Workshop:
Quantum Contextuality in Quantum Mechanics and Beyond

held in Prague on June 4 and 5, 2017

Abstracts

Samson Abramsky

University of Oxford, UK

Contextuality and quantum advantage

One way of understanding contextuality and its consequences is to see how it can be used to gain advantage in information-processing tasks. We show how quantum resources can be used to gain advantage in games witnessing the fundamental logical and computational concept of homomorphism between relational structures. This has wide applications, to constraint satisfaction, graph invariants, and beyond. This form of quantum advantage is equivalent to finding state-independent quantum witnesses for strong contextuality, in the hierarchy of notions of contextuality defined by Abramsky and Brandenburger.

Guido Bacciagaluppi

Utrecht University, The Netherlands

A proof of Specker's Principle

Specker's principle, the condition that pairwise orthogonal propositions should be jointly orthogonal, has been much investigated recently in the context of the programme of characterising quantum mechanics. It appears however to lack a transparent justification. In this paper, I present a variant of Specker's tale of the seer of Nineveh, using it as the basis for a surprisingly simple derivation of Specker's principle from the assumption of maximal entanglement, no-signalling, and one other assumption. I discuss connections with the work of Popescu and Rohrlich and prospects for further research.

Rui Soares Barbosa

University of Oxford, UK

The contextual fraction and contextuality as a resource

Recent work establishes the rôle of contextuality as a resource providing quantum advantages in information-processing and computational tasks. I'll discuss the contextual fraction as a quantitative measure of contextuality, defined in any measurement scenario. The contextual fraction bears a precise relationship to maximum violations of (generalised) Bell inequalities, it is monotone with respect to what can be thought as the "free" operations in a resource theory of contextuality, and is closely related to quantifiable advantages in certain informatics tasks, including cooperative games and a form of measurement-based quantum computation.

Howard N Barnum

University of Copenhagen, Denmark, and University of New Mexico, USA

On noncontextual "hidden variable" models for probabilistic theories

Quantum mechanics admits no "noncontextual hidden variable model", as greatly clarified by the work of Robert Spekkens, who has emphasized the requirement that both preparations and measurements be modelled noncontextually. Spekkens, and also Ferrie and Emerson, recently emphasized that such a model would be equivalent to a "positive quasiprobability representation": linear injections of the set of unnormalized quantum states, and of the set of quantum effects, into classical state and effect spaces, such that the probabilities of quantum effects evaluated on quantum states agree with those yielded by the classical evaluation pairing between their images. In this talk, I will explain that the nonexistence of such hidden variable models actually extends to all "general probabilistic theories" (GPTs) in which the set of effects is the full set of functionals whose values on normalized states are bounded between 0 and 1. Proving the result at this level of generality makes the conceptual essence of the quantum result more transparent. I will provide a self-contained proof, obtained in collaboration with Ludovico Lami. The result dates back at least to a 1995 paper by Bugajski, extending work by Busch and co-authors on the quantum version. A 1982 paper by Holevo formulated both contextual and noncontextual hidden variable models rigorously at the GPT level of generality, and argued, less formally than Bugajski, for the noncontextuality of quantum theory based on Kochen-Specker and on work of Srinivas.

Pawel Blasiak

Institute of Nuclear Physics Polish Academy of Sciences, Poland

Probability in the Plato's cave: Local model of a qudit

It is often argued that quantum interference, collapse of the wave function or contextuality are strictly quantum mechanical effects. While being certainly beyond the scope of our immediate intuitions, it is not at all clear to what extent these features discriminate against classical theories. Hence the interest in investigation of classical-like models reconstructing certain aspects or parts of the quantum formalism and its implications. Here, I will be concerned with a restricted setting of a single particle propagating in multipath interferometric circuits, that is physical realisation of a qudit. Notably, this framework is a host to many paradoxes and surprising effects raised in quantum foundations debates. Apart from quantum interference, they include e.g.: interaction-free measurements, quantum Zeno effect, Wheeler's delayed-choice experiment, violation of Leggett-Garg inequalities, pre- and post-selection paradoxes, etc. It is also a prototypical example of indivisible physical system for discussion and experimental tests of quantum contextuality, e.g. Klyachko inequalities. In this talk, I will describe ontological model which fully reconstructs behaviour of a single quantum particle in the interferometric circuits. The model is local in a sense that ontic variables propagate locally along the paths and the interferometric gates affect only the paths they are attached to. I will demonstrate that predictions of the model are indistinguishable from the quantum case of a qudit and allegedly 'non-local' effects arise only on the epistemic level of description by the agents whose knowledge is incomplete due to the restricted means of investigating the system (i.e. agents performing experiments on conceivable circuits built from the interferometric gates defined in the model). This illustrates importance of the distinction between the epistemic aspect of the description and the underlying 'true' ontological account (cf. Plato's allegory of the cave). Moreover, it demonstrates that many weird quantum mechanical effects, like quantum interference, collapse of the wave function or contextuality, have classical analogues which can be explained by incomplete knowledge, and hence are not reserved exclusively for the quantum realm. In particular, the model provides explicit counterexample showing that single-particle framework is not enough to establish non-locality, since in this case 'spooky action-at-adistance' could be understood as an epistemic effect, with the underlying ontology being local from the construction. The talk is based on preprint: P. Blasiak "Is single-particle interference spooky?" `https://arxiv.org/abs/1701.02552`

Adan Cabello

University of Sevilla, Spain

Why (Bell-Kochen-Specker) quantum contextuality?

Intentionally, I define contextuality as the violation of inequalities involving correlations between the outcomes of compatible sharp measurements, as defined in the framework of general probabilistic theories, and satisfied by ontological models where the assumption of outcome noncontextuality for sharp measurements is made, as it is the case for the hidden variable theories considered by Bell, Kochen and Specker. These noncontextuality inequalities (specifically, some of them called tight) provide necessary and sufficient conditions for the existence of joint probability distribution. The purpose of my talk is twofold. Firstly, I detail the reasons why I think this particular definition of contextuality teaches us much more about what is quantum theory and what the effectiveness of quantum theory tells us about the world than any other notion of non-classicality (including some, more general, proposed definitions of contextuality). Secondly, I introduce an alternative to the recently proposed ways of dealing with the inevitable finite precision, imperfect compatibility, and unsharpness of the measurements in actual experiments testing contextuality on quantum systems. I show that any experiment based on the assumptions that the measurements can have infinite precision, perfect compatibility and sharpness can be converted into a bipartite Bell inequality experiment in which none of these assumptions is needed. The interest of the method resides in that it does not only apply to state-independent experiments based on Kochen-Specker sets, but also to any state-dependent or state-independent violation of a noncontextuality inequality, including experiments of KCBS and Yu-Oh types. The method establishes a one-to-one correspondence between the initial state and the measurements tested in the contextuality experiment and the measurements tested in the Bell inequality experiment.

Giovanni Carù

University of Oxford, UK

All-vs-Nothing proofs of contextuality and the AvN triple theorem

(joint work with Samson Abramsky, Rui Soares Barbosa and Simon Perdrix)

All-vs-Nothing (AvN) arguments represent a powerful method for the detection of contextuality in quantum theory. Recent work by Abramsky et al.~has formalised and generalised Mermin's original formulation to a large class of contextuality proofs. We take advantage of this framework in conjunction with the graph state formalism to derive important implications on the nature of the quantum states admitting AvN arguments in Z_2. In particular, we show that every AvN argument for an n-qubit stabiliser state can always be reduced to an AvN proof for three qubits which are local Clifford-equivalent to the tripartite GHZ state. This result is achieved through a combinatorial characterisation of AvN arguments, which was previously known as the AvN triple conjecture. This enables the development of a computational method to identify all the AvN arguments in Z_2 on general n-qubit stabiliser states. We also present new insights into the stabiliser formalism and its connections with logic.

Gonzalo Carvacho
University of Rome "La Sapienza"

Integrated-Optics Circuits for Validation of Non-Classicality

Andrea Crespi[1,2], *Marco Bentivegna*[3], *Ioannis Pitsios*[1,2], *Davide Rusca*[1], *Davide Ponderini*[3], *Gonzalo Carvacho*[3], *Vincenzo D'Ambrosio*[3,4], *Adán Cabello*[5], *Fabio Sciarrino*[3] and *Roberto Osellame*[1,2]
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The intriguing behaviour of Quantum contextuality has been widely tested during the past years in different platforms including also single particles [1,2]. It was recently suggested that the non-classical power of quantum computing originates from contextuality [3] opening in this way a new scientific challenge with the aim of efficiently implementing this paradoxial aspect of nature.
Here we exploit a reconfigurable photonic circuit fabricated by femtosecond laser waveguide writing in order to experimentally test the quantum contextuality of a single photon on-a-chip.
The scheme of our experiment is depicted in Fig.1. Single photons generated by spontaneous parametric down conversion (SPDC) process are injected into the integrated photonic circuit through a single mode fiber, once the photon is coupled it will be prepared in a decolized state across four optical modes (first chip). The two-qubit state is characterized by a phase term (ϕ) which is actively controlled by a thermo-optic phase shifter [4]. The projective measurements (XX, XZ, ZX, ZZ) for the evaluation of a CHSH-like inequality will be performed by a second chip which is properly aligned with the first one depending on the desired projection. The inequality to be evaluated is given by:

$$S=\langle XX\rangle+\langle XZ\rangle+\langle ZX\rangle-\langle ZZ\rangle < 2+\epsilon$$

The parameter ϵ takes into account non-idealities of the experimental apparatus. Our work allows us to observe the first experimental evaluation of quantum contextuality within an integrated photonic circuit, paving in this way the evaluation of more complex test on-chip which undoubtedly will contribute to technological applications.

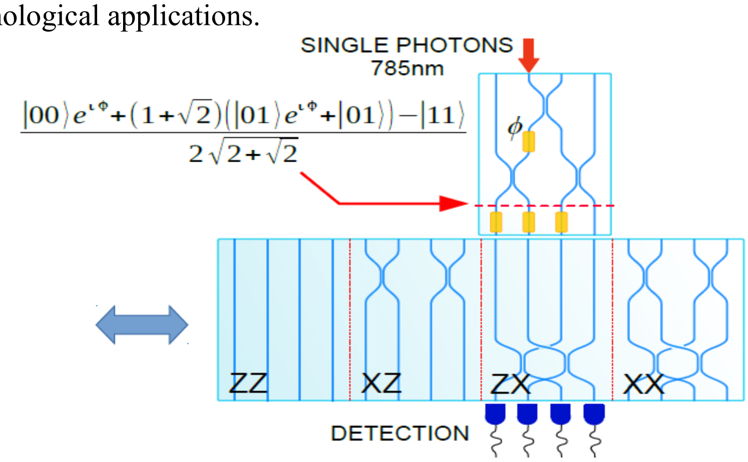

Fig.1- **Experimental apparatus:** Photons at 785 nm are injected into the preparation state chip to generate a delocalized state of a single photon (|00>, |01>, |10>, |11> in binary encoding). In order to perform the measurement another chip is employed with single photon detectors at the output of each mode. The second chip is able to be translated for the evaluation of different projections. Thermo-optic phase shifters (yellow rectangles) allow us to set the phase (ϕ) in the state and compensate for spurious phase terms given by misalignments between the two chips.

References:

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[3] Howard M. Et al. "Contextuality supplies the magic for quantum computation" Nature **510**, 351-355 (2014).
[4] Flamini, F. Et al. "Thermally reconfigurable quantum photonic circuits at telecom wavelength by femtosecond laser micromachining" Light Sci. Appl. **4**, e354 (2015)

Omid Charrakh

Ludwig Maximilian University, Germany

On the reality of the quantum wavefunction

Within the Ontological Models Framework (OMF), Pusey, Barrett, and Rudolph (PBR) have given an argument by which they claimed that the epistemic view on the wavefunction should be ruled out. My study highlights an incorrect conclusion in PBR's arguments, which was made due to inadequacies in the definitions of OMF. To be precise, OMF models the ontology of the preparation procedure, but it does not model the ontology of the measurement device. Such an asymmetric treatment becomes problematic, in scenarios in which measurement devices have a quantum nature. Modifying the OMF's definition such that the ontology of the measurement device becomes included, we will see how PBR's result disappears.

Nadish de Silva

University College London, UK

Contextuality and quantum gate injection

We present some recent results on the role of contextuality as a resource in magic state distillation for achieving universality in fault-tolerant quantum computation. These results build on the work of Howard et al and employ insights from the Abramsky-Brandenburger formalism for contextuality.

Ehtibar Dzhafarov

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Context-dependence and contextuality

I view contextuality analysis as a way of classifying and characterizing systems of random variables within classical (Kolmogorovian) probability theory. *Context-dependence* is a universal mathematical property of random variables, holding essentially by definition: a random variable is defined by its *content* (that which it measures) and its *context* (the conditions under which this content is measured). Hence the *identities* of two random variables measuring one and the same content in different contexts are different, even if their *distributions* coincide, and even if the contexts are defined by properties that cannot physically influence measurements of the given content (e.g., because they are space-like separated from this content). Any difference in the distributions of content-sharing random variables (again, essentially by definition) is the result of causal action ("signaling"). Contextuality proper is of non-causal, "correlational" nature. It is defined in terms of (in)compatibility of two sets of joint distributions: (1) the joint distributions of each set of *context-sharing* random variables, and (2) the *couplings* of (joint distributions imposed on) each set of *content-sharing* random variables. These couplings are chosen so that the joint distribution of any two content-sharing random variables satisfies a certain relation C, such as "are equal to each other with probability 1" (*identity coupling*, defining contextuality in the traditional sense of the term) or "are equal to each other with the maximal possible probability" (*maximal coupling*, allowing for noncontextual signaling system). In the uncomplicated version of the contextuality theory, such a C-coupling always exists and is unique (otherwise one has to consider the systems with partial contextuality or with undefined contextuality). For maximal couplings, to ensure this existence-and-uniqueness property, one has to present each system of random variables in a *canonical form*, one in which every random variables is replaced by a set of jointly distributed dichotomic random variables.

References:

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Samuel C. Fletcher

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On noncontextual, non-Kolmogorovian hidden variable theories

Bell's theorem and its generalizations restrict which kinds of hidden variable theories for quantum mechanics are consistent with empirical observation and other theoretical commitments. In particular, a version due to Pitowksy shows that these theories are just the ones whose hidden variable can be modeled using a classical (Kolmogorovian) probability space. Various authors (e.g., Suppes, Gudder, Hartle, Sorkin, and their collaborators) have suggested that slightly different types of hidden variable theory, relaxing one or another of the axioms of probability, does not run afoul of these theorems and thus may be a viable approach to the completion of quantum theory. Often using concrete examples, they have demonstrate the consistency of some such non-Kolmogorovian hidden variable theories for the EPR and GHZ experiments.

However, they have not typically considered the consistency of such a theory with another no-go result, the Kochen-Specker (KS) theorem, which rules out any hidden variable theory that assigns non-contextual, definite values to all physical quantities and preserves standard functional relationships between them. (A theory is non-contextual when it assigns values to all physical quantities independently of measurement context.) But on their face the aforementioned examples of non-Kolmogorovian hidden variable theories are non-contextual, definite value theories. We show that the KS theorem in fact implies a necessary condition for the possibility of a non-contextual hidden variable theory, namely that the theory has a finite null cover. In other words, there must be a cover of the hidden variable space by a finite number of measure-zero sets. Although the examples considered by the aforementioned authors do not contain enough possible measurements to be KS witnesses, the measurements they do concern are subsets of KS witnesses. Thus, one can avoid contradiction with the KS theorem by finding a finite null cover for the space.

One consequence of this result, however, is that any such theory must have a highly non-classical logic for its hidden states. In particular, the disjunction of a finite number of "self-contradictory" propositions can be a "tautologous" proposition. An agent with knowledge of these hidden states would not be able to use them as a guide to rational action, since she would accept a Dutch book, i.e., a series of bets for which she is guaranteed to lose money. These features cast doubt on the interpretation of non-Kolmogorovian probability spaces that avoid contradiction with KS as capturing any notion of coherent probability at all, despite the spaces' formal similarities with standard probability spaces. Consequently, it is unclear whether they have any advantages over contextual theories in their ability to explain puzzling features of quantum theory.

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Yuri Gurevich

Microsoft Research, USA

On hidden variables: Value and expectation no-go theorems

No-go theorems assert that hidden-variable theories, subject to appropriate hypotheses, cannot reproduce the predictions of quantum theory. We examine two species of such theo- rems, value no-go theorems and expectation no-go theorems. The former assert that hidden-variables cannot match the predictions of quantum theory about the possible values resulting from mea- surements; the latter assert that hidden-variables cannot match the predictions of quantum theory about the expectation values of measurements. We sharpen the known results of both species, which allows us to clarify the similarities and differences between the two species.

This is joint work with Andreas Blass of the University of Michigan.

Federico Holik

Institute of Physics La Plata, Argentine

Geometric probability theory in contextual probabilistic theories

We present recent advances in the study of the problem of generalizing geometric probability theory in order to apply it to the study of contextual theories. This means the development of a non-commutative version of geometric probability theory, in which states of probabilistic theories are considered as invariant measures over (posibly) non-Boolean event structures. In particular, we use this generalization to provide an improvement of the Jaynes' MaxEnt method. This allows us to include group theory within MaxEnt. This is a continuation of a previous work [1], which incorporates tools of algebraic geometry in order to expand the domain of applicability of our framework.

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Zhih-Ahn Jia

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Exclusivity principle determines the correlation monogamy

In this talk, I will explain the monogamy (or trade-off) relations of contextuality and many other kinds of quantum correlations. Adopting the graph theoretical approach to correlation experiments, I will explain that many kinds of monogamy relations can be recognised as a consequence of exclusivity principle(EP). I will provide an operational criterion for monogamy: if the fractional packing number of the graph corresponding to the union of event sets of several physical experiments does not exceed the sum of independence numbers of each individual experiment graph, then these experiments are monogamous. As an application of this observation, several examples are provided, including the monogamy for experiments of Bell type, Kochen-Specker type, and for the first time we give some monogamy relations of genuine nonlocality.

References:

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Polina Khrennikova

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Modelling agents' asset price expectations: a quantum theoretical paradigm

Nowadays, human decision making in economic and finance decisions is facing increasing complexity due to a) non-classical assessment of probabilities associated with states of the world and respective outcomes (cf. seminal works by Kahneman and Tversky (1979) and Prelec (1998)), b) non Bayesian updating of information and finally, the most difficult, albeit existent situation – the inability to assess the state space of all the outcomes and hence their probabilities . Situations, where no well-founded subjective probabilities can be devised, known as ambiguity could be well captured through probabilistic models of quantum mechanics (also known as ' Quantum-Like' models) by usage of complex- probability amplitudes that correspond to ambiguity in respect to the exact probabilistic distribution on the values of a random variable. In the setting of finance market, agents can exhibit information processing contextuality in respect the future distribution of assets' prices (returns) that can be manifest in the impossibility to form a classical probabilistic distribution (normal distribution of returns is the foundation of the key framework on portfolio selection- the 'Markowitz Modern Portfolio Theory') for the returns of financial assets. As highlighted in Shiller (2003), agents' expectations on future asset price are often based on non-classical probabilistic expectations and yield e.g., excess volatility on the stock market. We propose a model based on quantum probabilistic framework, where the price dynamics of some financial assets is created by the informational bath that consists of the agents' expectation about future asset prices (Khrennikova, 2016). One of the basic behavioural factors leading to quantum-like dynamics of forecasted prices is the 'irrationality' of expectations of the agents on the financial market. It leads to a deeper type of uncertainty than given by classical probability theory, e.g., in the framework of the classical financial mathematics, based on theory of stochastic processes. The quantum dimension of the uncertainty in price dynamics is expressed in the form of the price-states superposition and entanglement of different financial assets. The long-term equilibrium state in asset price dynamics can be well captured by a quantum master equation that has been successfully applied to decision-making problems under uncertainty, e.g. Asano et al (2010), Khrennikova et al (2014).

Kihwan Kim
Tsinghua University, China

Exponential gain of randomness certified by quantum contextuality

Mark Um, Pengfei Wang, Junhua Zhang, Ye Wang, Qi Zhao, Xiongfeng Ma and Kihwan Kim

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We demonstrate the protocol of exponential expansion of randomness certified by quantum contextuality in a trapped ion system. The intrinsic unpredictability of measurements in quantum mechanics can be used to produce genuine randomness. Recently, randomness expansion protocols based on inequality of Bell-text [1] and Kochen-Specker (KS) theorem [2], have been demonstrated. These schemes have been theoretically innovated to exponentially expand the randomness and amplify the randomness from weak initial random seed [3]. Here, we report the experimental evidence that the randomness can be exponentially expanded with the certification by KS theorem, in particular, the Klyachko-Can-Binicioglu-Shumovsky (KCBS) inequality. In the experiment, we use three states of a 138Ba+ ion between a ground state and two quadrupole states. In the 138Ba+ ion system, we do not have detection loophole and we apply a methods to rule out certain hidden variable models that obey a kind of extended noncontextuality [4]. And in order to make it sure that no-communication between two parties, we develop a time-separated sequence.

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[3] Carl Miller and Yaoyun Shi, arxiv:1411.6608v3 (2015).
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Minoru Koga

Nagoya University, Japan

On the non-existence of two-valued lattice homomorphisms of quantum logic

We show that there exists no two-valued lattice homomorphism from the lattice of all closed subspaces of a Hilbert space whose dimension is greater than or equal to three by using the notion of prime filters.

Adrian Kolodziejski

University of Gdansk, Poland

KCBS and CH inequalities revisited

We introduce a kind of Wigner function for KCBS and CH inequalities. The value of this function is negative for states violating these inequalities and positive otherwise. The construction is based on hermitian operators whose expectation values give the joint probability distribution for all possible measurement outcomes. Then we show that the joint distribution gets negative whenever KCBS or CH inequality is violated.

Ravi Kunjwal

Perimeter Institute for Theoretical Physics, Canada

Graph-theoretic approach to obtaining irreducible noncontextuality inequalities from Kochen-Specker uncolourability

Recent work (Kunjwal and Spekkens, Phys. Rev. Lett. 115, 110403 (2015)) has shown how operational noncontextuality inequalities robust to noise can be obtained from Kochen-Specker uncolourable (or KS-uncolourable) hypergraphs without assuming that measurement outcomes are fixed deterministically by the ontic state of the system in an underlying ontological model. This result was obtained by an explicit numerical enumeration of all the extremal points of the polytope of (measurement) noncontextual assignments of probabilities to the KS-uncolourable hypergraph. I'll focus on an analytical approach to deriving such noncontextuality inequalities that relies on constraints arising directly from the structure of the hypergraph without necessarily enumerating all the extremal probabilistic models on it. This cleanly identifies operational quantities that one can expect to be constrained (and why) by the assumption of noncontextuality instead of having to guess these quantities or obtaining them from brute-force numerical methods without any guiding principles to identify them. This approach relies on giving such noncontextuality inequalities and their upper bounds an interpretation in terms of the hypergraph structure. I'll demonstrate noncontextuality inequalities robust to noise for a whole family of KS-uncolourable hypergraphs, including many known KS constructions, using this method. To mark 50 years of the KS theorem, I'll also show how this analytical approach sheds light on the robust noncontextuality inequalities that one can obtain from the original 1967 construction of Kochen and Specker.

Marian Kupczynski

Université du Québec en Outaouais, Canada

Contextuality versus quantum nonlocality

Already in classical physics some properties of macroscopic bodies are attributive and some are contextual. For example an electric charge is attributive but a color is contextual because it depends on the experimental context used for its determination. Quantum mechanics provides probabilistic contextual models for various experiments performed in well-defined often incompatible experimental contexts. Many attempts have been made in order to reproduce quantum predictions using more detailed and more intuitive description of quantum experiments. Kochen-Specker theorem gave a strong argument against models in which a dynamical variable measured on a physical system has a definite (but unknown) value before being measured. It confirmed Bohr's ideas of wholeness and complementarity. Bohr insisted that outcomes of measurements are effect of the interaction of a physical system with a whole macroscopic experimental set-up. This conclusion was reinforced by Bell who proved that the outcomes of spin polarisation correlation experiments may not be explained using non-contextual hidden variable models. If contextual hidden variables are introduced correctly Bell-type inequalities may not be proven and intuitive description of spin polarization phenomena is possible. Therefore so called quantum nonlocality is a misleading terminology. Incompatible experimental contexts exist also in other domains of science what allows using quantum-like probabilistic models or context dependent classical probabilistic models. The importance of contextuality in various domains of science may not be underestimated.

Pawel Kurzynski

Adam Mickiewicz University, Poland

Contextuality and indistinguishability

There exist quantum phenomena that cannot be explained by noncontextual hidden-variable theories, yet the majority of them requires measurements that are performed on a single quantum system at a time. This fact constrains the phenomenon of contextuality to the microscopic domain. It is therefore natural to ask if quantum contextuality can be observed in measurements on collections of particles. Since particles in nature are identical, one can expect that such contextuality would be linked to bosonic and fermionic properties. Analysis of quantum contextuality in such scenarios would broaden our understanding of nonclassical effects in composite systems and perhaps would give us a hint on how to observe quantum phenomena in the macroscopic world. I propose a generalization of quantum contextuality to the case of many identical particles. I show that a type of contextuality exhibited by a collection of particles (state dependent, state independent, or noncontextual) depends on their type and their number. I also discuss further properties of this generalization and identify major open questions.

Carlos Lopez

University of Alcalá (UAH), Spain

A separable representation of the singlet beyond standard QM

The contextual character of quantum distributions of probability appears already in the paradigmatic two slit experiment. Formulations of QM in extended phase spaces are possible and consistent. The standard formalism is recovered through projection, calculation of marginal amplitudes (as $\Psi_L(x) + \Psi_R(x)$ in the two slit experiment), which gives account of the contextuality of the theory. An extended spin phase space is proposed where the singlet spin state is represented in separable form. A new isotropic spin state appears in the formalism and the correlations between particles of the pair happens at a subquantum level.

Archan S. Majumdar

S. N. Bose National Centre for Basic Sciences, India

Sharing of nonlocality of a single member of an entangled pair is not possible by more than two unbiased observers on the other wing

We address the recently posed question as to whether the nonlocality of a single member of an entangled pair of spin 1/2 particles can be shared among multiple observers on the other wing who act sequentially and independently of each other. We first show that the optimality condition for the trade-off between information gain and disturbance in the context of weak or non-ideal measurements emerges naturally when one employs a one-parameter class of positive operator valued measures (POVMs). Using this formalism we then prove analytically that it is impossible to obtain violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality by more than two Bobs in one of the two wings using unbiased input settings with an Alice in the other wing. (Mathematics 4, 48 (2016) [https://arxiv.org/abs/1604.08718]).

Shane Mansfield

University of Edinburgh, UK

A unified approach to contextuality and violations of macrorealism

I will present a unified approach to contextuality and violations of macrorealism in the sense of Leggett and Garg. This extends the sheaf theoretic framework for nonlocality and contextuality (Abramsky and Brandenburger, 2011) by introducing a notion of temporality which takes the form of a partial order on the set of measurements. Once care is taken to ensure that sheaf theoretic restrictions respect the partial order, this leads naturally to a general notion of causal empirical models as families of compatible probability distributions taking values in a certain presheaf. Nonlocality, contextuality and violation of macroscopic realism are characterised in a unified way by the absence of global sections. I will illustrate with the example of the double slit experiment which exhibits a logical violation of macrorealism.

Marcin Markiewicz

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New 3D geometrical visualization of a contextuality of a qutrit

Recently in the paper [1] there has been presented a surprisingly simple three-dimensional Bloch sphere representation of a qutrit, i.e., a single three-level quantum system. The construction starts with a symmetric state of a two-qubit system and relates it to the spin-1 representation. Using this representation one associates each qutrit state with a three-dimensional vector v and a metric tensor G which satisfy v*G*v <= 1. This resembles the well known condition for qubit Bloch vectors in which case G =1 . In our case the vector v corresponds to spin-1 polarisation, whereas the tensor G is a function of polarisation uncertainties. Alternatively, v is a local Bloch vector of a symmetric two-qubit state and G is a function of the corresponding correlation tensor. Qutrit is the simplest quantum system which reveals contextual behaviour [2]. Typically this kind of behaviour is visualized either using 3D vector representation of quantum projectors [2] or exclusivity graphs for all the experimental events in the contextuality test [3]. Both these approaches do not fully investigate the geometric structure of qutrit states. In this presentation we show how the state-dependent qutrit contextuality can be visualized exploring the entire geometric structure of the state space.

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Intersubjectivity of Quantum Measurement

Masanao Ozawa

The Kochen-Specker theorem denies the preexisting value of an observables. However, if we consider the process of measurement, we confront a difficult problem. Suppose that two observers simultaneously measure the same observable. Then, we can ask whether we can predict that they always obtain the same outcome, or we can predict only that their probability distributions are the same but the outcomes are uncorrelated. Here, we shall show that quantum mechanics predicts that only the first case occurs, provided that the two measurements reproduce the probability distribution correctly, or that they have the correct probability operator-valued measure (POVM). This suggests that the common outcome of the two measurements preexisted before the measuring processes start, in contrast to a common interpretation of the Kochen-Specker theorem.

Let A be an observable of a system $\mathbf{S}$ described by a Hilbert space $\mathcal{H}$ to be measured. Let M_1 and M_2 be two commuting observables representing the meters of the two remote observers in the environment $\mathbf{E}$ described by a Hilbert space $\mathcal{K}$. Then, the time evolution of the total system with the total Hamiltonian H on $\mathcal{H}\otimes\mathcal{K}$ determines the Heisenberg operators $A(0)$, $M_1(t)$, $M_2(t)$ with $0<t$, where

$$
\begin{aligned}
A(0) &= A\otimes I, && (1)\\
M_1(t) &= U(t)^\dagger (I\otimes M_1)U(t), && (2)\\
M_1(t) &= U(t)^\dagger (I\otimes M_2)U(t), && (3)\\
U(t) &= \exp(-iKtH/\hbar). && (4)
\end{aligned}
$$

Let $|\psi\rangle$ and $|\xi\rangle$ be the initial state of the system $\mathbf{S}$ and the environment $\mathbf{E}$. The POVMs of the two observers are defined by

$$
\begin{aligned}
\Pi_1(x) &= \langle\xi|P^{M_1(t)}(x)|\xi\rangle, && (5)\\
\Pi_2(y) &= \langle\xi|P^{M_2(t)}(y)|\xi\rangle, && (6)
\end{aligned}
$$

where for a general observable L, we denote by $P^L(\lambda)$ the spectral projection of L for an eigenvalue λ and we set $P^L(\lambda)=0$ otherwise; here, we assume that both $\mathcal{H}$ and $\mathcal{K}$ are finite dimensional for simplicity. The probability of obtaining the outcome x in the system state $|\psi\rangle$ is given by

$$
\Pr\{M_1(t)=x\|\,|\psi\rangle\} = \langle\psi|\Pi_1(x)|\psi\rangle \qquad (7)
$$

for observer 1, and

$$
\Pr\{M_2(t)=x\|\,|\psi\rangle\} = \langle\psi|\Pi_2(x)|\psi\rangle \qquad (8)
$$

for observer 2. Besides, the probability distribution of the measured observable A is given by

$$\Pr\{A(0) = x \| \, |\psi\rangle\} \quad = \quad \langle\psi|P^A(x)|\psi\rangle. \tag{9}$$

Then, our sole assumption is that the two measurements reproduce the probability distribution of A correctly; namely, we require

$$\Pr\{M_1(t) = x \| \, |\psi\rangle\} = \Pr\{M_2(t_2) = x \| \, |\psi\rangle\} = \Pr\{A(0) = x \| \, |\psi\rangle\}, \tag{10}$$

or equivalently $\Pi_1(x) = \Pi_2(x) = P^A(x)$ for all $x \in \mathbb{R}$.

Since M_1 and M_2 commute, their joint probability distribution in the system state $|\psi\rangle$ is given by

$$\Pr\{M_1 = x, M_2 = y \| \, |\psi\rangle\} = \langle\psi, \xi|P^{M_1(t)}(x)P^{M_2(t)}(y)|\psi, \xi\rangle \tag{11}$$

for all $x, y \in \mathbb{R}$.

Then, the following theorem holds.

Theorem. *Suppose that $\Pi_1(x) = \Pi_2(x) = E^A(x)$ for all x. Then, the outcomes of the measurements of A from M_1 and M_2 are always identical, namely, we have*

$$\Pr\{M_1(t) = x, M_2(t) = y \||\, |\psi\rangle\} = 0 \tag{12}$$

if $x \neq y$.

Thus, we conclude that if the two observer make two measurements of A both of which reproduce the probability distribution of A correctly, then their outcomes always coincides. A reasonable interpretation about the common value from the two measurements appears to be the value of A that has preexisted before the measurement.

The proof is based on the theory of quantum perfect correlations developed in [M. Ozawa, Quantum perfect correlations, Ann. Phys. (N.Y.) 321, 744-769 (2006)].

Michel Planat,
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FROM INFORMATIONALLY COMPLETE POVMS TO THE KOCHEN-SPECKER THEOREM

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Abstract of talk

It is possible to conciliate informationally complete (IC) measurements on an unknown density matrix and Kochen-Specker (KS) concepts (which forbid hidden variable theories of a non-contextual type). This was shown in [1] for qutrits and it is continued here for two-qubits (2QB), three-qubits (3QB) and two qutrits (2QT). Non symmetric IC-POVMs have been found in dimensions 3 to 12 starting from permutation groups, the derivation of appropriate non-stabilizer states (magic states) and the action of the Pauli group on them [2, 3]. For 2QB, 3QB and 2QT systems, Kochen-Specker theorem follows.

For $2QB$, the magic state is of type $(0, 1, \omega, \omega - 1)$, $\omega = \exp(\frac{i\pi}{3})$, and the IC-POVM manifests dichotomic trace products of projectors $\Pi_i = |\psi_i\rangle \langle\psi_i|$ as $\mathrm{tr}(\Pi_i\Pi_j)_{i\neq j} = |\langle\psi_i| \psi_j\rangle|^2_{i\neq j} \in \{\frac{1}{3}, \frac{1}{3^2}\}$. The triple products of projectors whose trace is $\pm\frac{1}{27}$, and simultaneously equal plus or minus the identity matrix, are organized as a (3×3)-grid. Taking the vertices of the grid as the 2QB Pauli group operators acting on the magic state instead of the corresponding projectors one recovers the standard form of Mermin square -that is used as an operator proof of the KS theorem.

For $3QB$, the Hoggar magic state $(-1\pm i, 1, 1, 1, 1, 1, 1, 1)$ leads to a SIC. Within the 4032 triples whose trace of triple products equal $-\frac{1}{27}$ [4], those whose product of projectors equal plus or minus the identity are organized into a geometric configuration $[63_3]$ whose automorphism group $G_2(2) = U_3(3) \rtimes \mathbb{Z}_2$ is of order 12096 and corresponds to the generalized hexagon $GH(2, 2)$ (or its dual). These configurations are related to the 12096 Mermin pentagrams that build a proof of the three-qubit Kochen-Specker theorem [5]. From the structure of hyperplanes of our $[63_3]$ configuration, one learns that we are concerned with the dual of G_2.

Finally for 2QT, a magic state such as $(1, 1, 0, 0, 0, 0, -1, 0, -1)$ may be used to generate an IC-POVM with dichotomic pairwise products $|\langle\psi_i| \psi_j\rangle|^2_{i\neq j} \in \{\frac{1}{4}, \frac{1}{4^2}\}$. Defining lines as triple of projectors with trace $\frac{1}{8}$, one gets a geometric configuration of type $[81_3]$ that split into nine disjoint copies of type $[9_3]$. Each of them can be seen as a 3QT proof of KS theorem since the product law for the eigenvalues of 2QT operators O_i, that is $\nu(\Pi_{i=1}^9 O_i) = \Pi_{i=1}^9[\nu(O_i)]$,

QUANTUM CONTEXTUALITY IN QUANTUM MECHANICS AND BEYOND, PRAGUE, CZECH REPUBLIC, JUNE 4-5 (2017).

is violated. The left hand side equals ω_3 while the right hand side equals ± 1 [3]. No non-contextual hidden variable theory is able to reproduce these results.

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Renate Quehenberger

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A Proposal for a psi-ontological model based on 5-dimensional geometry

There is an ongoing controversy about whether quantum states should be regarded as representing anything in physical reality. Here a truly Platonic approach shall oppose Copenhagen like interpretations which deny the need for any deeper description of reality beyond quantum theory. The presented proposal for an ontological model ties quantum mechanics to causal structures of higher dimensional space itself: It is the dynamic geometry of 5-dimensional space which works like a machine and seems to contain observables as described in some well-known contextuality proofs. Recently descriptions of noncontextual hidden variable (NCHV) theories were identified in visualizations of those evolving higher dimensional structures. The research asks for an epistemic access on the foundations of matter. Besides an overview of the assumed fundamental 5D model some geometrical structures matching the pentagram operators as proposed by Klyachko, Cabello and Bengtsson shall be presented together examples of contextuality proofs like Mermin's (magic) pentagram and the GHZ state visualized as 3D representations. Furthermore another 18 point KS proof related to a 24-dimensional configuration shall be put up for discussion.

Jeremy Steeger

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Betting on quantum objects

We prove a quantum version of the probabilists' Dutch book theorem: treating the projection lattice of a finite-dimensional Hilbert space as a quantum logic, if the possible ideal beliefs an agent should have regarding propositions in the lattice are given by the restrictions of unit vector states to the lattice, then all and only the Born-rule probabilities avoid Dutch books. We then demonstrate the implications of this theorem for several operational and realist quantum logics. In the latter case, we show the defenders of the eigenstate-value orthodoxy face a dilemma: either they can have no beliefs regarding ascriptions of property-values to quantum objects not in eigenstates of these properties, or they are susceptible to Dutch books. Contrariwise, those who favor vague properties admit all and only those beliefs about quantum objects that avoid Dutch books. On the vague properties approach, ontic states do not depend on what measurements are or are not performed by an observer. Vague properties thus afford a coherent, noncontextual ontology of objects for quantum systems.

William Sulis

McMaster University, Canada

Contextuality and Realism need not be incompatible: The Process Algebra Approach

Quantum contextuality (along with non-locality) is thought to distinguish the quantum domain from the classical. Biology and psychology, however, have long struggled with contextual dependencies in measurement and phenomenology. Khrennikov has extended the range of probability theories (much like non-Euclidean geometries extended geometry) to include contextual probability theories. These have been applied outside of physics to such classical fields as psychology, economics and social sciences. Since forms of contextuality appear classically, is it possible to find a rapprochement between contextuality and realism at the quantum level? Drawing on ideas from complex systems theory, emergence and process theory, the Process Algebra model proposes a quantum realm in which fundamental entities are generated by processes and fundamental particles and fields are emergent from these entities. These fundamental entities can be assigned definite values of selected properties, imparted to them by their generating process. It is not possible, however, to assign a complete set of properties to these entities because concatenation, which describes temporal sequences of processes and measurement, is non-commutative. Probabilities within the Process Algebra model are emergent and contextual in the sense of Khrennikov. The Process Algebra model has been shown to be compatible with Kochen-Specker results as well as both Bell and Leggett-Garg type inequalities. The Process Algebra model thus exhibits contextuality as well as a restricted form of realism, giving entities definite but incomplete properties.

Kohtaro Tadaki

Chubu University, Japan

A Refinement of Quantum Mechanics by Algorithmic Randomness

The notion of probability plays a crucial role in quantum mechanics. It appears in quantum mechanics as the Born rule. In modern mathematics which describes quantum mechanics, however, probability theory means nothing other than measure theory, and therefore any operational characterization of the notion of probability is still missing in quantum mechanics. We present an alternative rule to the Born rule based on the toolkit of algorithmic randomness by specifying the property of the results of quantum measurements in an operational way. Algorithmic randomness is a field of mathematics which enables us to consider the randomness of an individual infinite sequence. We then present an alternative rule to the Born rule for mixed states based on algorithmic randomness. In particular, we give a precise definition for the notion of mixed state. We then show that all of the alternative rules for both pure states and mixed states can be derived from a single postulate, called the principle of typicality, in a unified manner. We do this from the point of view of the many-worlds interpretation of quantum mechanics. Finally, we make an application of our framework to the BB84 quantum key distribution protocol in order to demonstrate how properly our framework works in a practical problem based on the principle of typicality. An extended abstract of this talk is available at `http://www2.odn.ne.jp/tadaki/4A1-2.pdf`.

Philip Walther

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Experimental superposition of causal orders and investigation of hyper-complex quantum theories

The advantages of the photons makes optical quantum system ideally suited for fundamental quantum physics experiments and a variety of applications in quantum information processing. Here I will discuss new experimental insights into concepts that superimpose the order of quantum gates as well as experimental benchmark values for hyper-complex extension of quantum mechanics that relay on quaternions (instead of complex numbers).

Linde Wester
University of Oxford, UK

Almost Equivalent Paradigms of Contextuality

Linde Wester

April 15, 2017

Two frameworks for contextuality in theories of physics have been proposed; one is a sheaf-theoretic approach by Abramsky and Brandenburger [2]; the second is an equivalence-based approach by Spekkens [5]. The two approaches share the goal of seeking to express the notion of contextuality in a manner that is independent from the quantum formalism, and hence applicable to any physical theory. The seemingly different approaches turn out to be equivalent for deterministic measurements, but differ in the non-deterministic case. Combining the advantages of the two formalisms results in an alorithmic method to detect contextuality in noise-free quantum circuits.

In the sheaf-theoretic approach, one defines contextuality as the non-existence of a joint probability distribution over the outcomes of joint measurements. It is formulated within the mathematical framework of sheaf theory. This framework provides methods to detect contextuality based on sheaf cohomology [3], as well as means of quantifying contextuality [1].

Contextuality in the equivalence-based approach is defined as the non-existence of ontological models for operational theories, which depend only on the statistical data of the experiment. The formalism distinguishes contextuality of preparations, transformations and of (unsharp) measurements. It provides an method to test contextuality experimentally, in a way that is robust to noise [4].

We show that any non-contextual scenario in the sheaf sense gives rise to a non-contextual ontological model in the equivalence-based sense. We call this the 'canonical' ontological model of the theory. We show that such a canonical model exists iff the theory can be realised by any factorizable non-contextual ontological model. For theories with deterministic measurements, the canonical model gives rise to a categorical isomorphism between suitable categories corresponding to the two formalisms. This isomorphism preserves non-contextuality. For the non-deterministic case, we give an explicit example that differentiates the two notions of contextuality. When applied to quantum circuits, the canonical ontological states provide an easy way to detect contextuality in a computational setting.

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An experimental test of entropic noncontextuality with single photons

Contextuality serves as a main discrepancy between quantum theory and the classical world, and supplies the power for quantum information processing. Previous tests of the contextuality focus mainly on the expectation values or the probabilities of the measurement outcomes. In this talk, we report the first experimental text of the contextuality of a three-level quantum system using classical conditional entropy of measurement outcomes derived by Kurzynski, Ramanathan, and Kaszlikowski [[Phys. Rev. Lett. 109, 020404 (2012)]. We observe the violation of the entropic contextual inequality with single-photon qutrits by more than 13 standard deviations. Our experiment gives the evidence that the classical properties of entropy also fail do describe measurements in contextual scenarios. The experimental confirmation of quantum contextuality in its simple and fundamental form sheds new light on the contradiction between quantum mechanics and noncontextual realistic models.

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University of Science and Technology, China

Experimental creation of superposition of unknown photonic quantum states

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As one of the most intriguing intrinsic properties of the quantum world, quantum superposition provokes great interest in its own generation. Though a universal quantum machine that creates superposition of two arbitrary unknown states has been shown to be physically impossible, a probabilistic protocol exists given that two input states have nonzero overlaps with the referential state. Here we report a probabilistic quantum machine realizing superposition of two arbitrary unknown photonic qubits as long as they have nonzero overlaps with the horizontal polarization state |H>, A total of 11 different qubit pairs are chosen to test this protocol and we obtain the average fidelity as high as 0.99, which shows the excellent reliability of our realization. This realization may have significant applications in quantum information and quantum computation, e.g., generating nonclassical states and realizing information compression in a quantum computation.

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