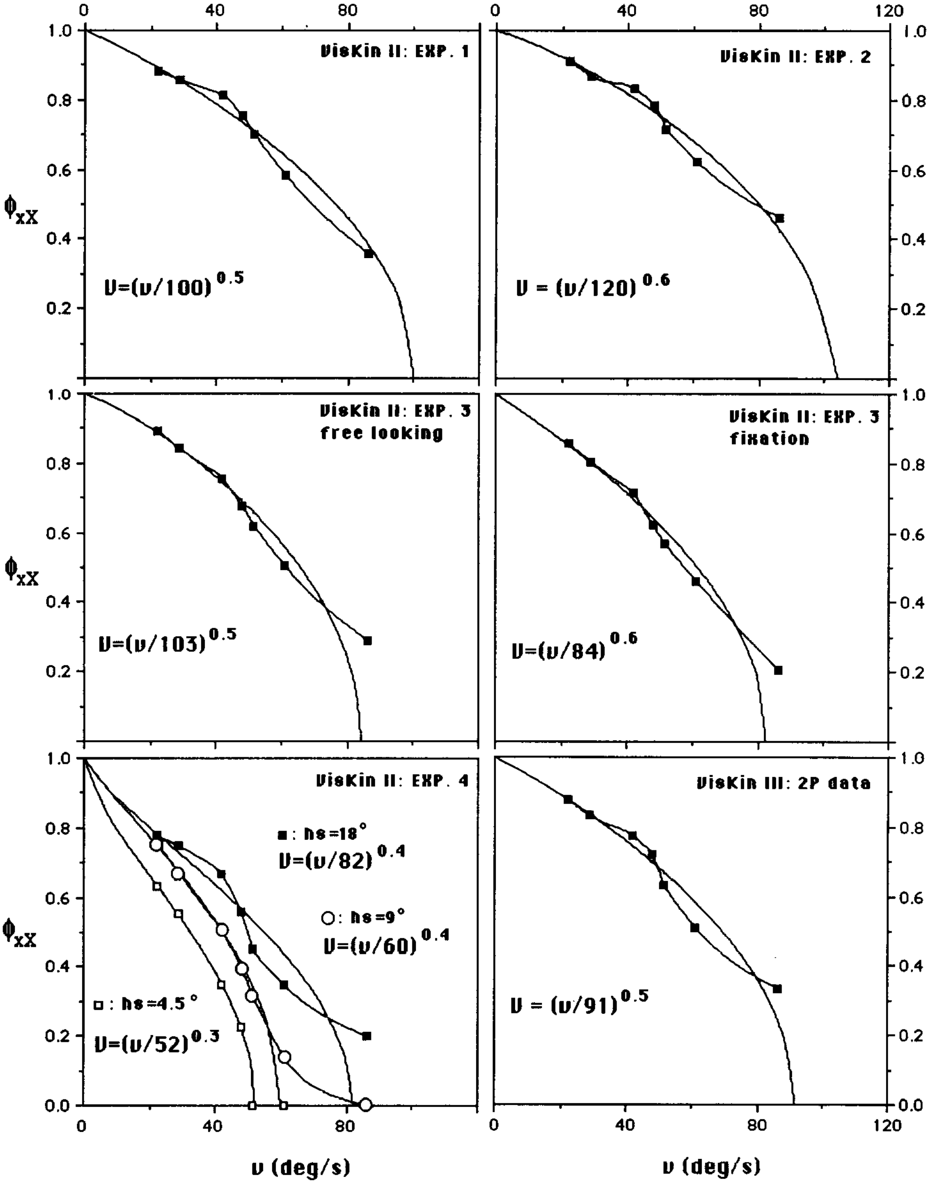

FIG. 3. The best fit of the Lorentzian model of visual kinematics to the experimental data presented in the Visual Kinematics papers (including the experiment presented in this paper). Each symbol represents an average across subjects and experimental factors that have been found irrelevant: 4 observers × 60 estimates per symbol (top); 2 observers × 60 estimates (middle); 5 observers × 20 estimates (left bottom); and 6 observers × 20 estimates per symbol for the 2P data of Fig. 5 (right bottom). The fitted formula is $\phi_{xX} = (1 - V^2)^{1/2}$; $V = (v/\text{const})^{\kappa}$.

explicit in their paper.)[5] Such a model predicts, of course, the existence of SCM (Lorentz–Fitzgerald contraction), but the predicted dependence of ϕ_{xX} on v is very different from the one obtained in the experiments reported in this and the preceding Visual Kinematics papers (Eq. (2)). The experimental data presented in Caelli *et al.* (1978) are on the estimated length of a single light segment; this paradigm will be shown below to deal with a mixture of geometric and distributional deformations (due to the integration–interaction mechanisms, such as luminance summation), and therefore it cannot provide decisive evidence.

I have tested the Lorentzian model on the results of all experiments reported in Visual Kinematics I and II, after having replaced the restriction $V \propto v$ implicitly imposed by Caelli *et al.* (which does not work obviously) with a more general assumption: $V \propto v^{\kappa}$. For every empirical SCM curve the optimal values of κ and $\Omega^{-1/\kappa}$ were found to provide the best least-square fit (in linearizing transformations of the plots). Parameter $\Omega^{-1/\kappa}$ is the hypothetical "maximum perceivable velocity" (Ω^{-1} is its perceived value). The fit, as one can see in Fig. 3, was quite poor. Only pooled data are shown in Fig. 3, but the fit is even worse when applied to individual data.

One can conclude that there is neither empirical nor theoretical support for the assumption that visual kinematics is Lorentzian. One must add to this that there seems to be no grounds for the very concept of the "maximum perceivable velocity," at least not for attaching to this concept a kinematic meaning. A light distribution can move fast enough to be completely smeared, but this hardly can be considered a velocity perception limit: an increase in contrast or distance traversed may be sufficient to restore the perception of motion.

The conclusion that visual kinematics does not form a uniparametric group has obvious negative implications for the general theory of Lie transformation groups in visual perception (Hoffman, 1966, 1978; Dodwell, 1983).

3. Geometric and Distributional Transformations Combined

3.1. *A Mixture of Geometric and Distributional Transformations*

According to the theory just presented, the transformations of the visual space metric in visual motion are universal, in the sense that they affect spatial intervals between any two visual points in a state of common motion. As a result, the geometric transformations should affect the appearance of any moving light

[5] This assumption is essential: the "subjective Lorentz transformations" of Caelli *et al.* cannot be derived otherwise. At the same time their analysis of velocity halving is equivalent to the assumption that $V = \operatorname{arctanh}(v\Omega)$. This is not an internal contradiction in their model, but an isolated error: the arctanh-formula has no substantiation in Lorentzian or any other kinematic transformations (ratios and/or differences of speeds should be computed algebraically, not according to the "relativistic addition of velocities"). Curiously, I found that their velocity halving data are better approximated by straight lines than by their curves, suggesting a power function, $V = \lambda v^{\alpha}$, with exponents between 1.20 and 1.68.

distribution, irrespective of what other, non-geometric, perceptual deformation the distribution was subjected to. These deformations in motion can be called distributional because they change distributions of color/brightness in visual space, rather than the metric of the space itself. That distributional deformations do occur in motion is a well-known fact, traditionally described in terms of luminance summation/integration and visual masking (see Visual Kinematics I for a more detailed discussion).

As a simple demonstration, consider a "point-size" luminance perturbation. When stationary, and if the contrast is within certain limits, this stimulus is mapped into a "point-size" color/brightness perturbation. If SCM were the only factor involved, the apparent shape/size of the same stimulus in motion would be practically the same as that when at rest: the contraction of a close to zero value should be negligible. It is known, however, that a moving dot stimulus generally looks elongated along the direction of motion. It is not obvious a priori in what logical order a moving luminance profile is subjected to the geometric and distributional transformations: whether the distributional changes take place in the geometrically shrunken space or whether the geometric shrinkage is applied to smeared or otherwise deformed visual objects.

The experiment described below does not answer this question, but merely demonstrates the complexity of the mixture of geometric and distributional deformations in moving single-perturbation stimuli. Viewed from a methodological standpoint, the experiment shows that the use of the 2P paradigm in the experiments presented in the preceding Visual Kinematics papers was critical for the geometric/kinematic interpretation of the SCM phenomenon. Even in a crude approximation the geometric transformations could not be identified if the observers were to judge the shape/size parameters of a single moving stimulus. An immediate motivation for the experiment to be presented is related to the work by Caelli *et al.* (1978), who attempted to reconstruct visual kinematics from the length-in-motion estimates of a single light segment, and to the work by Burr (1980), who demonstrated that the visual system can effectively minimize visual smear in motion.

3.2. *2P versus 1P Stimulation*

Figure 4 schematically shows the *2P* and *1P* stimuli used in the experiment: two identical rectangular luminance increments on a uniform background moving along their longer dimension with a common velocity (2P), and a single rectangular segment (1P). The moving stimuli appeared from behind the left screen border, uniformly moved toward the right one, and disappeared behind it (the borders were clearly seen because the luninance outside the screen was practically zero). The experiment was carried out under free looking observation conditions: no fixation point was present, and the observers were allowed to move their eyes in a natural way (see Visual Kinematic II); viewing was binocular; the head movements were restrained by a chin rest with a forehead support.

The parameters characterizing the light segments and their spatial separation in

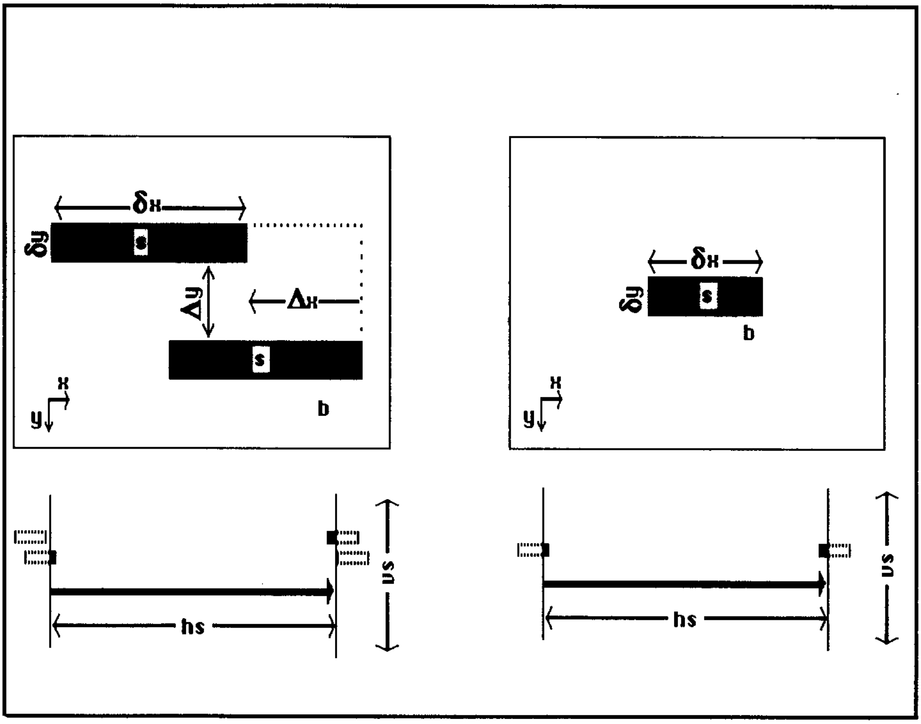

FIG. 4. 2P stimulation (left) and 1P stimulation (right) used in the experiment. $\delta x:\delta y$, shape parameters (actual value was 12.9°:0.5° in 2P stimuli and 4.2°:0.5° in 1P stimuli); $s:b$, segment/background contrast (30 $cd \cdot m^{-2}$:3 $cd \cdot m^{-2}$); hs · vs, screen size parameters (36.8°:10.7°); $\Delta x:\Delta y$, shift parameters in 2P stimuli (4.2°:1.2°). Note that $\delta x_{1P} = \Delta x_{2P} = 4.2°$. The bottom panel schematizes the "gradual" appearance–disappearance mode of presentation used. The velocity varied at seven levels from 22.1°/s to 86.4°/s.

the 2P stimulation should be clear from the figure (see Visual Kinematics I for a detailed discussion). The single light segment was identical to the constituting segments of the 2P stimuli, except that its length, δx, was equal to Δx in the 2P stimuli. In the 2P trials the dependent variable was the estimated perceived spatial separation, ΔX, between two segments constituting a 2P stimulus. In the 1P trials the task was to estimate the apparent length of the moving segment (δX). The estimates of both types were made by adjusting the length of a stationary light segment to match ΔX or δX.

The 2P and 1P trials with different velocities were completely randomized, with the total of 20 match-estimations per stimulus type per velocity per observer. The adjustments of the stationary length segment (match-estimates) were made after a moving stimulus had been presented four times in brief succession.

The experimental setup (an optical–mechanical system) was identical to that described in Visual Kinematics I (Experiments 8 and 9f of that paper).

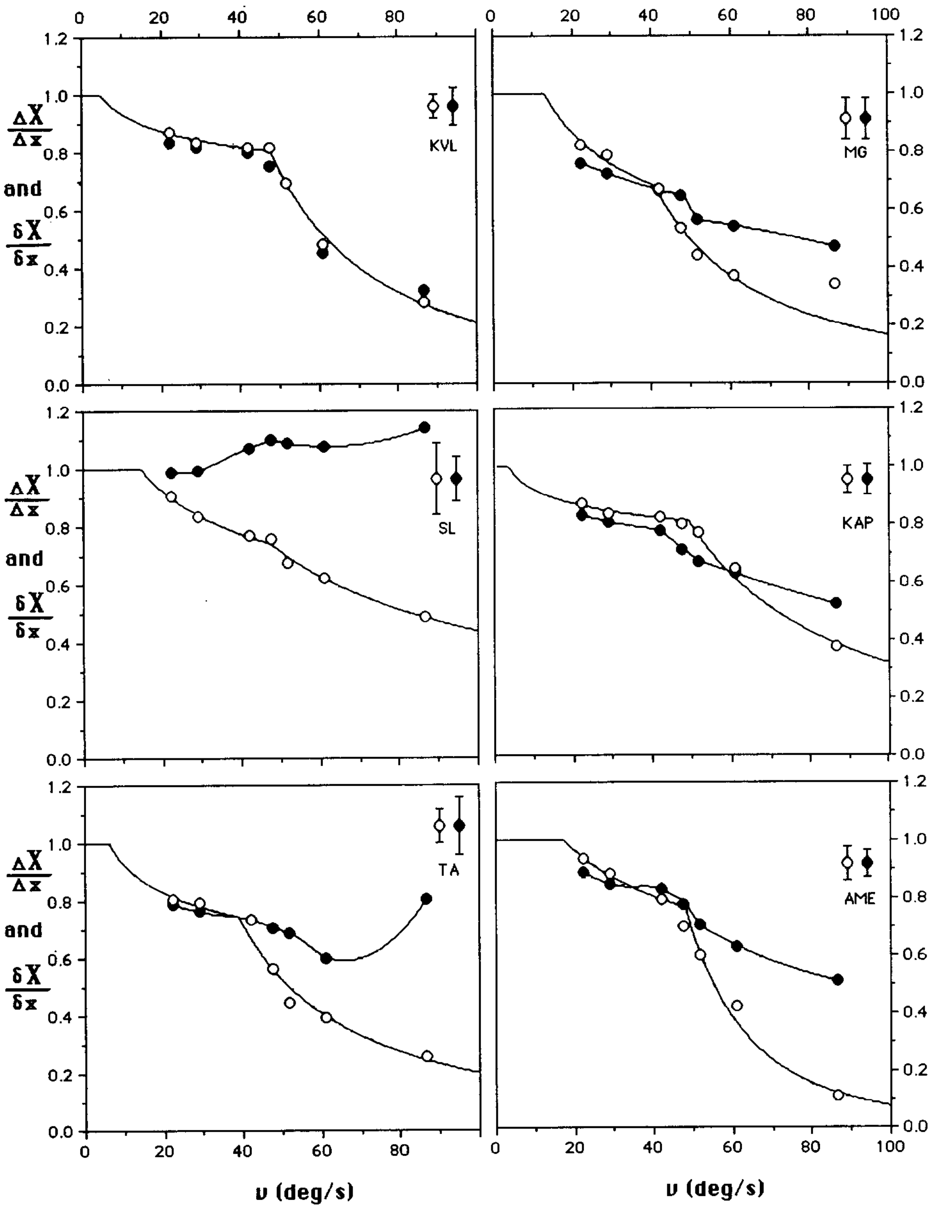

FIG. 5. ΔX in units of Δx for 2P stimuli (○) and δX in units of δx for 1P stimuli (●) as functions of angular velocity. Matching, free looking, $\Delta x_{2P} = \delta X_{1P} = 4.2°$. 20 estimates per symbol. Solid lines for 1P data are spline-interpolations. Solid lines for 2P data are obtained by fitting Eq. (2).

Six observers with normal or corrected-to-normal acuity participated in the experiment. All observers, except for KVL, were naive to the aims and design of the experiment.

3.3. *Discussion of the Results*

The results are presented in Fig. 5. The match-estimates of $\Delta X(v)_{2P}$ and $\delta X(v)_{1P}$ for every value of angular velocity v have been normalized by the physical value of $\Delta x_{2P} = \delta x_{1P}$. This is equivalent to normalizing by $\Delta X(0)_{2P}$, or $\delta X(0)_{1P}$, the perceived separation (length) at rest. The vertical bars attached to a symbol show ± 1 standard deviation averaged over all conditions represented by this symbol.

Inspection of the SCM curves for the 2P stimuli reveals a pattern the same as that in the experiments described in the preceding Visual Kinematics papers: for all observers the dependence of $\Delta X/\Delta x$ on v can be reasonably approximated by (2), with $v_0 \approx 10°/s$ (the intersection of the extrapolated upper branch of the curve with the no-contraction level) and $v_1 \approx 45°/s$ (the intersection of the two descending branches). The occasional deviations are clearly nonsystematic and disappear in the across-subject pooling shown in Fig. 6 (bottom curve, geometric averaging). Note the change of the vertical scale when visually comparing Figs. 5 and 6 with Fig. 3.

The 1P curves in Fig. 5 are presented in the format $\delta X/\delta x$ versus v, by analogy with the 2P curves. In striking contrast to the latter, the 1P curves exhibit substantial qualitative differences between different observers, from practically coinciding with the 2P curves (observer KVL) to monotonically increasing with v (observer

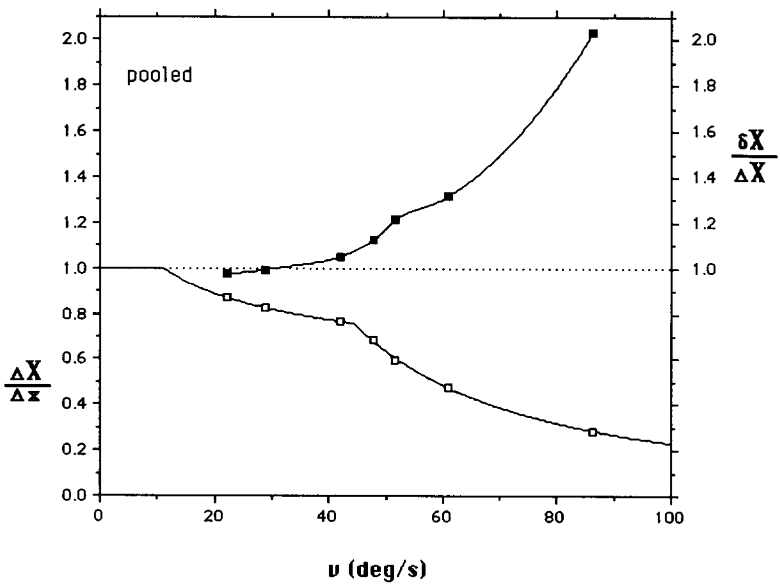

FIG. 6. Open symbols: pooled 2P stimulation data from Fig. 5. Solid line is obtained by fitting Eq. (2). Solid symbols: geometrically averaged $\delta X/\Delta x$ ratio from Fig. 5. Solid line is a spline-interpolation.

SL). No across-subject pooling algorithm in this situation would be representative of the individual data. One should conclude from these data that even if it might be possible to separate the distributional and geometric components in the δX-transformations (by comparing them to the ΔX-transformations), it would be grossly misleading to use a 1P paradigm alone for investigation of the visual geometry in motion.

One might suggest that a curve of "purely distributional deformations" for each observer could be computed by dividing the values of $\delta X/\delta x$ by the corresponding values of $\Delta X/\Delta x$ (or, equivalently, δX_{1P} by ΔX_{2P}, because $\delta x_{1P} = \Delta x_{2P}$). This procedure is based on the assumption that the geometric transformations take place after the moving contours have been deformed by the light integration–interaction mechanisms of vision. In other words, these mechanisms change the apparent length from how it looks at rest (δx) to how it would have looked in motion if the frontoparallel geometry did not change (δX^*); then this length is geometrically transformed into

$$\delta X(v) = \phi_{xX}(v)\, \delta X^*(v). \tag{11}$$

The division procedure that restores δX^* follows then from the fact that $\phi_{xX}(v)$ is estimated by $\Delta X/\Delta x$. Actual computations of $\delta X_{1P}/\Delta X_{2P}$ yield very dissimilar curves for different observers, with only one feature in common: no curve shows a considerable decreasing trend. The pooled values of $\delta X/\Delta X$ (geometric averaging) are shown in Fig. 6 (top curve).

The individual variability observed in the $\delta X_{1P}/\Delta X_{2P}$-curves indicates a flaw in the assumptions that led to (11): either the δX-estimates are influenced by high-level cognitive factors or the logical order of the distributional and geometric transformations is different from the one assumed. The involvement of high-level cognitive factors is more than just plausible: in fact it was phenomenologically obvious that the "length" of a moving 1P stimulus is a poorly defined concept, due to the brightness non-uniformity along the moving shape. Oversimplifying, the moving brightness profile is composed of a "body," a segment of relatively uniform brightness, and a low-contrast decaying "comet tail," with no clear endpoint. My informal observations suggest that the distance from the frontal edge to the "comet tail end," increases with v, whereas the distance to the "body end" decreases. This makes the δX-estimates critically dependent on the observer's criterion of the segment's trailing end (compare, e.g., the curves for observers MG and SL, Fig. 5). Moreover, different criterion positions can be adopted for different velocities. As velocity increases, the boundary between the "body" and the "tail" becomes less pronounced, thus inducing the observer to shift the criterion to the "tail end" (see Fig. 5, observer TA). It is remarkable that, judging by the observer KVL's data (Fig. 5), a highly experienced observer can maintain such a criterion position (probably at the "body end") that the transformations of the corresponding length are almost entirely geometric.

A general conclusion seems to be that in order to make δX-estimates theoretically tractable, one needs detailed measurements of the moving brightness profile, and a way to determine the observer's criterion of the endpoints. Note that neither is needed in the 2P paradigm. The possibility that the two "corresponding" points are in fact placed asymmetrically within the two visual shapes is formally equivalent to the assumption that the shapes themselves are different (mapping inhomogeneity) and is ruled out by the same arguments (see Visual Kinematics I).

SCM, as measured in the 2P paradigm, is a robust effect that can be replicated with any visual display that allows a perceptually continuous motion with velocities exceeding 10–15°/s and is free from crude geometric distortions. Other (non-geometric) characteristics of a display are irrelevant by the very logic of the 2P paradigm: they would transform both moving images equally, without changing physical distance between them. Here again the 2P paradigm is in a sharp contrast with the "length measurements" in the 1P stimuli. For example, a moving segment presented on a CRT screen, as in the experiments of Caelli *et al.* (1978), in addition to the visual distributional deformations will also be subjected to physical deformations due to the phosphorus decay time.

Finally, even if detailed measurements of a moving brightness profile were available, it still remains to be found by what algorithm one can "subtract" the geometric transformations (2P curves) from their combination with distributional deformations (1P curves). This returns us to the possibility mentioned earlier, that distributional deformations do not necessarily precede geometric transformations. There is nothing a priori implausible in assuming that distributional deformations occur on different successive levels, partly preceding and partly following the geometric transformations.

4. Detection and Discrimination of Spatial Intervals in Motion

In this section I will consider the consequences of the geometric transformations in motion for detection and discrimination of moving spatial intervals. The problem is important because of the considerable interest in dynamic acuity in the psychophysical literature. Another motivation for the discussion is that "class A" measurements are traditionally considered to be superior to magnitude estimation and sensory–physical matching (Marks, 1974). Finally, the discussion provides a good example of how an "obvious" derivation from the kinematic theory might turn out to be wrong.

Indeed, it seems almost obvious that due to the SCM effect detectability and discriminability of spatial intervals should change in motion too. Applied to the dynamic vernier acuity, a prediction seems to be that the vernier thresholds should increase in motion by a factor of $1/\phi_{xX}(v)$. The fact is, however, that although this possibility is not excluded, neither is it predicted by the kinematic theory.

A typical vernier stimulus can be viewed as a 2P stimulus with a very small value of Δx (usually $\Delta y \approx 0$, and δy is much larger than δx; cf. Fig. 4). A typical dynamic vernier acuity task is to determine the sign of the Δx-misalignment in a moving vernier target. Consider first the following simple model bringing dynamic vernier acuity conceptually close to the SCM effect. Let $\Delta X(v)$ be a random variable and let Δx be judged to be positive if and only if $\Delta X(v) > 0$. Let, for simplicity, $\Delta X(v)$ be distributed normally with parameters μ_v and σ_v. Then, for $\Delta x > 0$, the probability of correct response will be

$$P_c = \text{Prob}[\text{“}\Delta x > 0\text{”}] = \Phi(\mu_v/\sigma_v), \tag{12}$$

where Φ is a standard normal integral. Now $\mu_v = E[\Delta X(v)] = \phi_{xX}(v)\,\Delta x$ is a monotonically decreasing function, but the probability P_c depends also on σ_v. If and only if the latter is a velocity-independent constant, one comes to the prediction mentioned earlier: the vernier threshold (taken as the value of Δx corresponding to a fixed P_c) should increase in motion by factor $1/\phi_{xX}(v)$. If, however, σ_v itself is a decreasing function of v, then the vernier threshold predictions would depend on the comparative rates of decrease in μ_v and σ_v. Thus, if σ_v decreases faster than μ_v does, the predicted thresholds will actually decrease with increasing v. The most natural possibility is, of course, that σ_v is a fixed proportion of μ_v. Indeed, if each "instantaneous" value of the random variable $\Delta X(v)$ is shrunken by the factor of $1/\phi_{xX}(v)$ with respect to the values of the random variable $\Delta X(0)$, then both μ_v and σ_v will be shrunken by the same factor; $\mu_v/\sigma_v = \text{const}$. The thresholds will then not be affected by SCM at all.

Assumptions about the decision rule are as important for predictions as are the assumptions concerning the distribution of $\Delta X(v)$. To give only one example, consider the following decision rule: the observer says "$\Delta x > 0$" if $\Delta X(v) > \varepsilon$, says "$\Delta x < 0$" if $\Delta X(v) < -\varepsilon$, and guesses with probability p that "$\Delta x > 0$" if $|\Delta X(v)| < \varepsilon$, where ε is a velocity-independent constant. Unlike in the preceding model, the predicted thresholds in this case would increase with v, even if σ_v/μ_v is constant. By changing the assumptions concerning the decision rule and distribution of $\Delta X(v)$, one can generate different models covering all possible dependencies of dynamic vernier acuity on v.

In addition, one has to take into account the possibility that all these models might be wrong in assuming that vernier judgements are based exclusively on the value of ΔX. Although the problem is far from being settled, empirical evidence suggests that several different misalignment cues can be utilized for judging vernier offsets (Watt, Morgan, & Ward, 1983; Westheimer & McKee, 1977b). If so, then in addition to constructing a detection model, one has to find out in what logical order and how the different cues are modified by geometric and distributional transformations in motion. The available experimental data on dynamic vernier acuity impose at most weak limitations on the spectrum of theoretical possibilities. One of the few firmly established facts is that at low velocities (below 10–15°/s for perceptually continuous undirectional motion) dynamic vernier acuity is practically independent of velocity (Fahle & Poggio, 1981; Westheimer & Mckee, 1975, 1977a). Unfortunately for these theoretical considerations (but perhaps not coincidentally; see footnote 3), this is the very velocity range in which no or little SCM seems to occur.

With only minor modifications all of the above analyses can be repeated for spatial discrimination tasks, like same–different or greater–less judgements concerning moving spatial intervals. Again, the kinematic theory provides only one component of a psychophysical model, rather than a complete model.

5. Conclusion

It has been established in this work (Visual Kinematics I–III) that the fronto-parallel geometry of the visual space changes in visual motion as a function of motion velocity: perceived spatial intervals contract in the direction of motion, but not transversely, the contraction coefficient being independent of the contracted value. The possibility to speak of the visual geometry in motion unambiguously, and to measure its properties experimentally, isolated from all distributional deformations in motion, has been provided by the Mapping Homogeneity Principle, the central theoretical construct of the Visual Kinematics papers. The MHP has all the characteristics of a fundamental principle: it is general, simple, empirically falsifiable, and consistent with actual experimental data. At the same time, the principle itself and the kinematic theory based on it are only reasonable approximations. One cannot exclude the possibility, for example, that more precise and detailed measurements of the SCM effect would reveal small but systematic departures from the proportionality relation for near-threshold or even large spatial intervals. I do not think that this possibility diminishes the heuristic value of the MHP.

The proposed theory of visual kinematics places the SCM effect in a broader theoretical context: the interconnection between (relative) spatiotemporal localization of visual events and their state of motion. The theory shows also a direction of research whose results would be necessary and sufficient for a complete identification of visual kinematics, i.e., specification of the transformation coefficients and the psychophysical function for perceived velocity. On the basis of the experiments reported, and in agreement with the MHP, visual kinematics has been identified as approximately linear (with respect to spatiotemporal coordinates), but this seems to be the only property it shares with the two kinematic structures considered in mechanics (Galileian and Lorentzian). In particular, the existence of spatial transformations in motion in no way implies the existence of time and simultaneity transformations (as defined in Section 2.4): this is a strictly empirical question. The logical independence of the transformations of space ($A - VB_1 = \phi_{XX}$), time (B_2), and simultaneity (B_1), seen clearly when constructing a kinematic theory ab ovo, rather than uncritically copying it from mechanics, has broader implications than for visual psychophysics alone. Even in very good treatises on relativistic kinematics, for example, attempts are made to derive two of the three transformations from the remaining one (usually simultaneity), interpreting the latter as a "cause" of the former two (e.g., Arzelier, 1966). In fact kinematic properties do not have causes any more than Euclidean geometry does: one can say only that the "cause" of the very derivability of any two transformations from the third one is in the Galileian relativity principle (which is fundamental for mechanics, but cannot be substantiated in vision).

Comparison with physical kinematics helps one also to realize the importance of the non-Galileian structure of visual kinematics for the traditional question of whether visual motion is a sensory attribute or a "subconscious inference" from the difference in spatiotemporal positions of the same visual object. With very few exceptions (e.g., Kinchla & Allan, 1969; Mandriota, Mintz, & Notterdam, 1962), it has been maintained throughout the history of experimental psychophysics that motion perception is "direct" rather than "inferred" (e.g., Gibson, 1965; Sekuler, 1975). At least some arguments, however, advanced to support this view are quite weak and "indirect," mainly because the question is ambiguously formulated and, to a large extent, terminological (see Lappin, Bell, Harm, & Kottas, 1975, for a brief overview of some arguments). One such argument, that I find faulty, has been mentioned earlier: it claims that visual motion is not inferred because visual velocity does not equal the ratio of spatial and temporal intervals traversed. Even the commonly accepted dissociation between visual motion and visual displacement in a motion aftereffect (Wolgemuth, 1911) needs further investigation to be demonstrated convincingly and formulated rigorously.

The way I understand the meaning of the proposition "Visual motion is inferred" is that continuous visual motion can be equivalently presented as a succession of instantaneously vanishing visual events along the motion path (as in Zeno's famous aporia). In other words, a visual event appears and disappears in position (X, T), a similar visual event appears and disappears in position $(X+dX, T+dT)$, and so on. The question is now whether this conceptual removal of visual motion vectors is consistent with the spatiotemporal properties of visual scenes.

It might come as a surprise, but there is a clear and well-known physical analogue for the dichotomy of "sensory vs inferred," if understood in the way just described. It is the distinction between a real, energy-transmitting, motion and what is called "geometric motion." To give a simple example, the motion of a light source is real, whereas the motion of the optical image formed by the light source on the retina is "geometric": the light flux forming the image at moment t_1 is different from the light flux forming the image at moment t_2, however close the two moments are. The principle kinematic difference between real and "geometric" motions is that the transformation formulae (for spatiotemporal coordinates) associated with the latter are necessarily Galileian. One can imagine different physical worlds with different kinematic structures associated with real motions (including Galileian), but it is logically contradictory to imagine a non-Galileian kinematics associated with "geometric" motions (e.g., Ugarov, 1969). From this point of view, the non-Galileian structure of visual kinematics established in this work is a direct indication that visual motion is real, rather than "geometric," or, in psychophysical terms, is sensory rather than inferred.

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