Betting on Quantum Objects

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Objectives

- Prove a quantum probabilism theorem for projection-lattice logics $\mathcal{P}(\mathcal{H})$ to assess beliefs about measurement outcomes *and* beliefs about unseen objects.
- Apply the theorem to an operational interpretation of such logics, recovering a result due to Randall and Foulis.
- Show the coherence of vague-property semantics for such logics.
- Show a trilemma for eigenstate-value link semantics.

Generalized Dutch Books

Data for generalized Dutch books:

- A propositional language L
- A set $\mathbb{T}$ of (partial) truth valuations $T : L \to N$
- A set $\mathbb{V}$ of (partial) ideal beliefs $V_T : L \to [0,1]$
- An agent's actual (partial) beliefs $B : L \to [0,1]$

B is *Dutch-bookable* if there is a finite set $\{\phi_i\}$ in the domain of some $V \in \mathbb{V}$ and stakes $s_i \in \mathbb{R}$ such that for all $V \in \mathbb{V}$ defined for $\{\phi_i\}$,

$$\sum_{i=1}^{n} s_i(V(\phi_i) - B(\phi_i)) < 0. \qquad (1)$$

Kühr and Mundici prove the following [1].

Theorem 1. *For $\mathbb{V}$ pointwise-closed in $[0,1]^L$, if $A \subseteq [0,1]^L$ is pointwise-closed and convex, $\mathbb{V} \subseteq A$ and $\partial A \subseteq \mathbb{V}$, then $B \in [0,1]^L$ avoids Dutch books if and only if $B \in A$.*

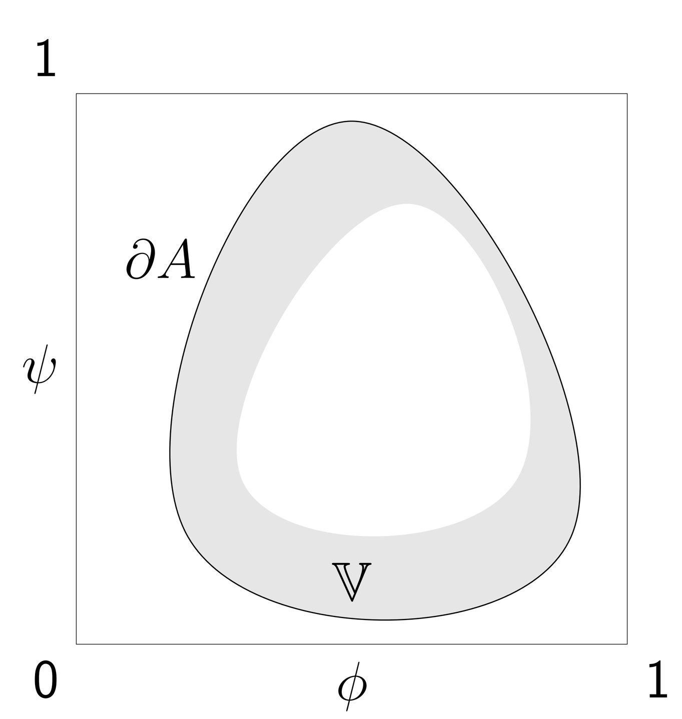

Figure: An illustration of Theorem 1 for L containing two propositions ϕ and ψ.

Proof of the Theorem

A sketch of the proof of QPF:

- By $\dim(\mathcal{H}) < \infty$, states $\mathcal{E}$ on $\mathcal{B}(\mathcal{H})$ are normal states $\mathcal{N}$, and $\mathcal{E} = \mathcal{N}$ is convex and weakly* closed and $\partial\mathcal{E} = \partial\mathcal{N}$ is weakly* closed [2].
- The map from states to the Tychonoff cube
$$r : \mathcal{E} \to [0,1]^{\mathcal{P}(\mathcal{H})} :: \omega \mapsto \omega\restriction_{\mathcal{P}(\mathcal{H})}$$
is continuous, linear (convexity-preserving), injective, and closed.
- Thus $r(\mathcal{N})$ is convex and pointwise-closed and $r(\partial\mathcal{N}) = \partial r(\mathcal{N})$ is pointwise-closed; Theorem 1 completes the proof.

The Language $\mathcal{P}(\mathcal{H})$

For $\phi \in \mathcal{P}(\mathcal{H})$, $\phi = P^{A_\alpha}(a_\alpha)$ via spectral theorem:

- **Operationalist.** ϕ is
  After measurement, the values of $\{A_\alpha\}$ for the system lie in $\{a_\alpha\}$, respectively.
- **Classical-realist.** ϕ is
  The values of $\{A_\alpha\}$ for the system lie in $\{a_\alpha\}$, respectively.
- **Dispositional-realist.** ϕ is
  The system is disposed to yield a value of $\{A_\alpha\}$ in $\{a_\alpha\}$ upon measurement of $\{A_\alpha\}$, respectively.

(QPF) Quantum Probabilism in Finite Dimensions

For finite-dimensional $\mathcal{H}$, if ideal beliefs are restrictions of vector states to the lattice $\mathcal{P}(\mathcal{H})$, then Born-rule beliefs (restrictions of normal states to $\mathcal{P}(\mathcal{H})$) are all and only the total beliefs avoiding Dutch books.

Belief-Fixing Strategy

- **Born-fixing.** Pick an appropriate $B \in r(\mathcal{N})$.
- **Truth-fixing.** Pick a convex sum of $T \in \mathbb{T}$.

Operationalist Semantics

$$\mathbb{T}_O := \left\{ T_\eta(\phi) := \begin{cases} \langle \eta, \phi\eta \rangle & \langle \eta, \phi\eta \rangle \in \{0,1\} \\ \text{undf.} & \text{otherwise} \end{cases} \right\}$$

for $\eta \in \mathcal{H}$, $||\eta|| = 1$.

Weak truth-fealty. *For all $T \in \mathbb{T}$:*

- *$T(\phi) = T(\psi) \Rightarrow V_T(\phi) = V_T(\psi)$ or both undf.*
- *If $T(\phi)$ is undf. then $V_T(\phi)$ is undf.*
- *If $T(\phi)$ is 1 (0) then $V_T(\phi)$ is 1 (0).*

Weak truth-fealty fixes $\mathbb{V}_O$.

Coherence of operationalism. *Given Born-fixing, a corollary of QPF yields all beliefs avoid Dutch books.*

This recovers a result due to Randall and Foulis in the setting of test spaces [3].

Vague-Property Semantics

$$\mathbb{T}_V := \{T_\eta(\phi) := \langle \eta, \phi\eta \rangle\}$$

Strong truth-fealty. *$V_T = T$ for $T \in \mathbb{T}$.*

Strong truth-fealty fixes $\mathbb{V}_V$.

Coherence of vague properties. *Given Born-fixing or truth-fixing and $\dim(\mathcal{H}) < \infty$, QPF yields that agents' beliefs are all and only those total ones that avoid Dutch books.*

E-V Link Semantics

$$\mathbb{T}_E := \left\{ T_\eta(\phi) := \begin{cases} \langle \eta, \phi\eta \rangle & \langle \eta, \phi\eta \rangle \in \{0,1\} \\ 2 & \text{otherwise} \end{cases} \right\}$$

Weak truth-fealty fixes $\mathbb{V}_E$ for $V_T(\phi) = c$ when $T(\phi) = 2$, for either $c \in [0,1]$ or c undefined.

Incoherence of E-V link. *Given Born-fixing and $c \in [0,1]$, a corollary to Theorem 1 yields that agents' beliefs are Dutch-bookable (in the Bohm-EPR case).*

E-V Link Trilemma

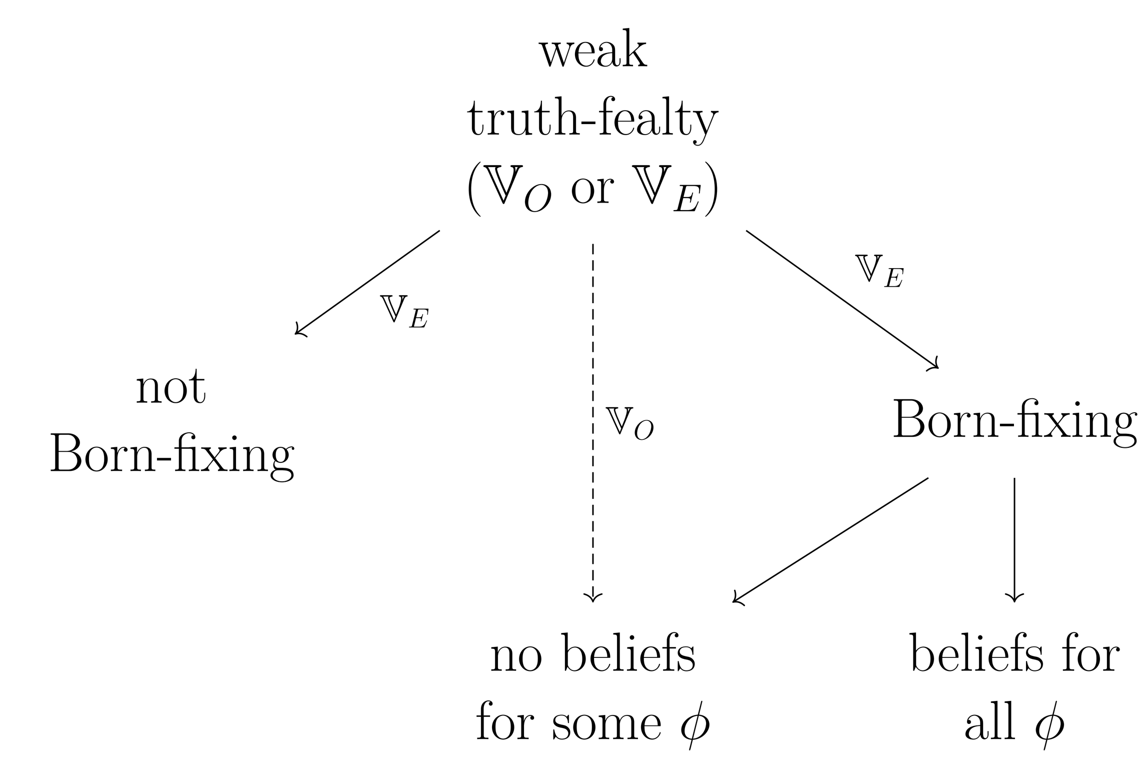

Figure: A sketch of the trilemma facing the defender of the eigenstate-value link; the rightmost leg leaves agents susceptible to Dutch books.

References

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