Quantum Contextuality in Quantum Mechanics and Beyond, Prague, 4-5 June 2017

Probability in the Plato's cave

Local model of a qudit

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Plan of Talk

It is often argued that quantum interference, collapse of the wave function or contextuality are strictly quantum mechanical effects which defy classical explanation. In this talk, we give explicit counterexample demonstrating that these features are present in classical models too. We show that single-particle phenomena in the interferometric circuits can be explained as epistemic effects in a local hidden variable model, thereby pushing the real mystery to the multi-particle behaviour.

1. **Motivation:** *Plato's cave & quantum ontology*
2. **QM framework:** *Single-particle interferometry (qudit)*
3. **Local model of a qudit:**
   - *Ontology + action of the gates*
   - *Analysis of the model*
4. **Conclusions**

Allegory of the Cave

… are we living in a MATRIX?

Constrained access to information ⇒ *Different picture of the world*

Ontic vs. Epistemic

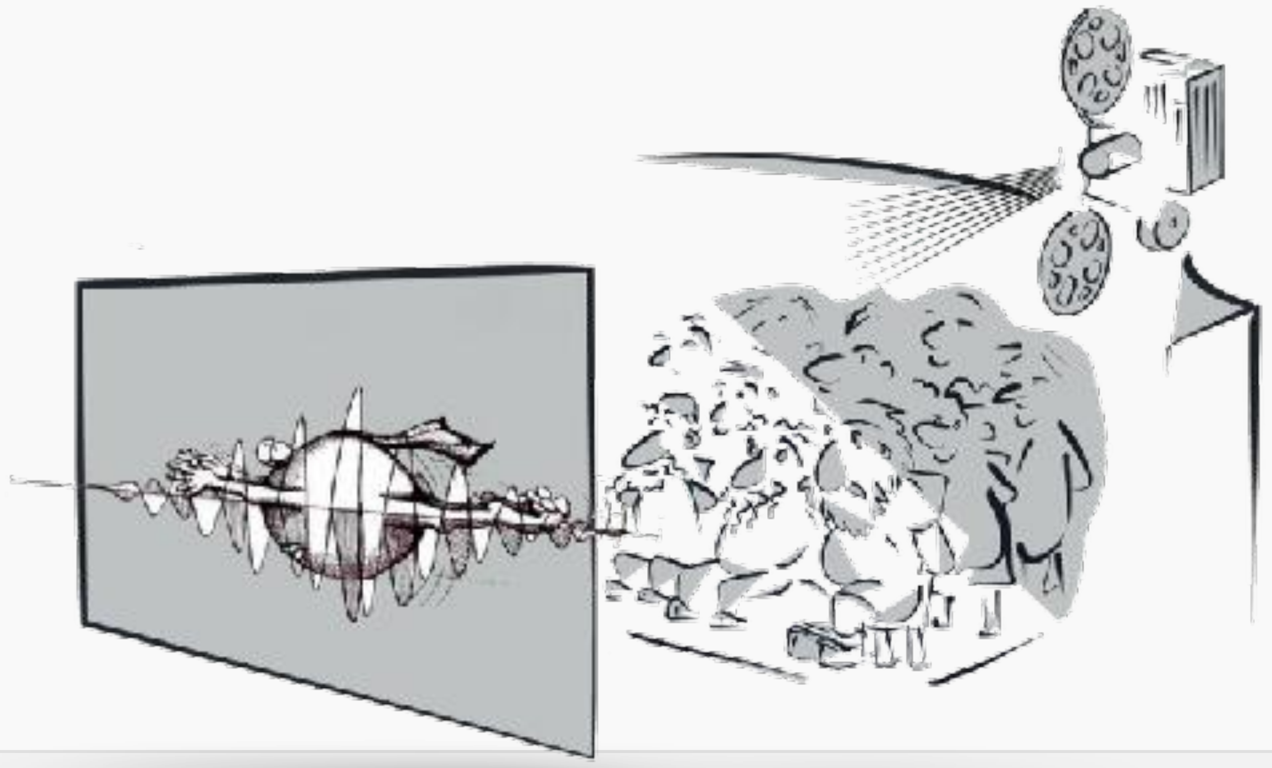
Best theory we've ever had … But …

*"But our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature — all **scrambled up** by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the **unscrambling** is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple."*

Edwin T. Jaynes
"Probability in Quantum Theory"
in: "Complexity, Entropy, and the Physics of Information" (1990)

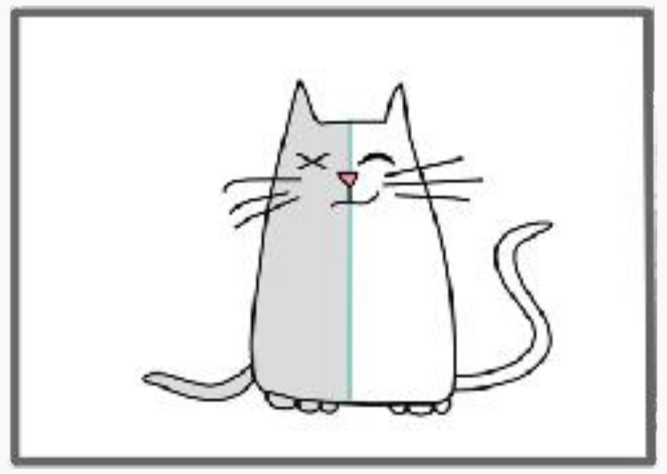
Edwin T. Jaynes
(1922 - 1988)

Ontology + incomplete information ⇒ QM formalism



What is the wave function ?

What it the ontology? Information … about what?



Observer

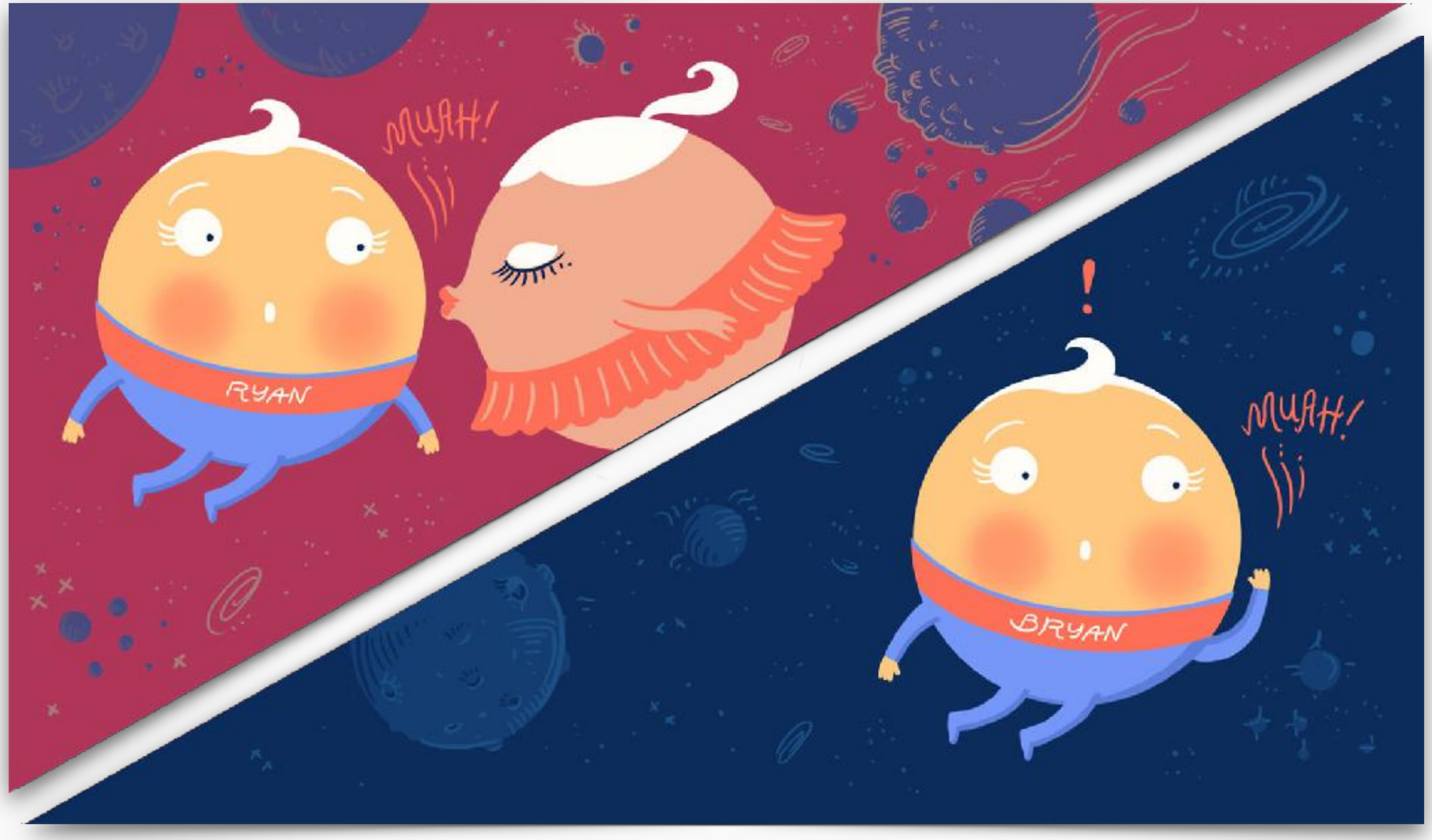
System

Where it belongs? Observer/System?
Observer… What is the system?
System… Where in the system?
A kind of probability distribution?
Probability of what?
Information? About what?
Whose information? Mine, yours ?
If QT is a fundamental theory,
then what is the ontology?
etc…

Quantum mechanics

What this talk is (not) about

Multi-particle phenomena



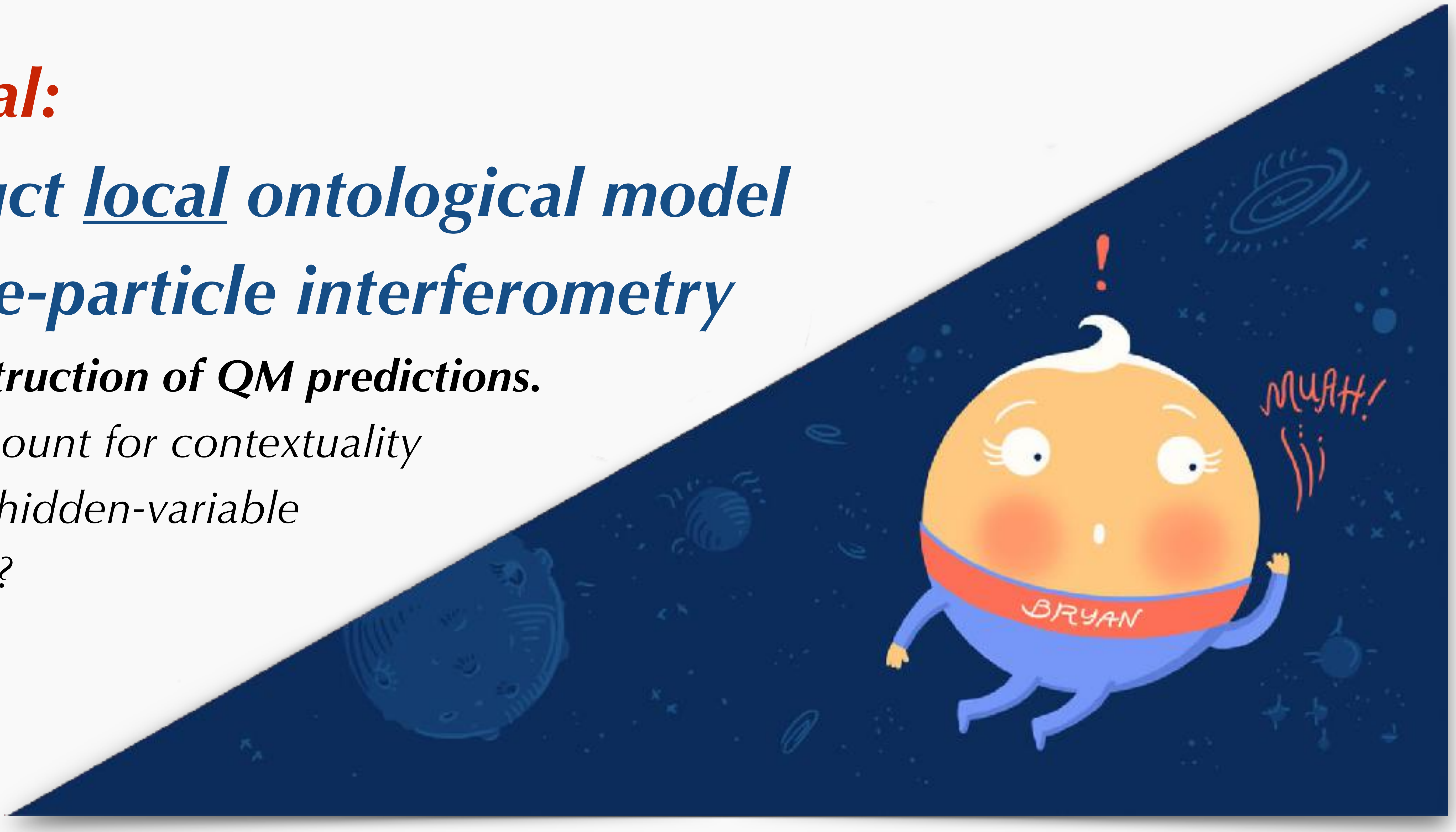
Single-particle phenomena

Correlations between
<u>many</u> quantum particles:
QM in Hilbert space
<u>with</u> tensor products
see on the right
+
entanglement
Bell's inequalities
indistinguishability
…

<u>Single</u> quantum particle
interacting with apparatus:
QM in Hilbert space
<u>without</u> tensor products
measurement
collapse of wave function
quantum interference
contextuality
…

Quantum mechanics

What this talk is (not) about



Our goal:

Construct <u>local</u> ontological model of single-particle interferometry

Full reconstruction of QM predictions.

How to account for contextuality in classical hidden-variable framework ?

Single quantum particle interacting with apparatus:

QM in Hilbert space without tensor products

measurement

collapse of wave function

quantum interference

contextuality

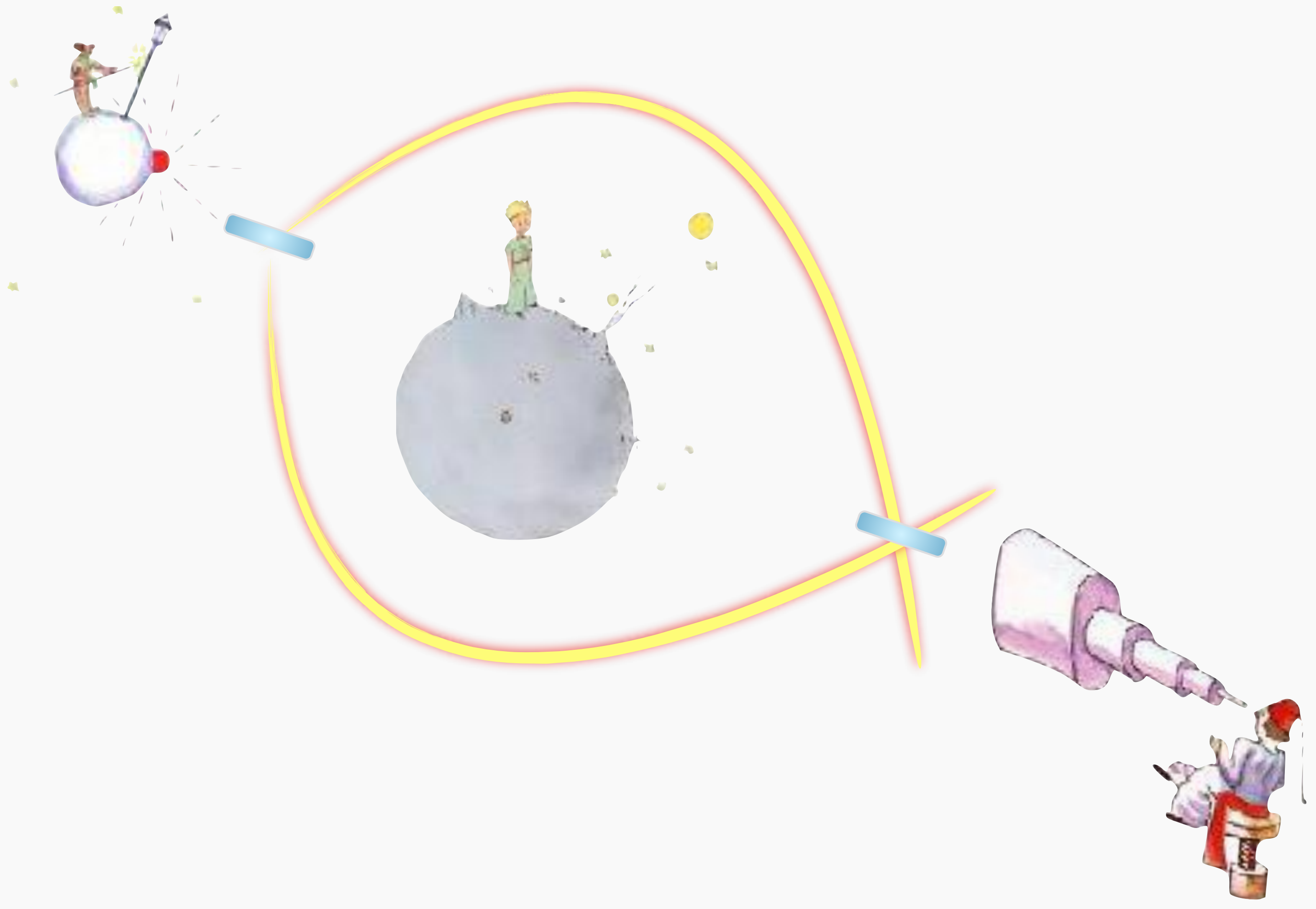
…

Single-particle phenomena

Problems with the ontology

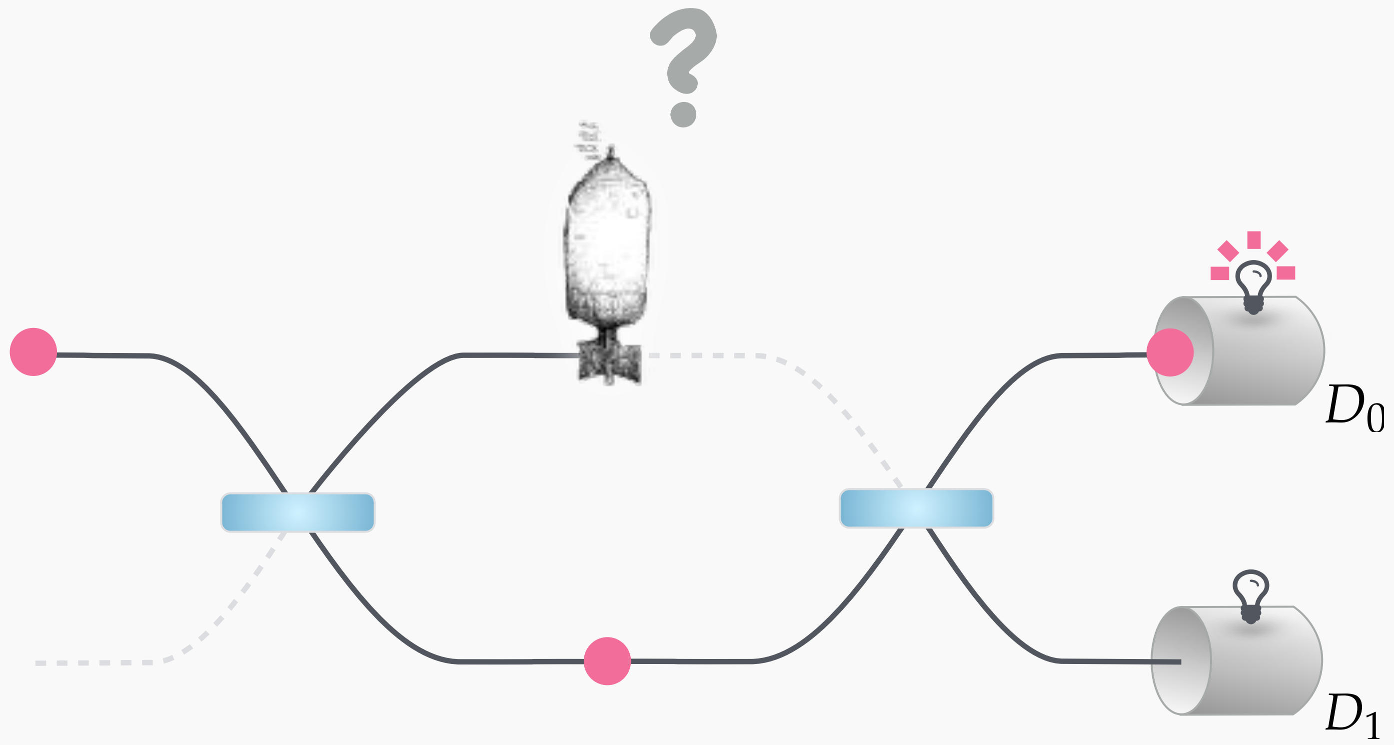
Single-particle framework (QUBIT)

Wave-particle duality
Wheeler's delayed-choice experiment



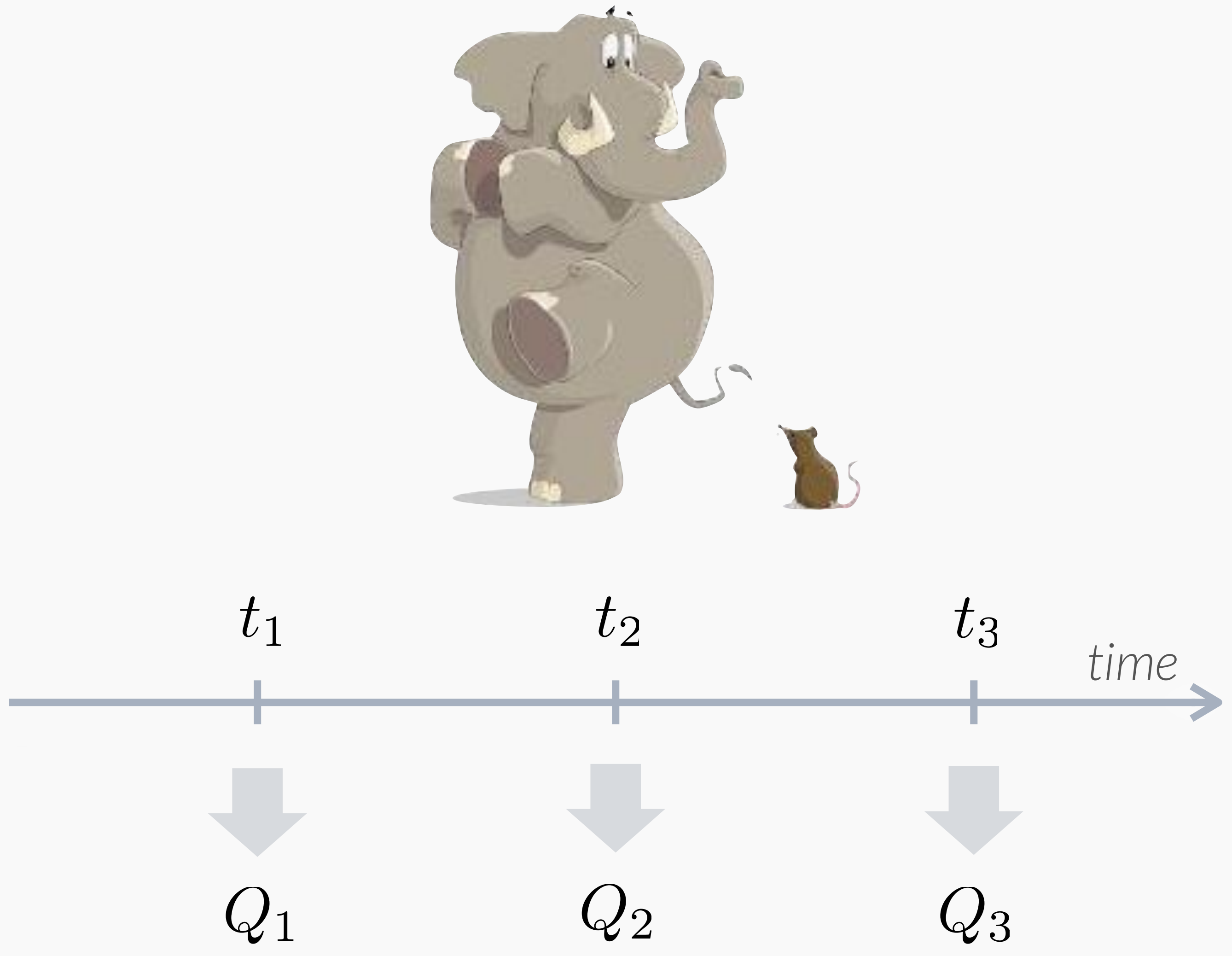
*How the particle particle '**changes**' its past ?*

Non-locality and interaction-free measurements
Elitzur-Vaidman bomb testing problem



*How the particle '**feels**' the other path ?*

Micro vs. macroscopic realism
Leggett-Garg inequalities



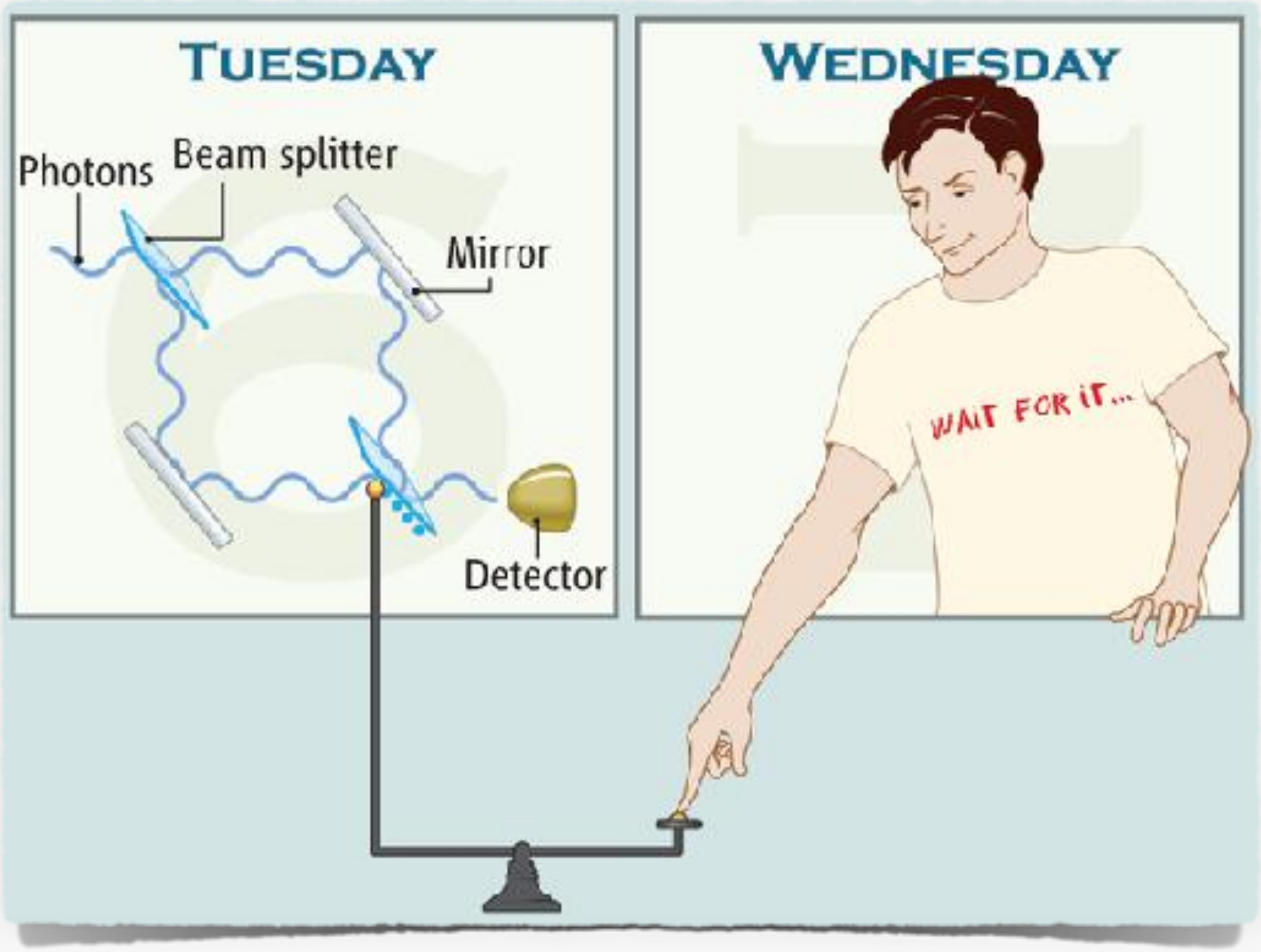
*How the world becomes '**macro**' ?*

Problems with the ontology

Single-particle framework (QUBIT)

Wave-particle duality

Wheeler's delayed-choice experiment



PHYSICS

Quantum Procrastination

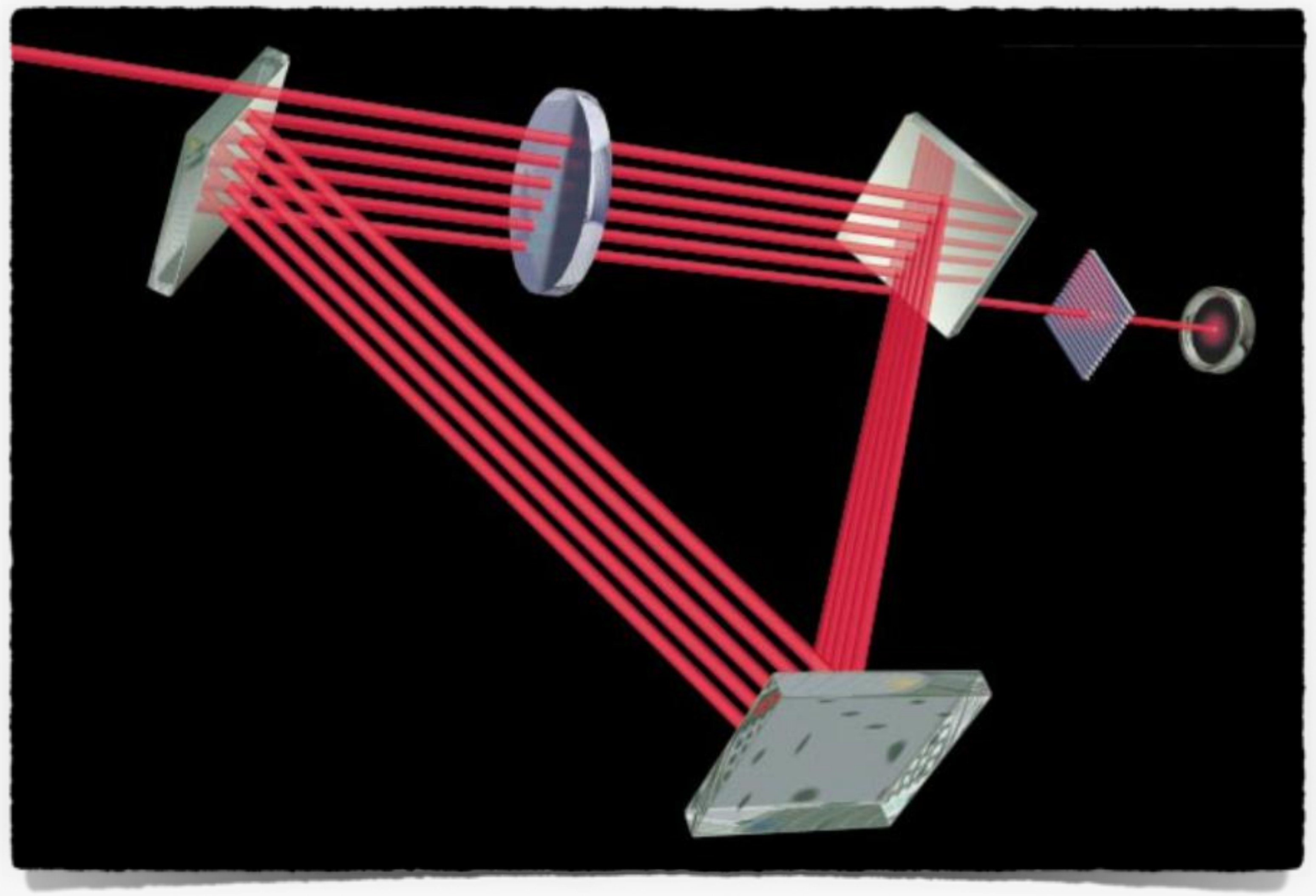
Seth Lloyd

Entangling two photons allows the wave and particle nature of light to be interchanged even after the light has already been detected.

Science ***338*** *621 (2012)*

Non-locality and interaction-free measurements

Elitzur-Vaidman bomb testing problem



Quantum Seeing in the Dark

Quantum optics demonstrates the existence of interaction-free measurements: the detection of objects without light—or anything else—ever hitting them

by Paul Kwiat, Harald Weinfurter and Anton Zeilinger

Scientific American ***275*** *72 (1996)*

Micro vs. macroscopic realism

Leggett-Garg inequalities

QUANTUM MECHANICS

No moon there

An experiment reveals that micrometre-sized superconducting circuits follow the laws of quantum mechanics, and thus defy common experience of how macroscopic objects should behave.

Johan E. Mooij

Nature Physics ***6*** *401 (2010)*

Problems with the ontology

Single-particle framework (QUDIT)

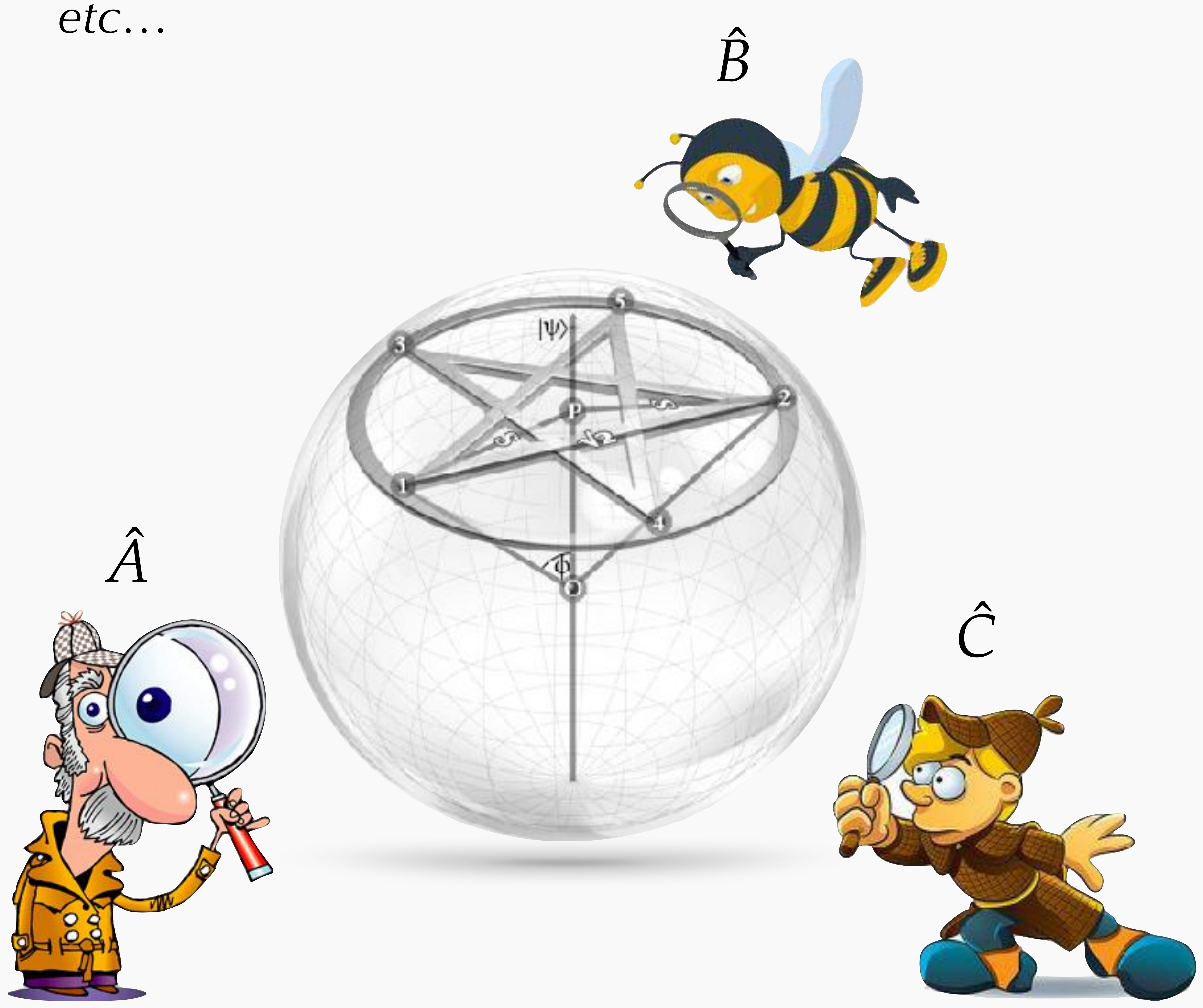
Contextuality

Kochen - Specker theorem (1967)

Klyachko inequalities (2008)

Cabello's state-independent tests (2008 - …)

etc…



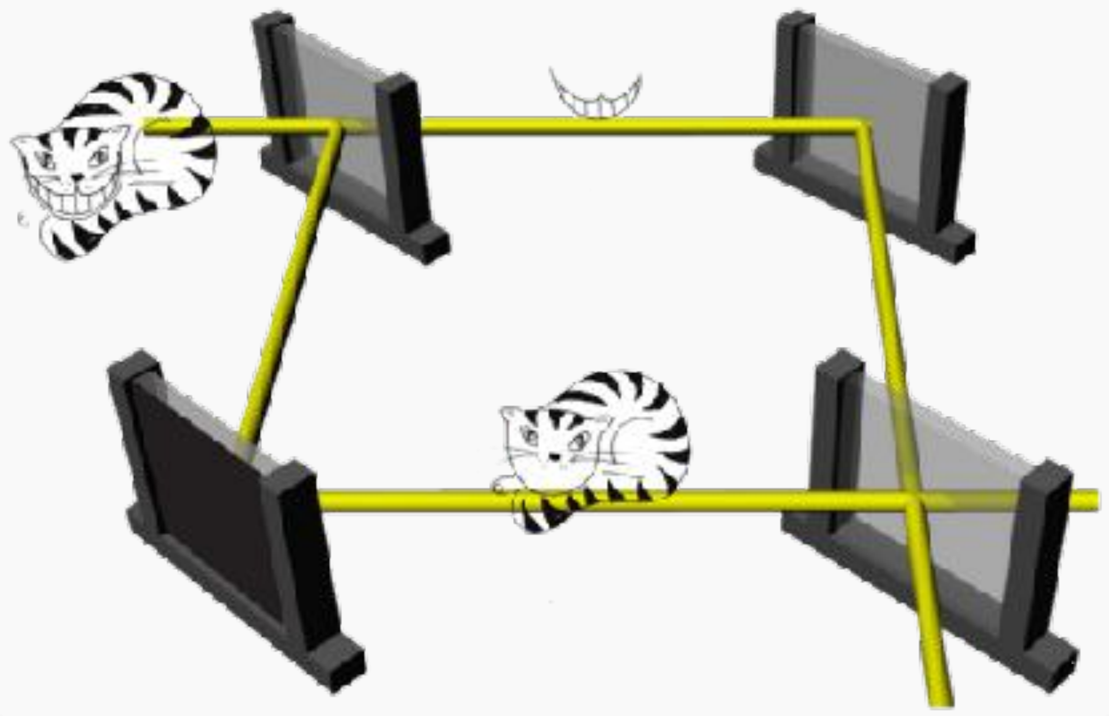
Quantum paradoxes

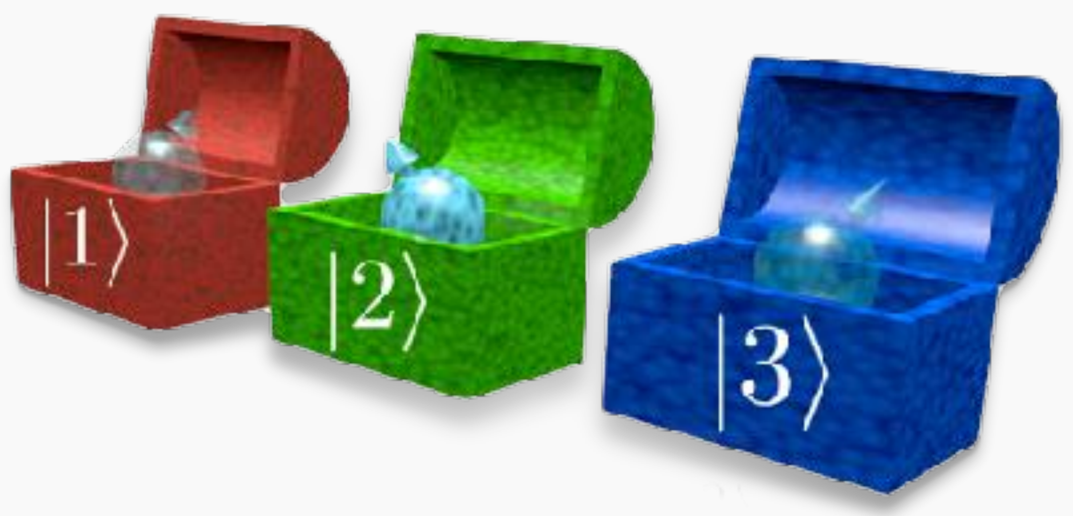
Pre- and post-selection:

Three-box paradox, Cheshire cat paradox, …

Weak measurements,

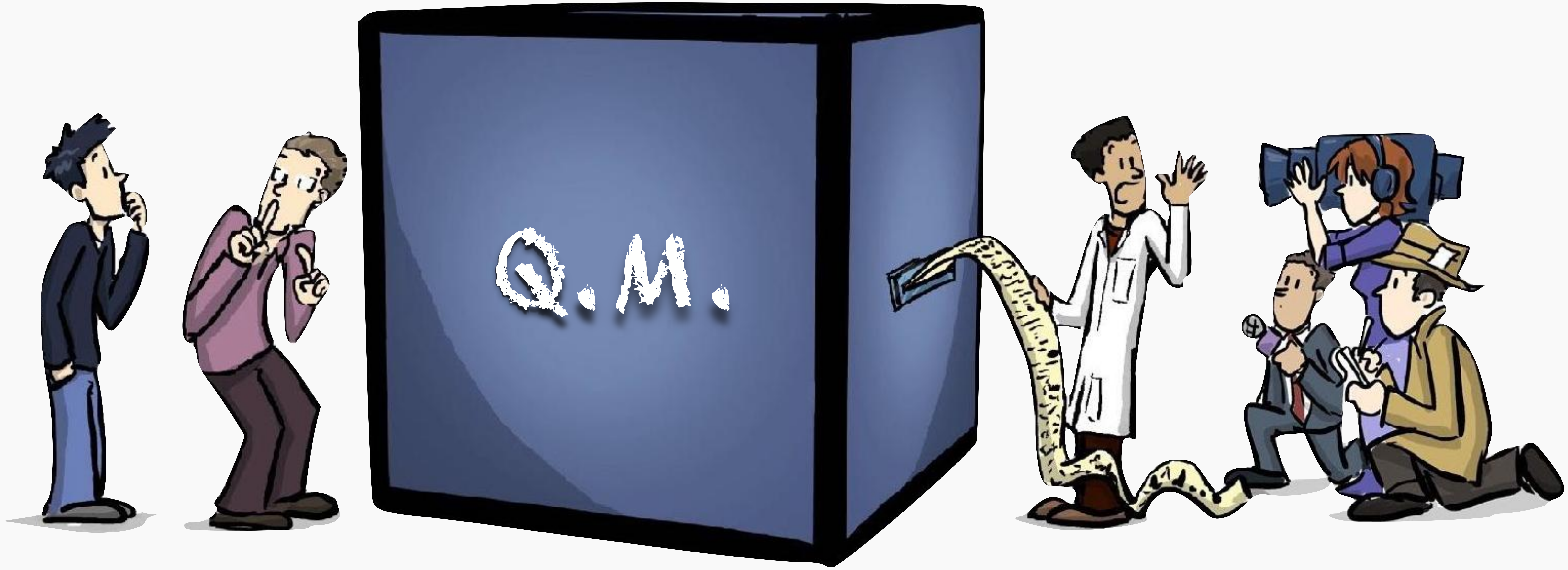
etc…





Quantum mechanics

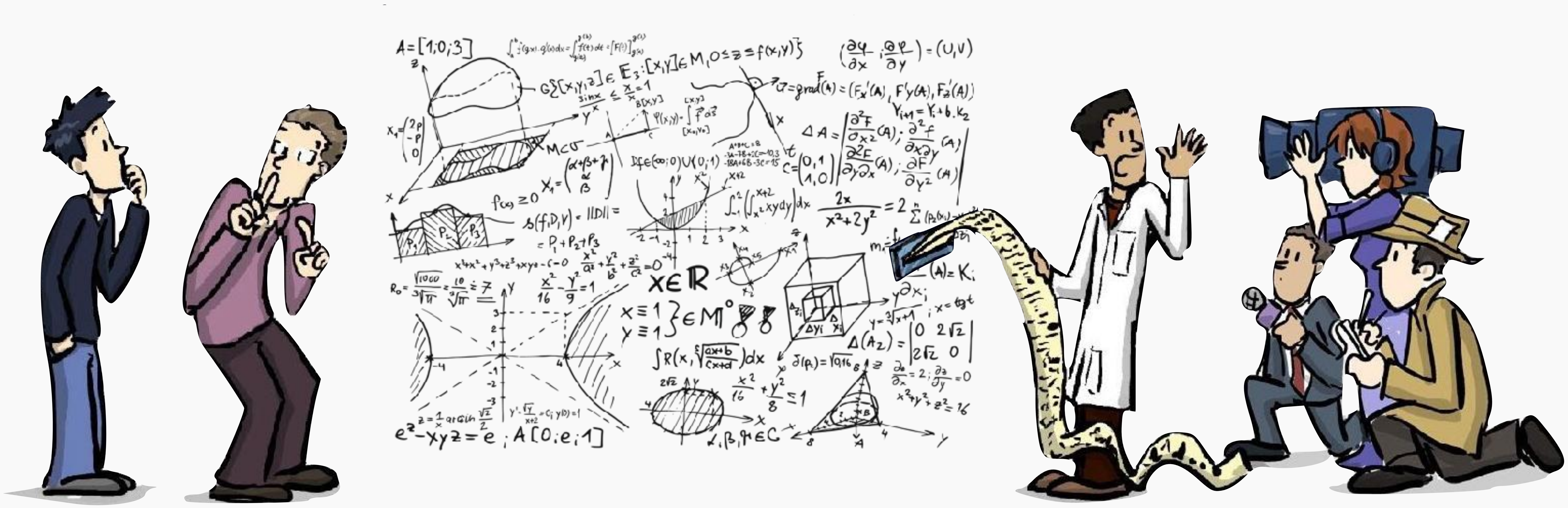
… as we have it …



Quantum mechanics

… as we have it …

Mathematical formalism ✔

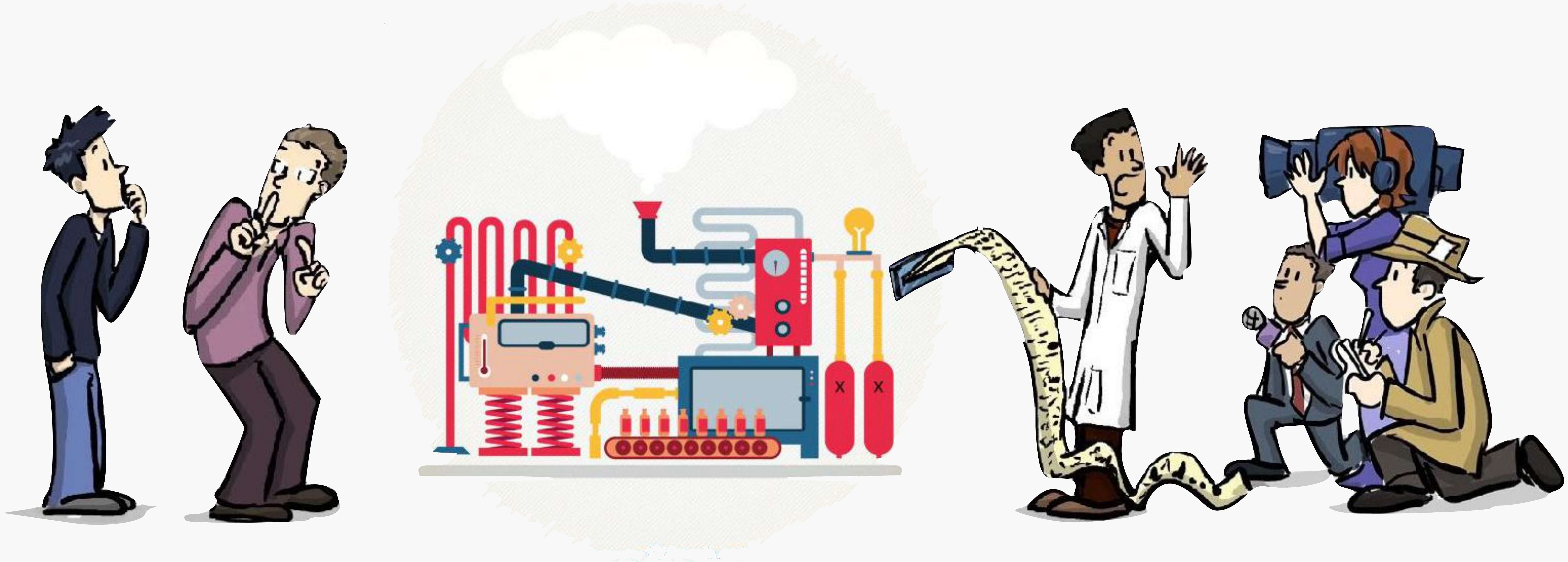


Quantum mechanics

… as we have it …

Mathematical formalism ✓

Operational description ✓



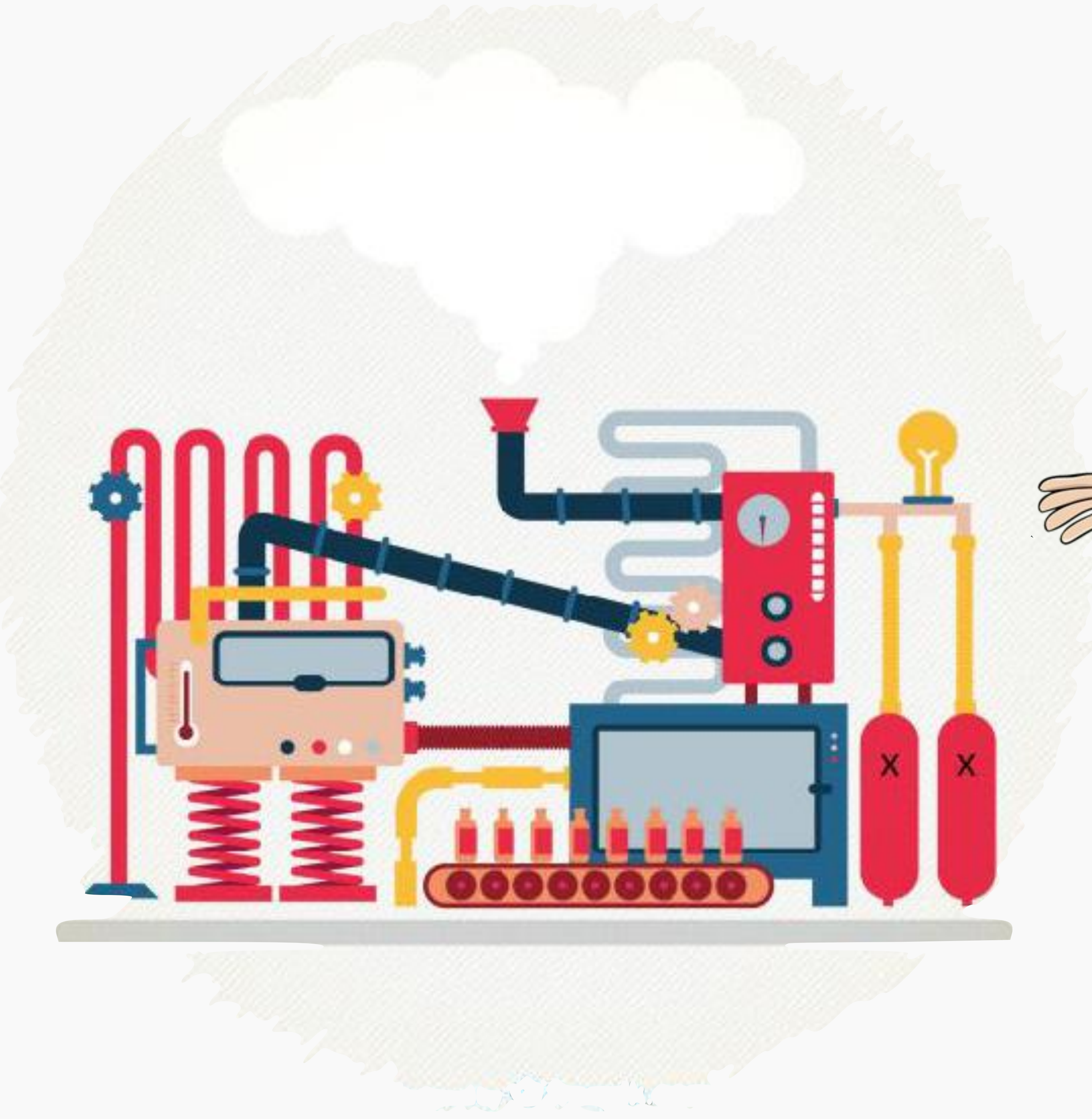
Quantum mechanics

… as we have it …

Mathematical formalism ✓

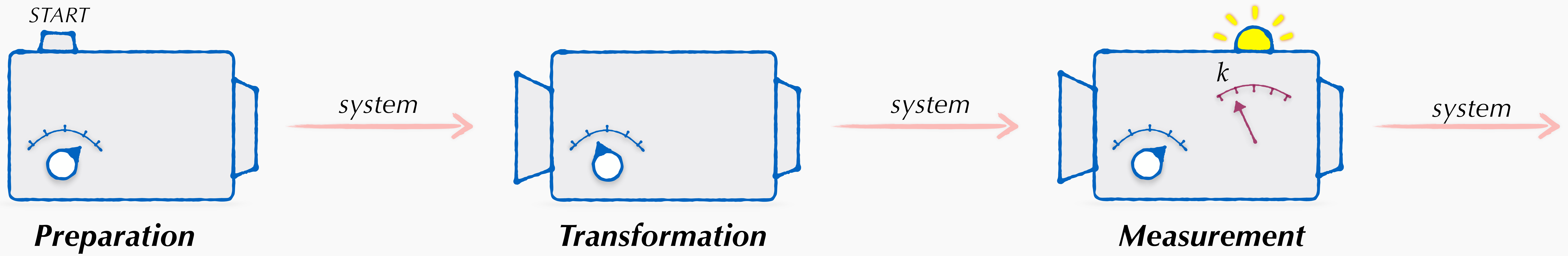
Operational description ✓

… but what is the ontology ?



Operational framework

Postulates of QM



$$|\psi\rangle \in \mathcal{H}$$

$$|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$$

$$|\psi\rangle \longrightarrow |\psi'\rangle = U\,|\psi\rangle$$

$$\hat{A} = \sum_k a_k P_k$$

$$\mathrm{Pr}(k|\psi) = \langle\psi|\, P_k \,|\psi\rangle$$

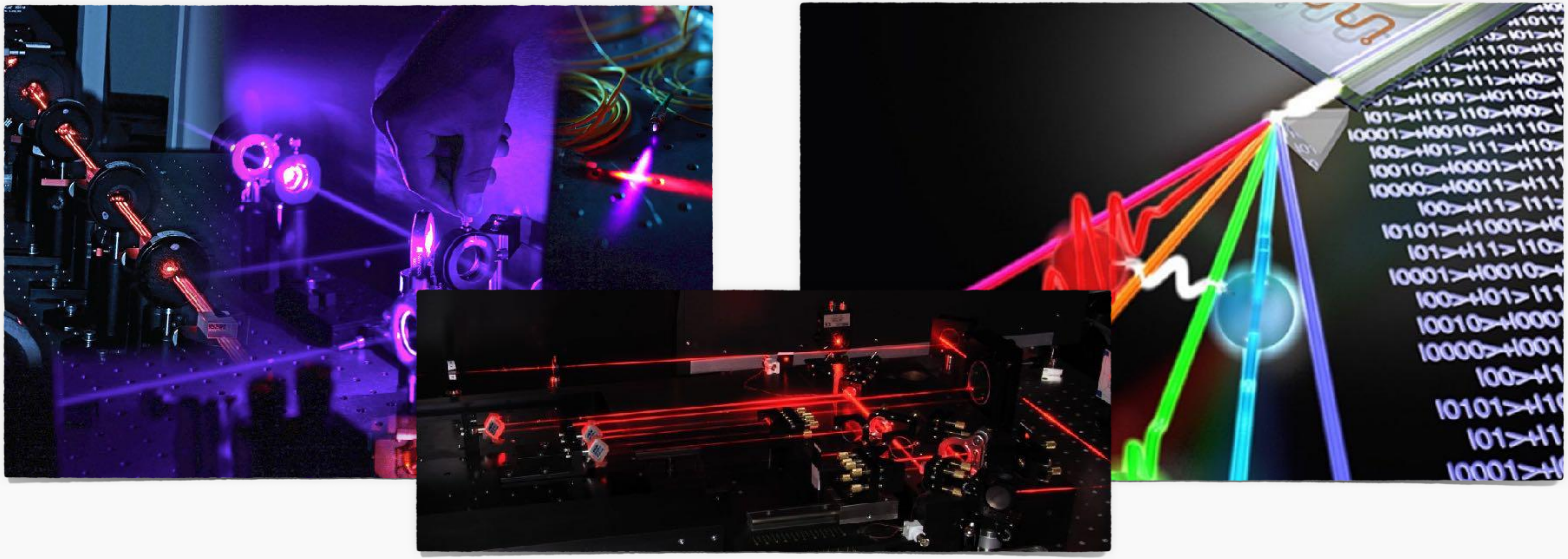
$$|\psi\rangle \xrightarrow{k} \frac{P_k\,|\psi\rangle}{\sqrt{\langle\psi|\, P_k \,|\psi\rangle}}$$

Collapse of the wave function

> *In a strict sense, quantum theory is a **set of rules** allowing the computation of **probabilities for the outcomes** of tests which follow specified preparations.*
>
> *Asher Peres in "Quantum Theory: Concepts and methods" (1995)*

Real physics in the lab

How to interpret the formalism



Quantum phenomena ***do not occur*** *in a Hilbert space. They* ***occur*** *in a laboratory.*

Asher Peres in "Quantum Theory: Concepts and methods" (1995)

Quantum interferometry

Single particle in a circuit

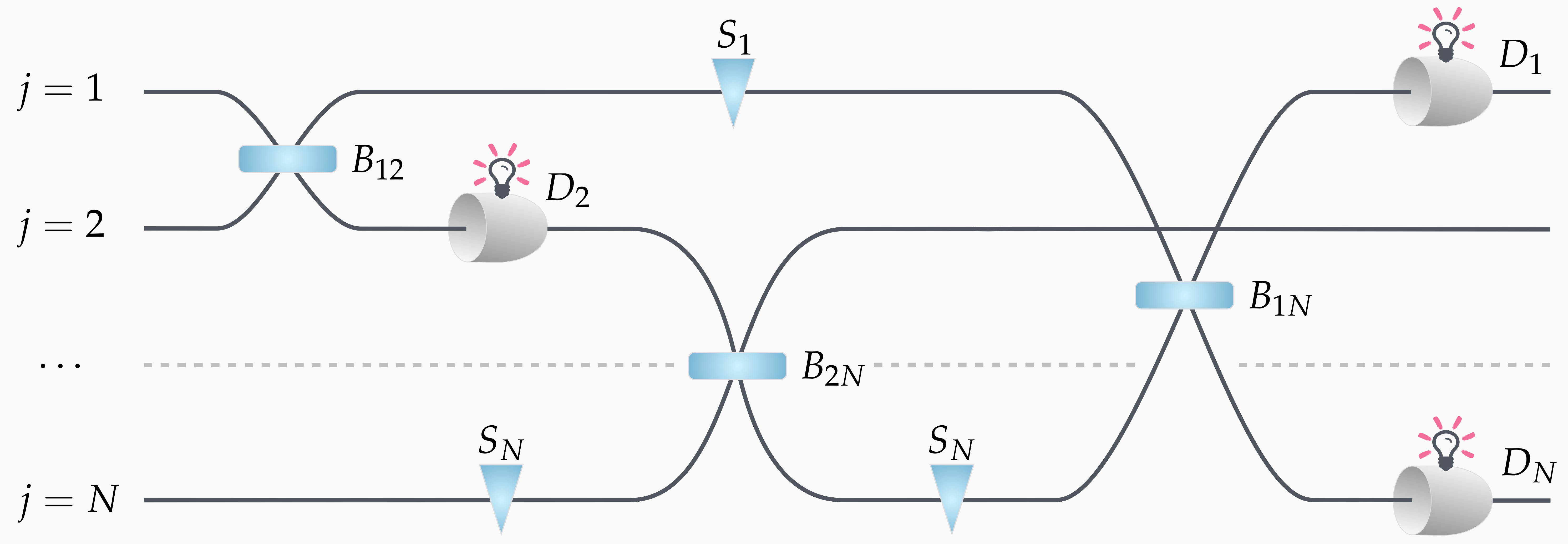
Quantum states:

$\mathcal{H} = \mathbb{C}^N$ - *Hilbert space*

$|1\rangle, \dots, |N\rangle$ - *comput. basis*

$|j\rangle$ - *particle in j^{th} path*

$$|\psi\rangle = \sum_{j=1}^{N} \psi_j |j\rangle = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix} = \vec{\psi}$$



Interferometric circuit

Interferometric gates:

$$\psi_j \xrightarrow{free} \psi_j \qquad \psi_j \xrightarrow{S_j} e^{i\omega}\psi_j$$

$$\begin{pmatrix} \psi_s \\ \psi_t \end{pmatrix} \xrightarrow{B_{st}} \begin{pmatrix} \psi'_s \\ \psi'_t \end{pmatrix} = \begin{pmatrix} i\sqrt{R} & \sqrt{T} \\ \sqrt{T} & i\sqrt{R} \end{pmatrix} \begin{pmatrix} \psi_s \\ \psi_t \end{pmatrix}$$

Collapse of the wave function

$$|\psi\rangle \xrightarrow{D_j} \begin{cases} |j\rangle & \text{'CLICK',} \\ \frac{(\mathbb{1}-\mathbb{P}_j)|\psi\rangle}{\|(\mathbb{1}-\mathbb{P}_j)|\psi\rangle\|} & \text{'NO CLICK',} \end{cases}$$

where: $Pr(D_j|\psi) = |\langle j|\psi\rangle|^2 = |\psi_j|^2$ *and* $\mathbb{P}_j \equiv |j\rangle\langle j|$

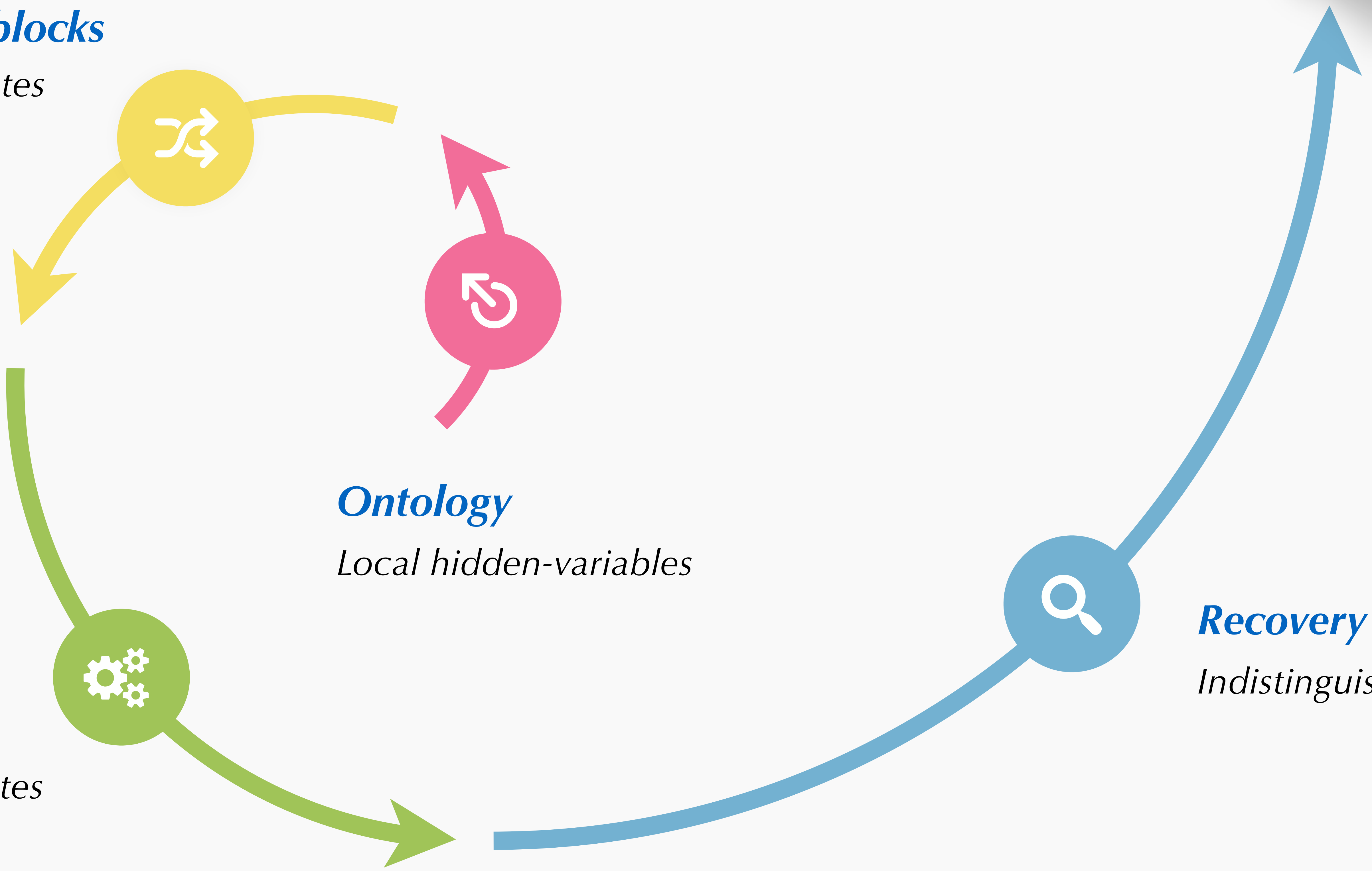
Born's rule

Theorem:

M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani. "Experimental Realization of Any Discrete Unitary Operator" Phys. Rev. Lett., ***73*** *58–61 (1994)*

Building the model

Plan of action



Basic building blocks

Interferometric gates

Ontology

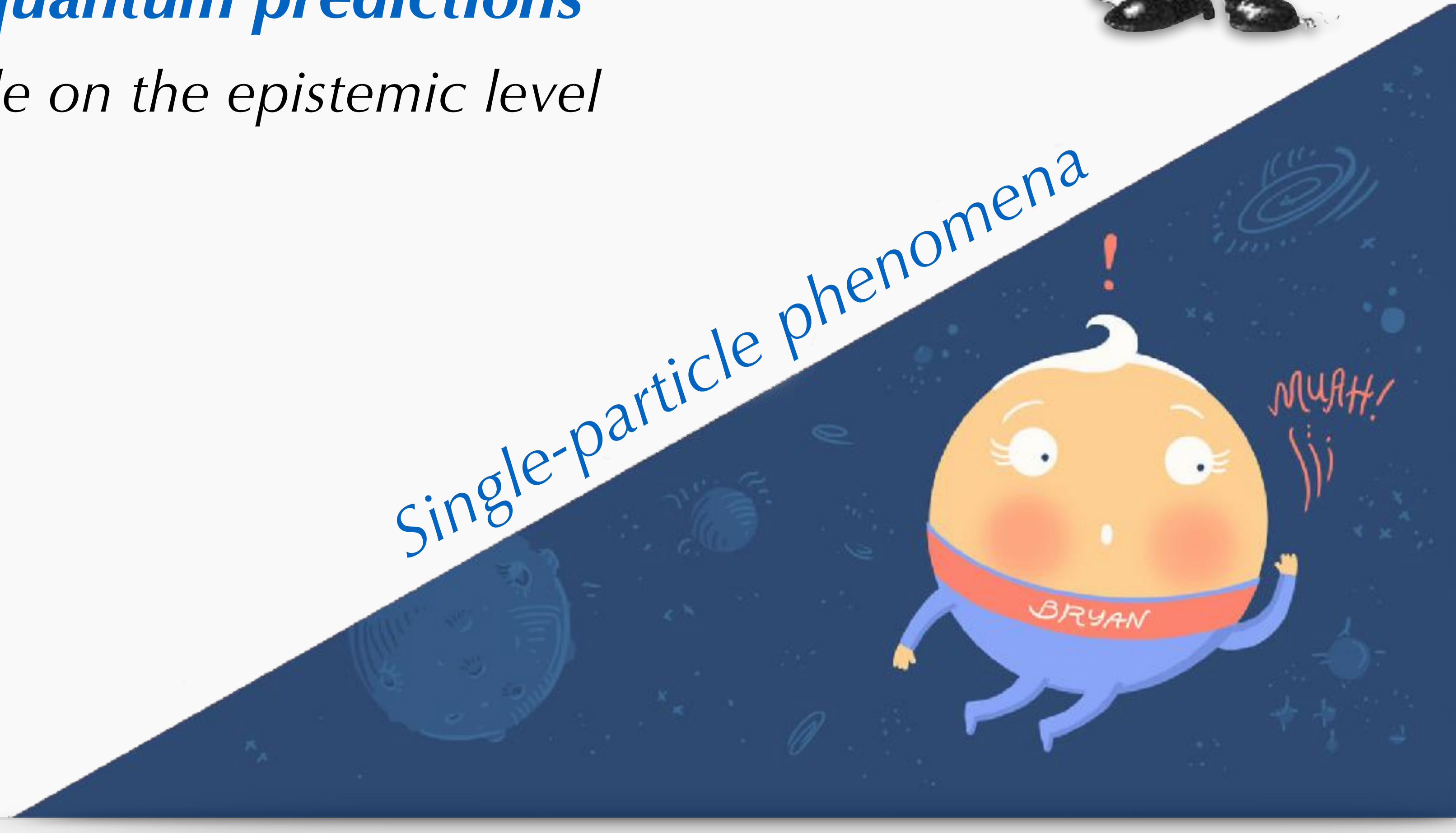
Local hidden-variables

Analysis of the model

Geometry of accessible states

Recovery of quantum predictions

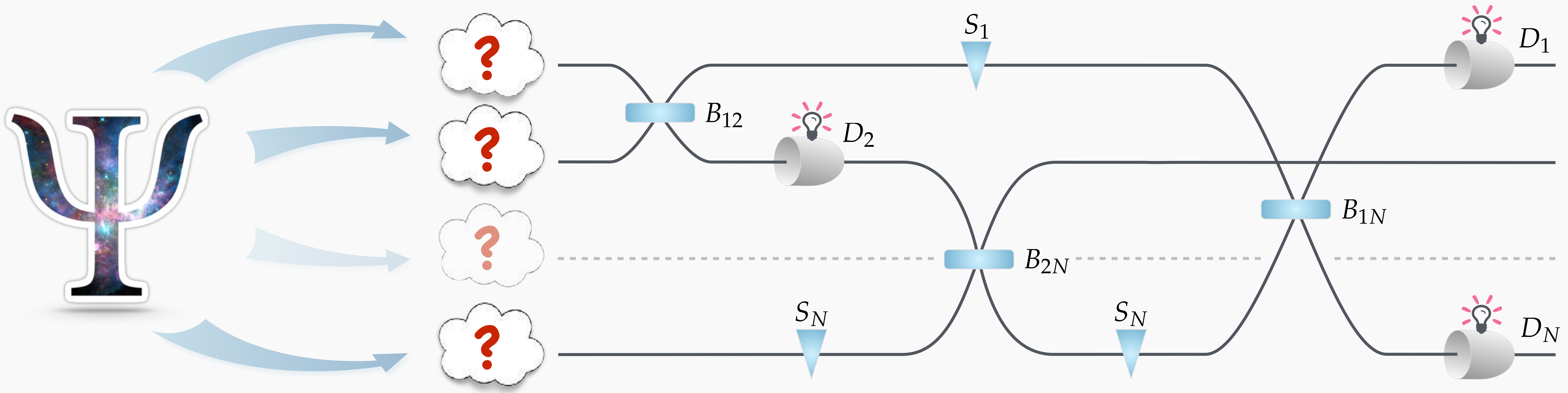
Indistinguishable on the epistemic level



Our goal: *Construct **local ontological model** of single-particle interferometry*

What is the ontology?

Issues with locality



Interferometric circuit

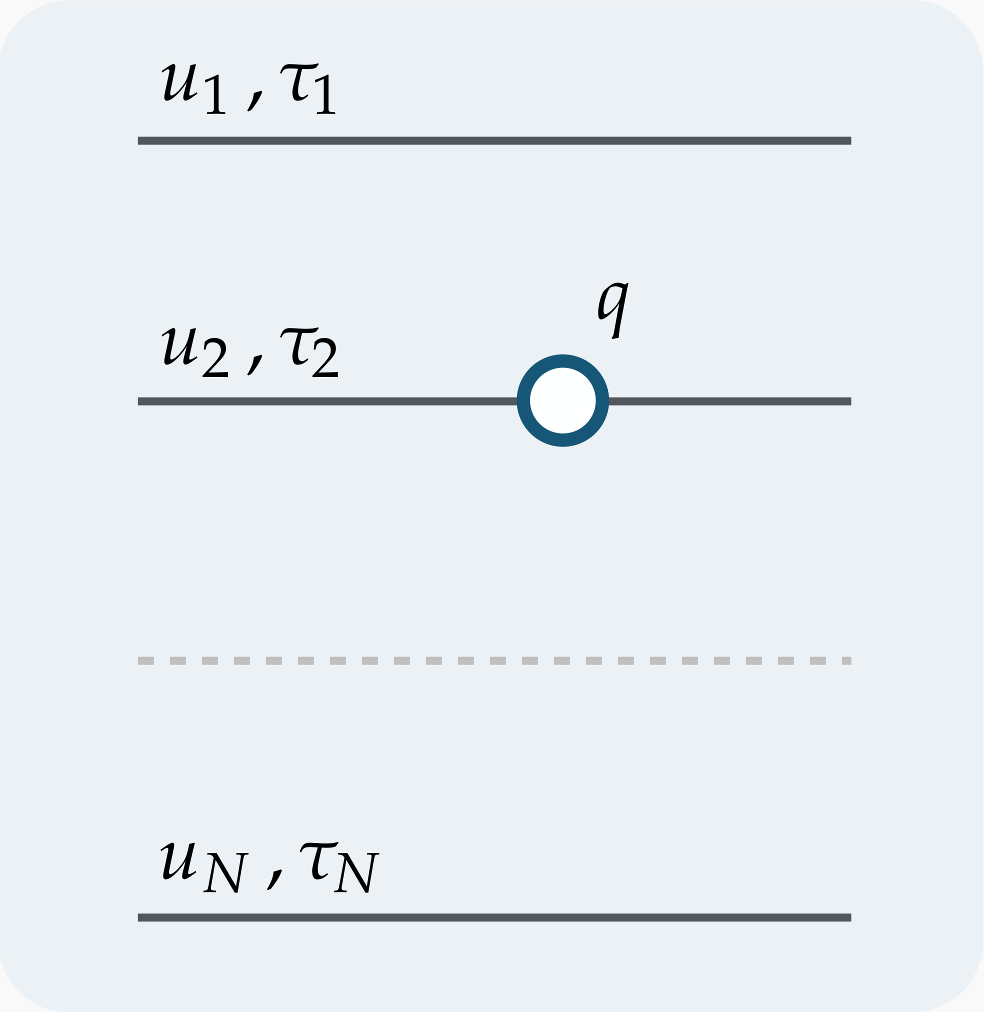
Relevant questions:

- *What is propagating in the paths ?*
- *What is action of the gates ?*
- *Is the propagation/action of the gates local ?*
- *How the model perceived by the agents ?*

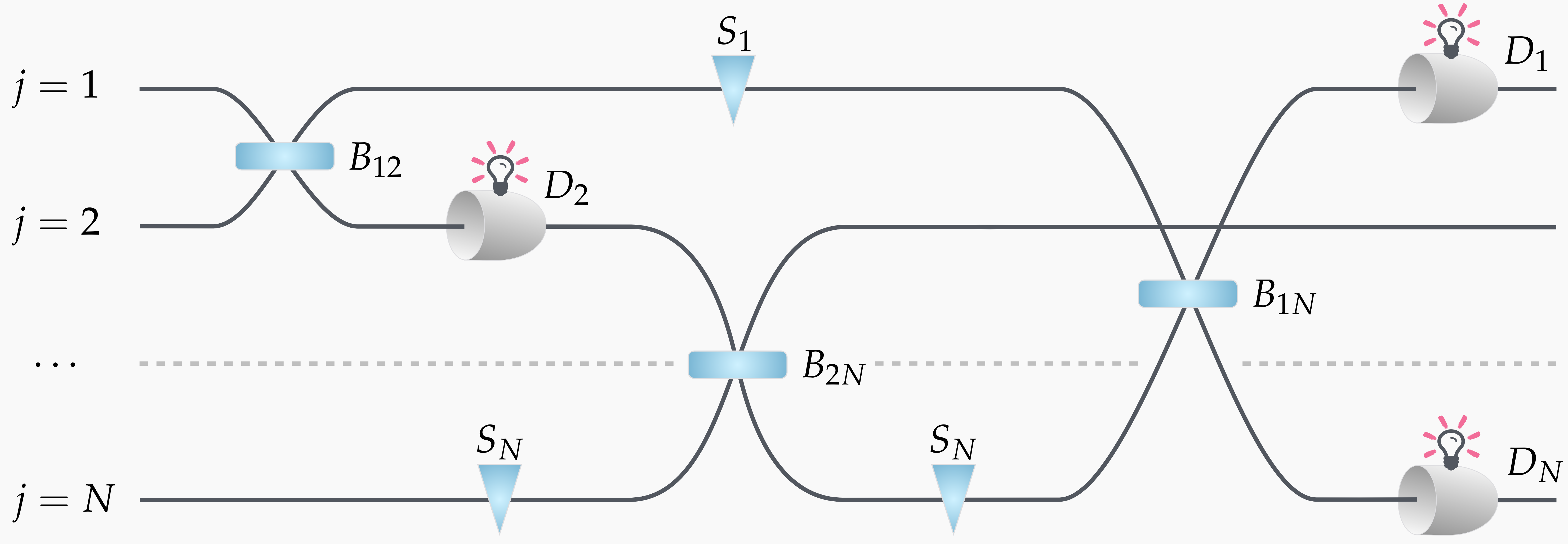
  Where does the weirdness come from ?

Ontology of the model

Hidden variables



Ontology



Interferometric circuit

q - *position* of the *particle*
u_j - *amplitude* of the *field* in j^{th} path (complex)
τ_j - *strength* of the *field* in j^{th} path (real)

where is the particle — vector of field amplitudes — vector of field strengths

Ontic state space:

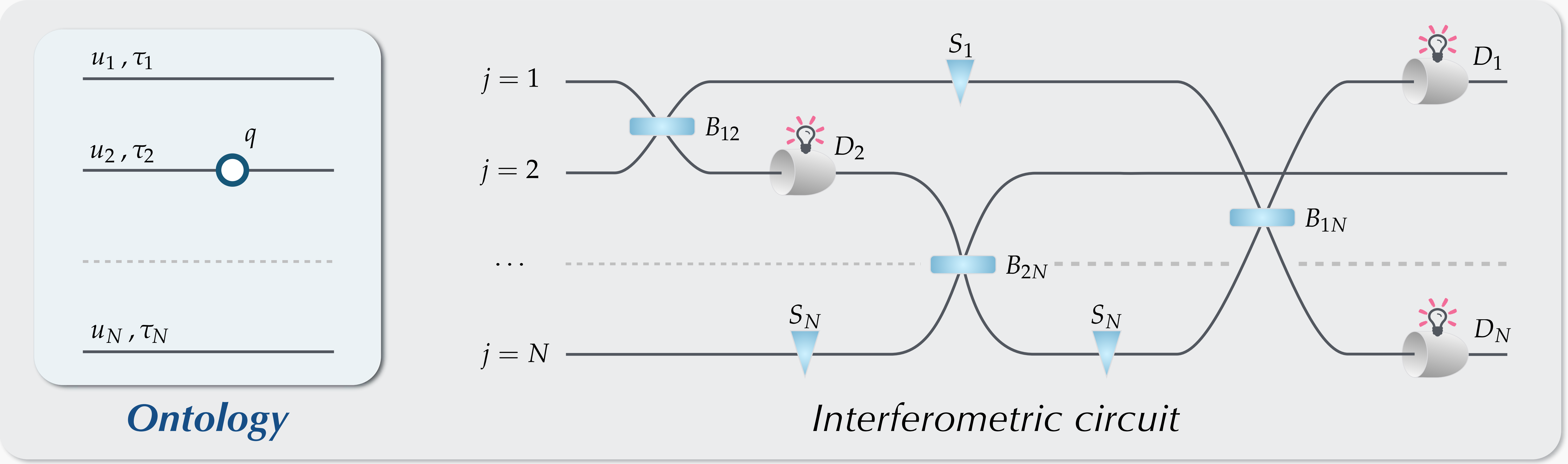
$$\Lambda = \{ q : q = 1, \dots, N \} \times \{ \vec{u} \in \mathbb{C}^N : |u_j| \leqslant 1 \} \times \{ \vec{\tau} \in \mathbb{R}^N : 0 \leqslant \tau_j \leqslant 1 \}$$

Epistemic state space:

$$\mathcal{P}(\Lambda) = \left\{ \boldsymbol{p} : \Lambda \longrightarrow [0,1] \; : \; \int_\Lambda \boldsymbol{p}(\lambda) \, d\lambda = 1 \right\}$$

Interferometric gates

Definitions & locality constraints



Free evolution:

(local, deterministic)

$$u_j \xrightarrow{\text{free}} u_j \quad \& \quad \tau_j \xrightarrow{\text{free}} \tau_j/2$$

"Ageing"

Phase shifter:

(local, deterministic)

$$u_j \xrightarrow{S_j} e^{i\omega} u_j \quad \& \quad \tau_j \xrightarrow{S_j} \tau_j/2$$

Detector:

(local, deterministic)

Measurement disturbance

$$u_j \xrightarrow{D_j} \begin{cases} 1 \\ u_j \end{cases} \quad \& \quad \tau_j \xrightarrow{D_j} \begin{cases} 1 & \text{if } q=j \\ 0 & \text{if } q\neq j \end{cases}$$

(*) **no jumps !!!**

Beam splitter:

(local, stochastic)

Amplitude-strength interaction

$$\begin{pmatrix} u_s \\ u_t \end{pmatrix} \xrightarrow{B_{st}} \begin{pmatrix} u'_s \\ u'_t \end{pmatrix} = \begin{pmatrix} i\sqrt{R} & \sqrt{T} \\ \sqrt{T} & i\sqrt{R} \end{pmatrix} \begin{pmatrix} \delta_{\tau_s \tau^{(st)}} & 0 \\ 0 & \delta_{\tau_t \tau^{(st)}} \end{pmatrix} \begin{pmatrix} u_j \\ u_k \end{pmatrix}$$

$$\tau_s\,,\tau_t \xrightarrow{B_{st}} \tau^{(st)}/2 \qquad \text{where: } \tau^{(st)} = \max\{\tau_s\,,\tau_t\}$$

"Level up"

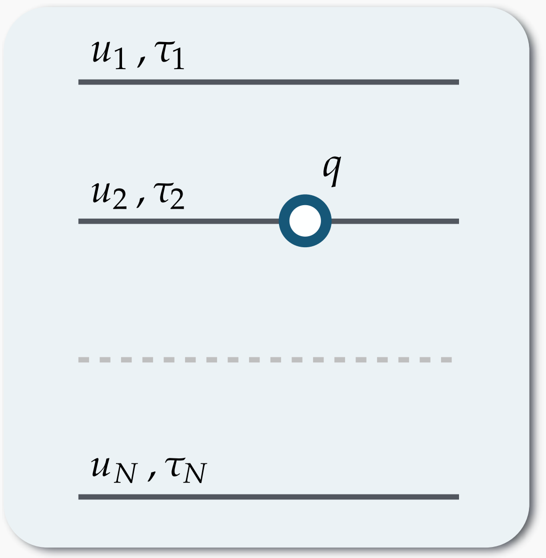
$$q \xrightarrow{B_{st}} \begin{cases} q' = s & \text{with probab. } \dfrac{|u'_s|^2}{|u'_s|^2+|u'_t|^2}\,, \\ q' = t & \text{with probab. } \dfrac{|u'_t|^2}{|u'_s|^2+|u'_t|^2}\,. \end{cases}$$

Stay or jump

Analysis of the model

Special subsets of ontic states

Ontic state space: $\Lambda = \{\, q \,:\, q = 1, \ldots, N \,\} \times \{\, \vec{u} \in \mathbb{C}^N \,:\, |u_j| \leqslant 1 \,\} \times \{\, \vec{\tau} \in \mathbb{R}^N \,:\, 0 \leqslant \tau_j \leqslant 1 \,\}$



Ontology

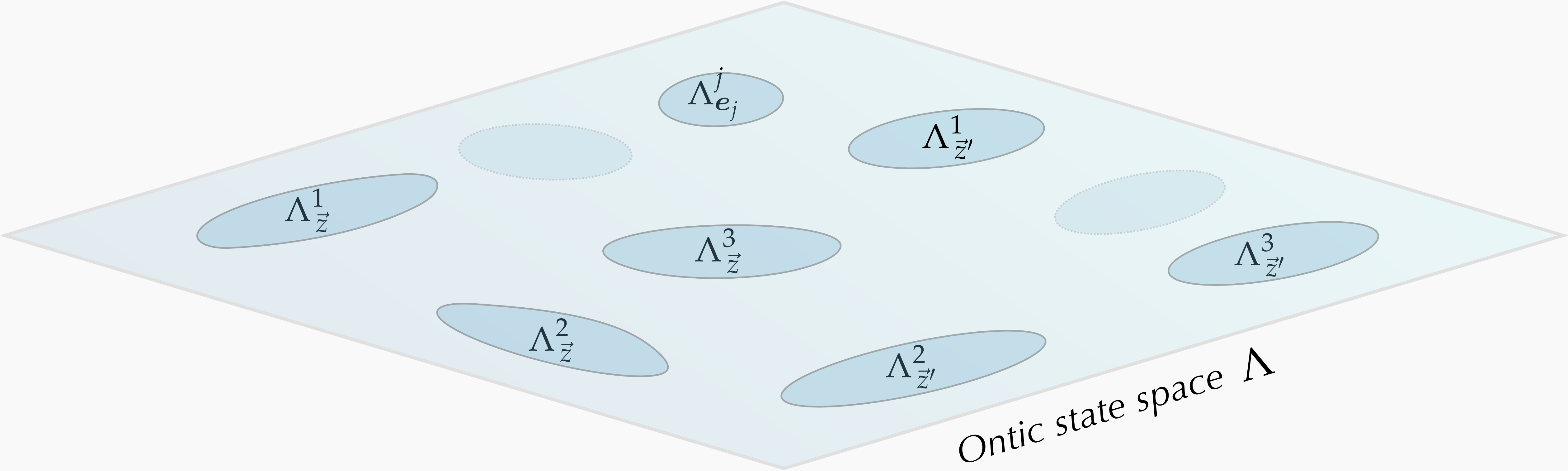
Definition

Let $i \in \{1, \ldots, N\}$ *and* $\vec{z} \in \mathbb{C}^N$.
Construct the following ***subsets*** $\Lambda^i_{\vec{z}} \subset \Lambda$ *of the ontic states:*

$$(q, \vec{u}, \vec{\tau}) \in \Lambda^i_{\vec{z}} \overset{df}{\Longleftrightarrow} \begin{cases} a) & q = i \\ b) & \tau_i = \tau > 0 \\ c) & \Delta_\tau \vec{u} \sim \vec{z} \end{cases}$$

where:

$$\tau := \max \{\tau_1, \ldots, \tau_N\}$$
$$\Delta_\tau := \operatorname{diag}(\delta_{\tau_1 \tau}, \ldots, \delta_{\tau_N \tau})$$



i.e.: a) *particle* ***present*** *in* i^{th} *path,*
b) *field in* i^{th} *path has* ***highest strength*** *(non-vanishing),*
c) *vector of field amplitudes with highest strength* $\Delta_\tau \vec{u}$ *is* ***proportional*** *to* $\vec{z}$.

Observation

For different labels i *and* $\vec{z}$ *(up to proportionality) these subsets are* ***disjoint***.

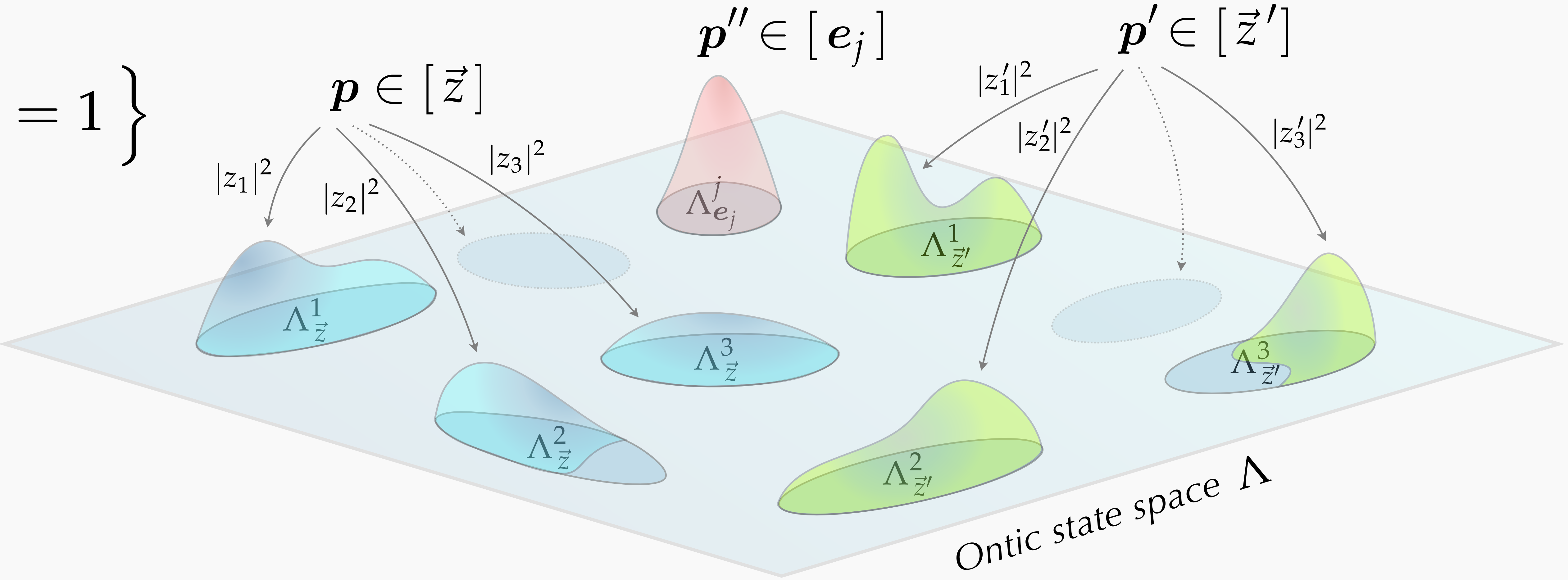
Analysis of the model

Special classes of distributions

Ontic state space:
$$\Lambda = \{\, q \,:\, q = 1,\ldots,N \,\} \times \{\, \vec{u} \in \mathbb{C}^N \,:\, |u_j| \leqslant 1 \,\} \times \{\, \vec{\tau} \in \mathbb{R}^N \,:\, 0 \leqslant \tau_j \leqslant 1 \,\}$$

Special subsets: $\Lambda^i_{\vec{z}} \subset \Lambda$ *with* $i \in \{1,\ldots,N\}$ *and* $\vec{z} \in \mathbb{C}^N$.

Epistemic states:
(probability distributions)
$$\mathcal{P}(\Lambda) = \left\{ \boldsymbol{p} : \Lambda \longrightarrow [0,1] \,:\, \int_\Lambda \boldsymbol{p}(\lambda)\, d\lambda = 1 \right\}$$

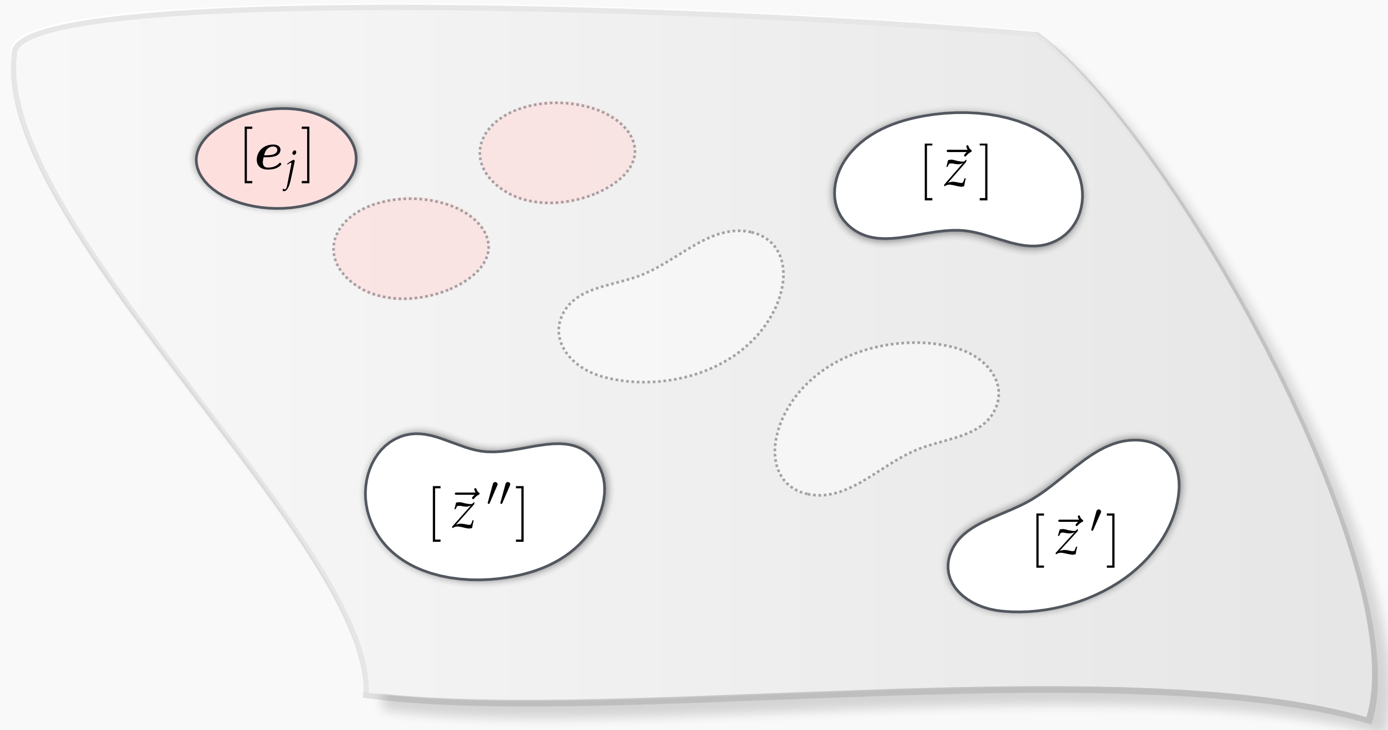


__**Definition**__

For each normalised $\vec{z} \in \mathbb{C}^N$ construct the following **class of distributions** $[\,\vec{z}\,] \subset \mathcal{P}(\Lambda)$ *over the ontic state space:*

$$\boldsymbol{p} \in [\,\vec{z}\,] \quad \overset{df}{\Longleftrightarrow} \quad \boldsymbol{p} = \sum_{i=1}^{N} |z_i|^2\, \boldsymbol{p}_i \qquad s.t. \qquad supp\,(\boldsymbol{p}_i) \subset \Lambda^i_{\vec{z}}\ .$$

Cumulative prob. over $\Lambda^i_{\vec{z}}$



__**Observation**__

For different label $\vec{z}$ *(up to proportionality) these classes are* ***disjoint*** *in* $\mathcal{P}(\Lambda)$.

Analysis of the model

Congruence of classes

Theorem

*Transformations implemented by **any configuration of gates** act **congruently** on the family of classes*

$$\left\{ [\vec{z}\,] \subset \mathcal{P}(\Lambda) \;:\; \vec{z} \in \mathbb{C}^N ,\ \|\vec{z}\| = 1 \right\}$$

i.e. classes transform as a whole

$$[\vec{z}\,] \ni \boldsymbol{p} \longrightarrow \boldsymbol{p}' \in [\vec{z}\,']$$

*with mapping $\vec{z} \longrightarrow \vec{z}\,'$ **determined** by the configuration of gates implemented in the circuit with the following rules:*

Free evolution: $\quad z_j \xrightarrow{free} z_j$

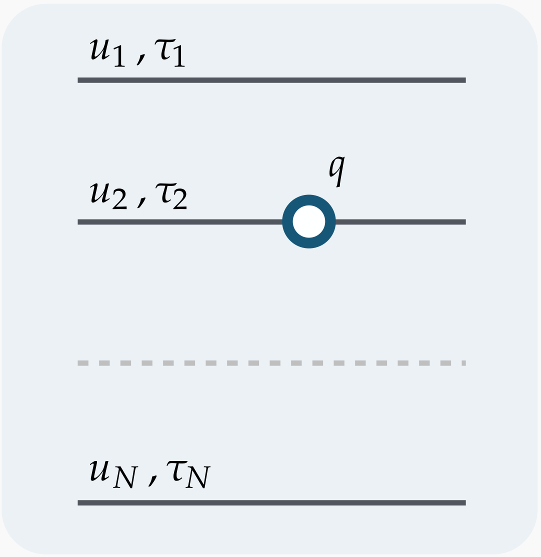
Phase shifters: $\quad z_j \xrightarrow{S_j} e^{i\omega}\, z_j$

Detectors: $\quad \vec{z} \xrightarrow{D_j} \begin{cases} \boldsymbol{e}_j & \text{'CLICK'}, \\ \dfrac{(\mathbb{1}-\mathbb{P}_j)\,\vec{z}}{\|(\mathbb{1}-\mathbb{P}_j)\,\vec{z}\,\|} & \text{'NO CLICK'}. \end{cases}$ *with probability* $Pr(D_j\,|\,\vec{z}\,) = |z_j|^2$

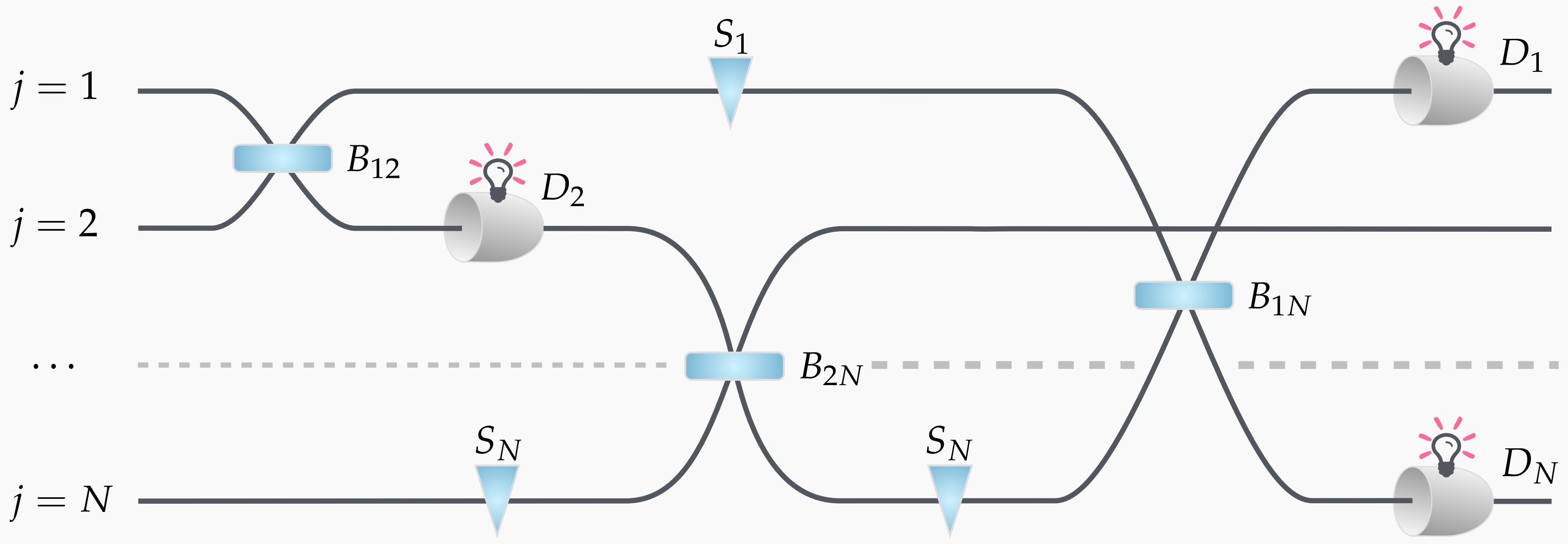
Beam splitters: $\quad \begin{pmatrix} z_s \\ z_t \end{pmatrix} \xrightarrow{B_{st}} \begin{pmatrix} i\sqrt{R} & \sqrt{T} \\ \sqrt{T} & i\sqrt{R} \end{pmatrix} \begin{pmatrix} z_s \\ z_t \end{pmatrix}$

Fact

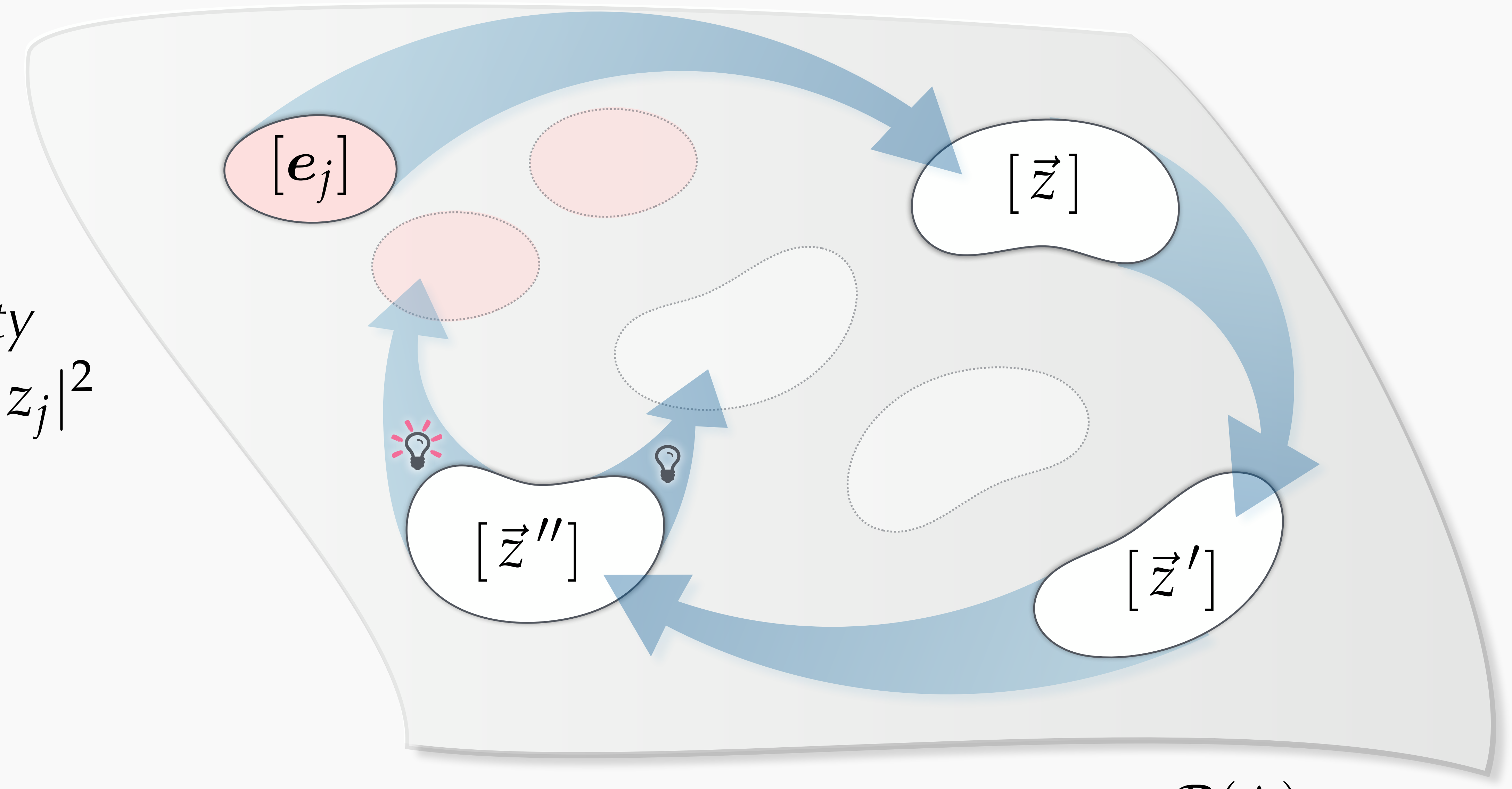
Initial preparation *of the system with a particle in j^{th} path starts off in a state* $\boldsymbol{p} \in [\,\boldsymbol{e}_j\,]$.



Ontology



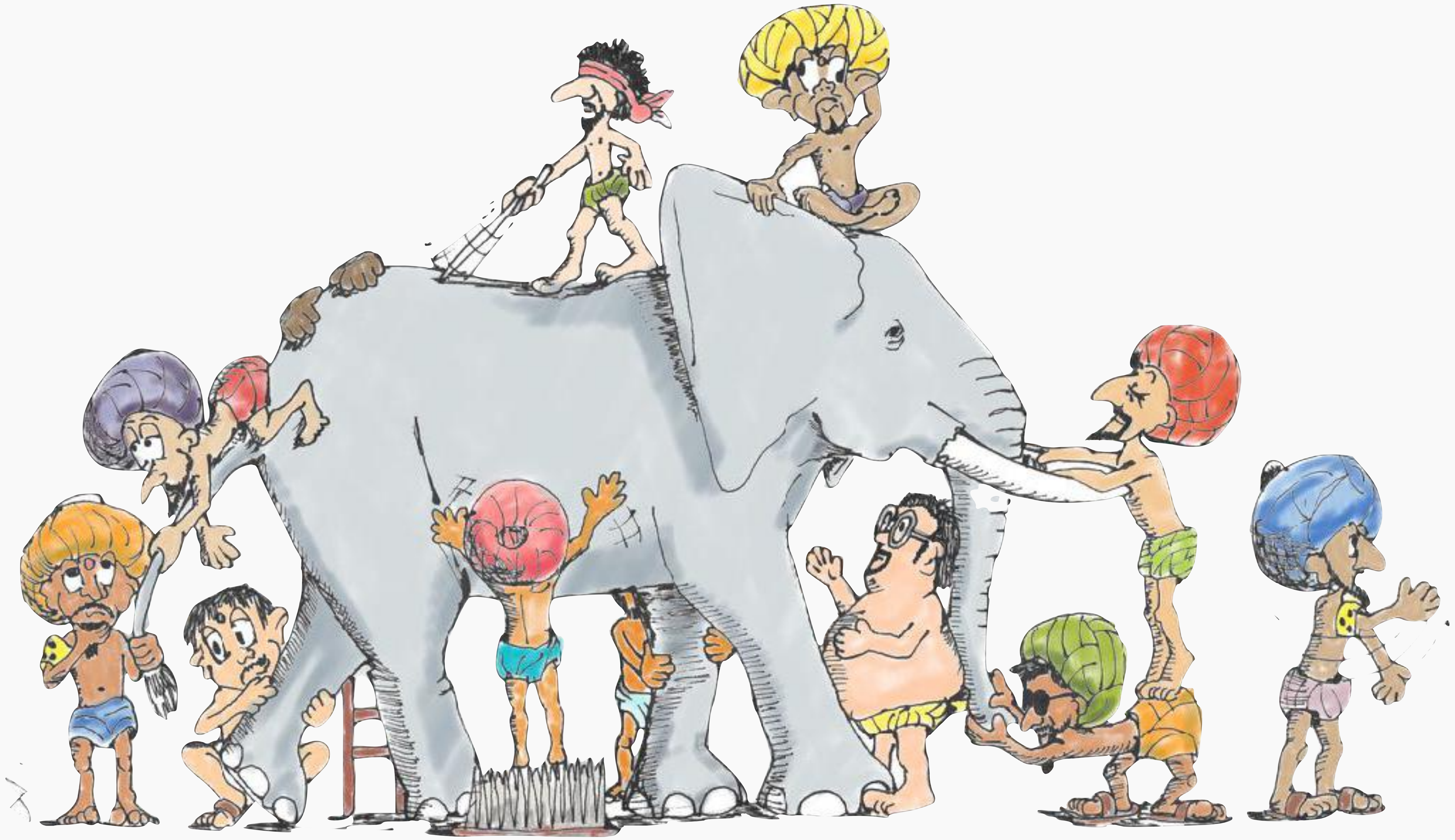
Interferometric circuit

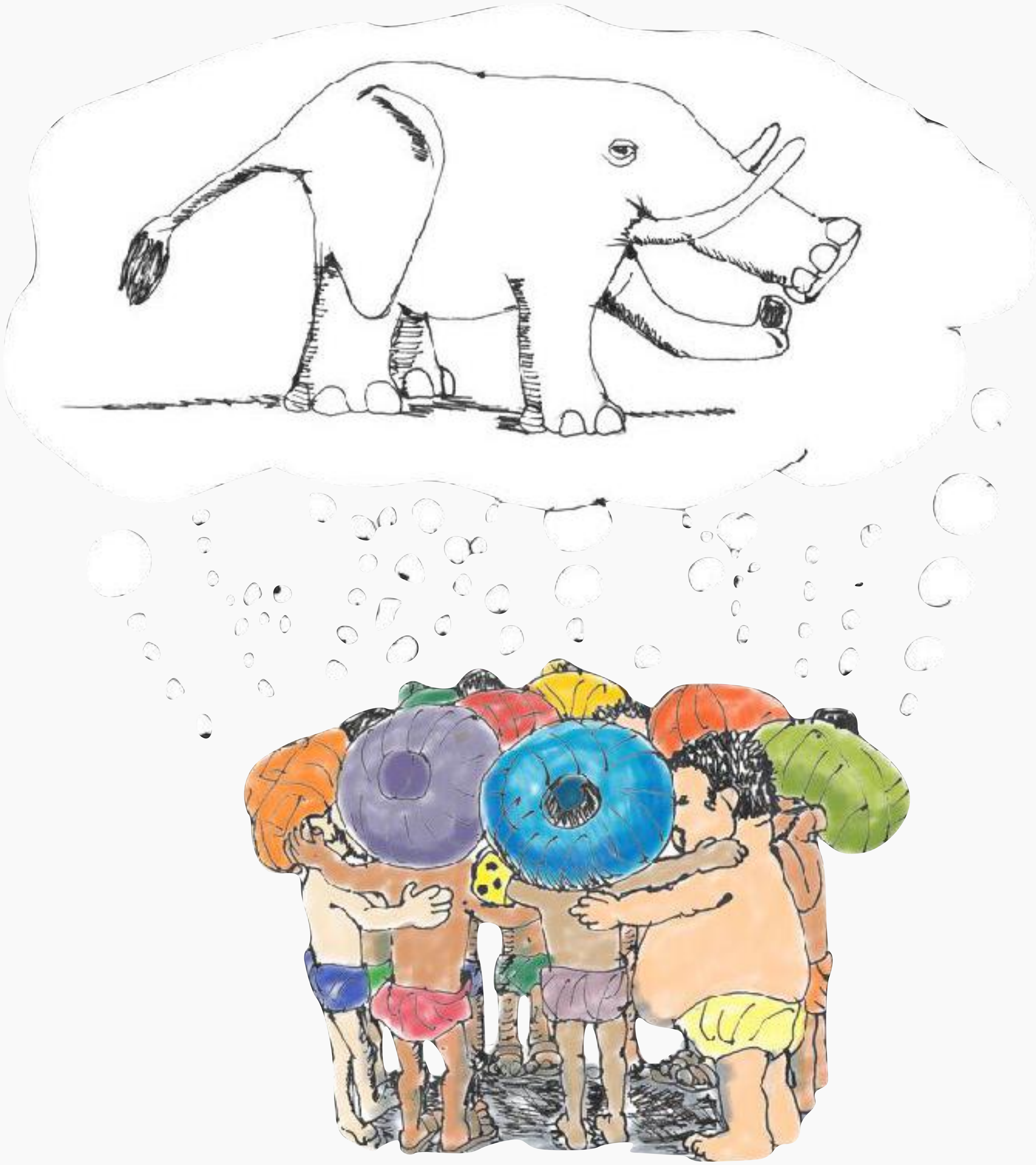


Epistemic state space $\mathcal{P}(\Lambda)$

Ontic vs. Epistemic

Blind man and an elephant





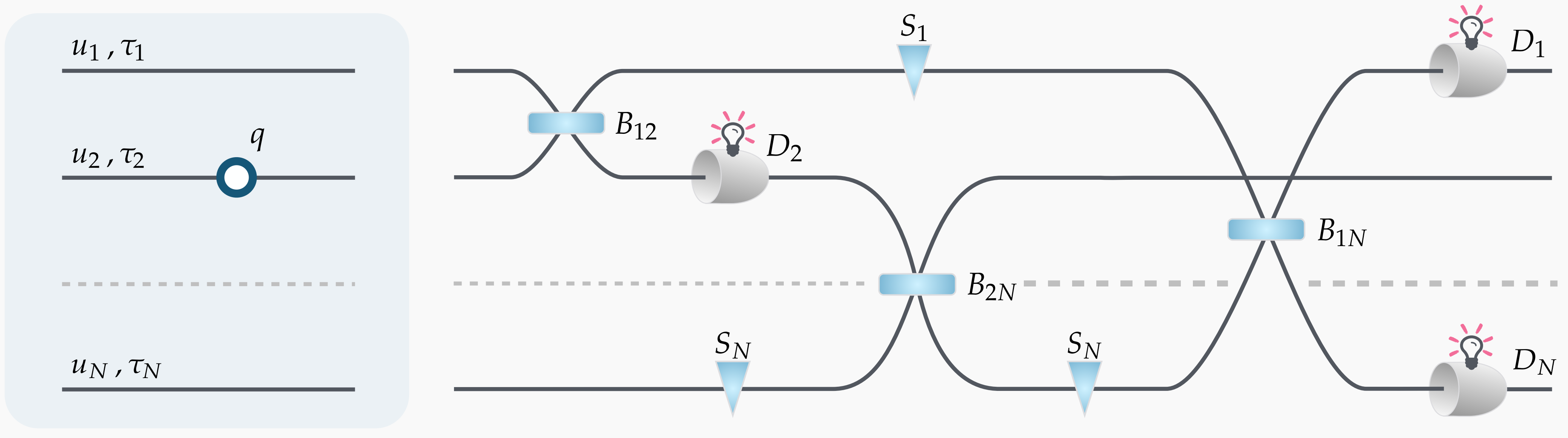
"We have to remember that what we observe is not nature in itself, but ***nature exposed to our method of questioning****."*

— Werner Heisenberg

Epistemic desideratum

Agent under constraints

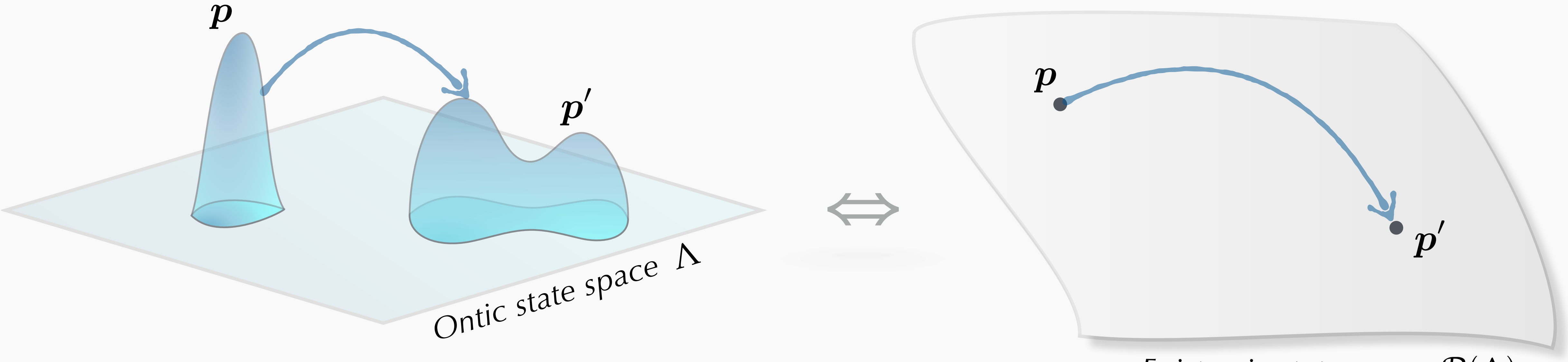
Ontic *perspective*



Ontology Interferometric circuits

$$\Lambda = \{ q : q = 1, \ldots, N \} \times \{ \vec{u} \in \mathbb{C}^N : |u_j| \leqslant 1 \} \times \{ \vec{\tau} \in \mathbb{R}^N : 0 \leqslant \tau_j \leqslant 1 \}$$

where is the particle — vector of field amplitudes — vector of field strengths

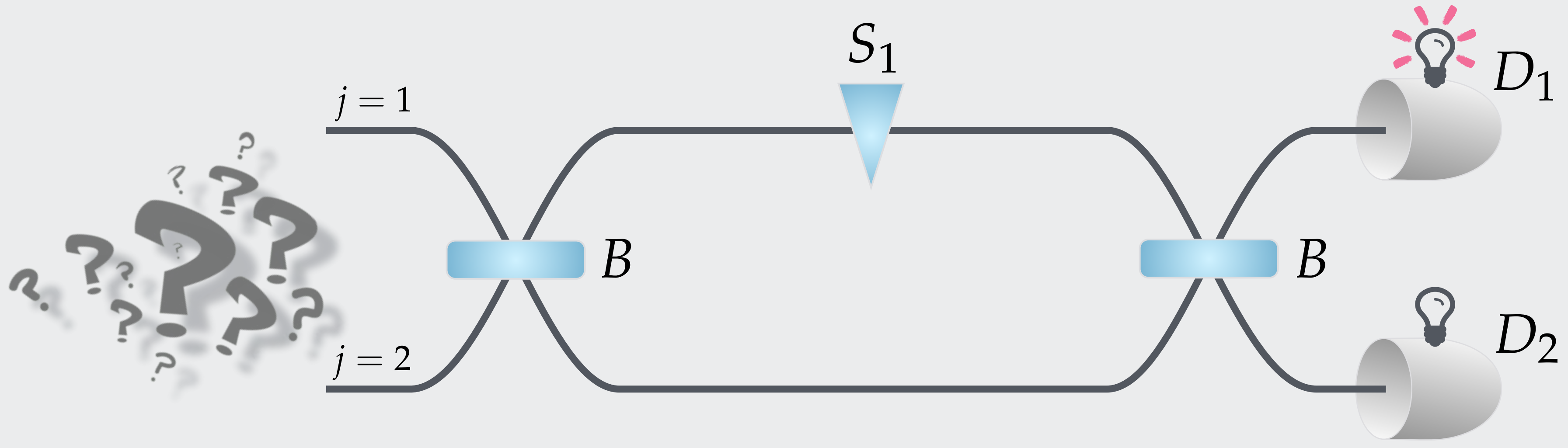


Epistemic state space $\mathcal{P}(\Lambda)$

Epistemic *perspective*

Available tools:

- *Phase shifters*
- *Beam splitters*
- *Detectors (post-selection)*
- *Probabilistic mixing*



*The agent '**sees**' the model only through experiments i.e. using only a **limited choice of gates**.*

Epistemic desideratum

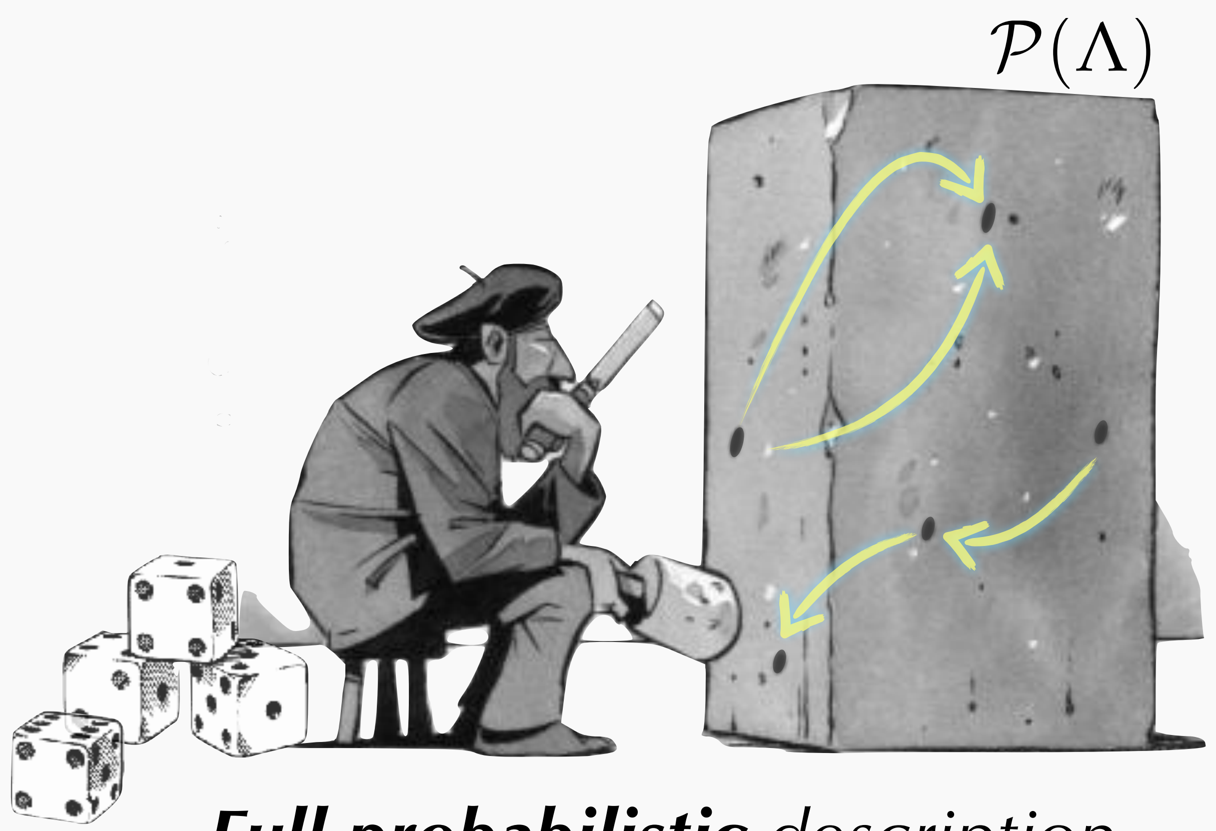
Agent under constraints

***Operational description** of the model*

*(i) Which distributions in $\mathcal{P}(\Lambda)$ **can be prepared** by the agent with limited tools at hand?*

*(ii) How do these distributions **transform under action of the gates** in the model?*

*(iii) What is the **minimal (operational) structure** which correctly describes predictions of the model?*



***Full probabilistic** description*

*What is the **geometry** of **accessible states***

***Epistemic** perspective*

Available tools:

Phase shifters

Beam splitters

Detectors (post-selection)

Probabilistic mixing

***Operational indifference principle*:**

*Distributions that are **not distinguishable** by means available to the agent, that is give the same probabilistic predictions for any conceivable experiment (circuit), are **equivalent** from the operational point of view.*

Epistemic desideratum

Operational account

Questions relevant for the description:

(i) *Which distributions in $\mathcal{P}(\Lambda)$* ***can be prepared*** *by the agent with limited resources at hand?*

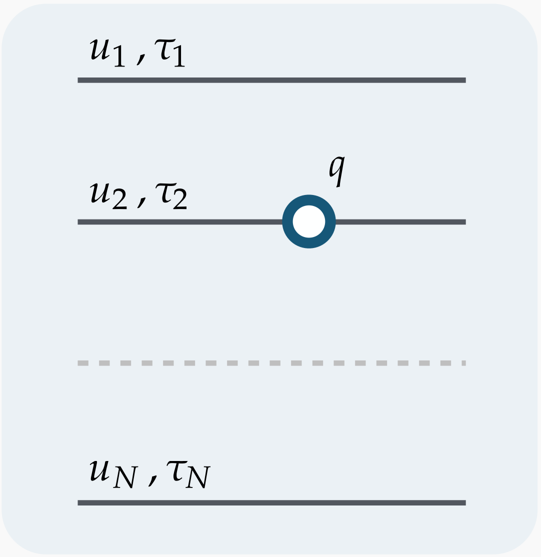
Answer: $$\bigcup \left\{ [\vec{z}] \, : \, \vec{z} \in \mathbb{C}^N , \, \|\vec{z}\| = 1 \right\} \subsetneq \mathcal{P}(\Lambda) \, .$$

(ii) *How do these distributions* ***transform under action of the gates*** *in the model?*

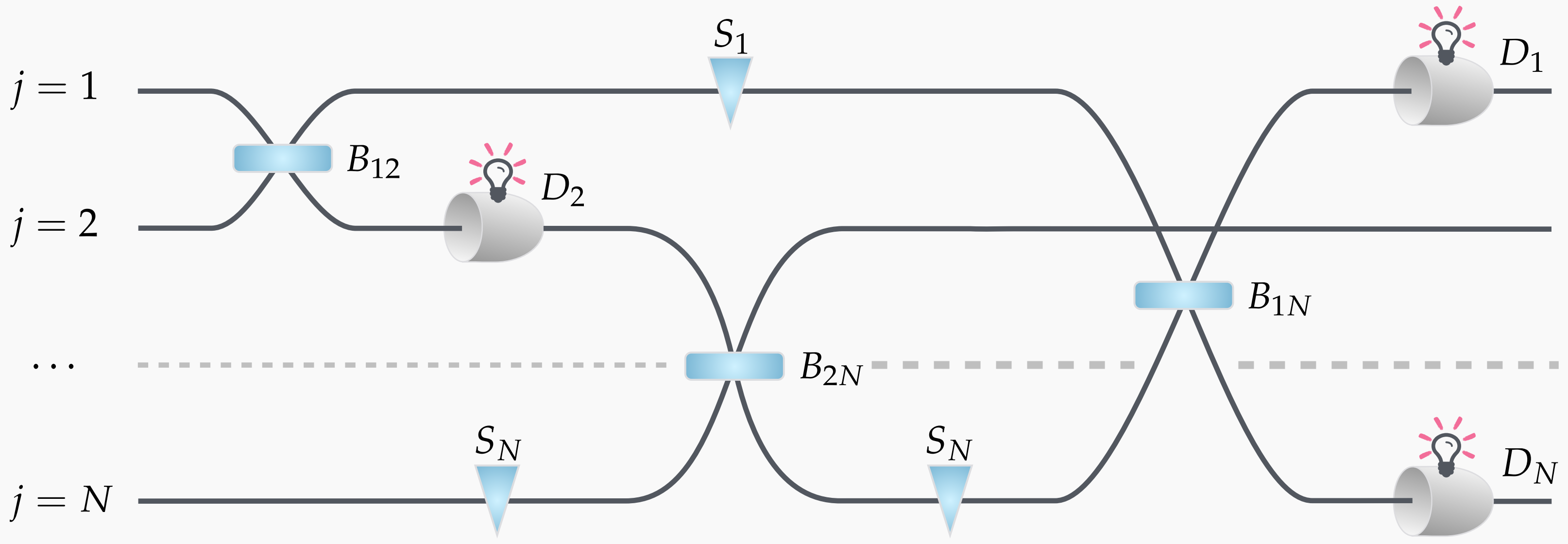
Answer: $[\vec{z}] \longrightarrow [\vec{z}\,']$ *like in QM.*

(iii) *What is the* ***minimal (operational) structure*** *which correctly describes predictions of the model?*

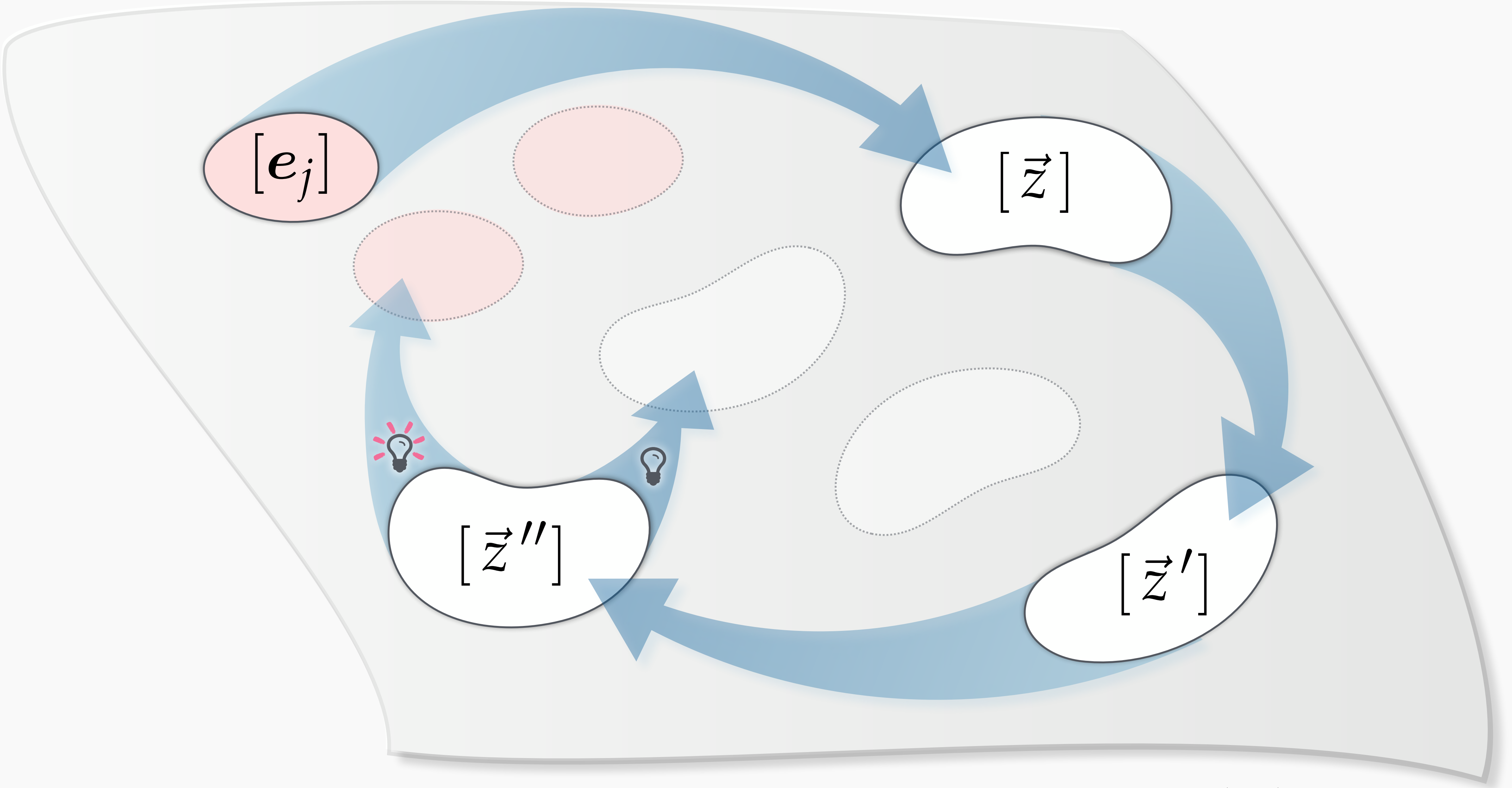
Answer: *Class label* $\vec{z} \in \mathbb{C}^N$ + *QM rules for* $\vec{z} \longrightarrow \vec{z}\,'$.



Ontology



Interferometric circuit

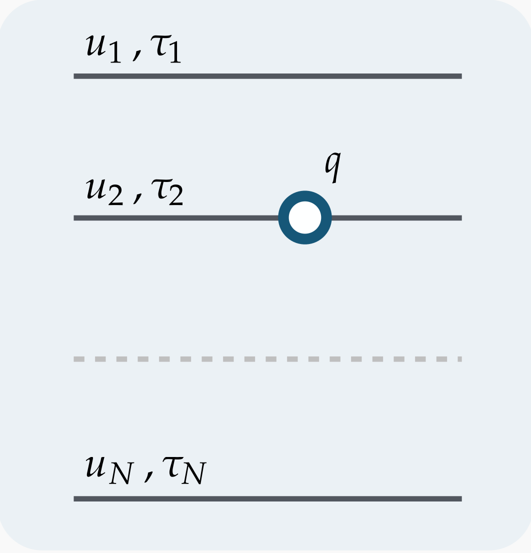


Epistemic state space $\mathcal{P}(\Lambda)$

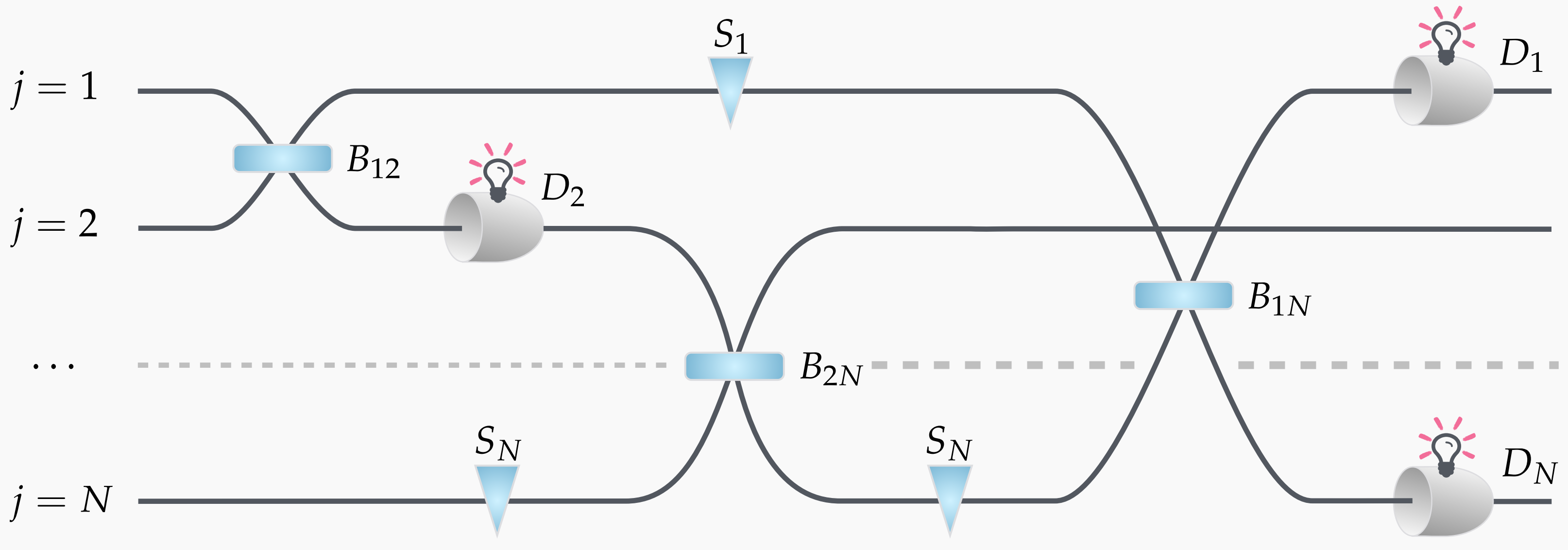
Questions relevant for the description:

(i) *Which distributions in $\mathcal{P}(\Lambda)$* ***can be prepared*** *by the agent with limited resources at hand?*

Answer: $\bigcup \left\{ [\vec{z}] \, : \, \vec{z} \in \mathbb{C}^N , \, \|\vec{z}\| = 1 \right\} \subsetneq \mathcal{P}(\Lambda) \, .$



Ontology



Interferometric circuit

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(iii) *What is the* ***minimal (operational) structure*** *which correctly describes predictions of the model?*

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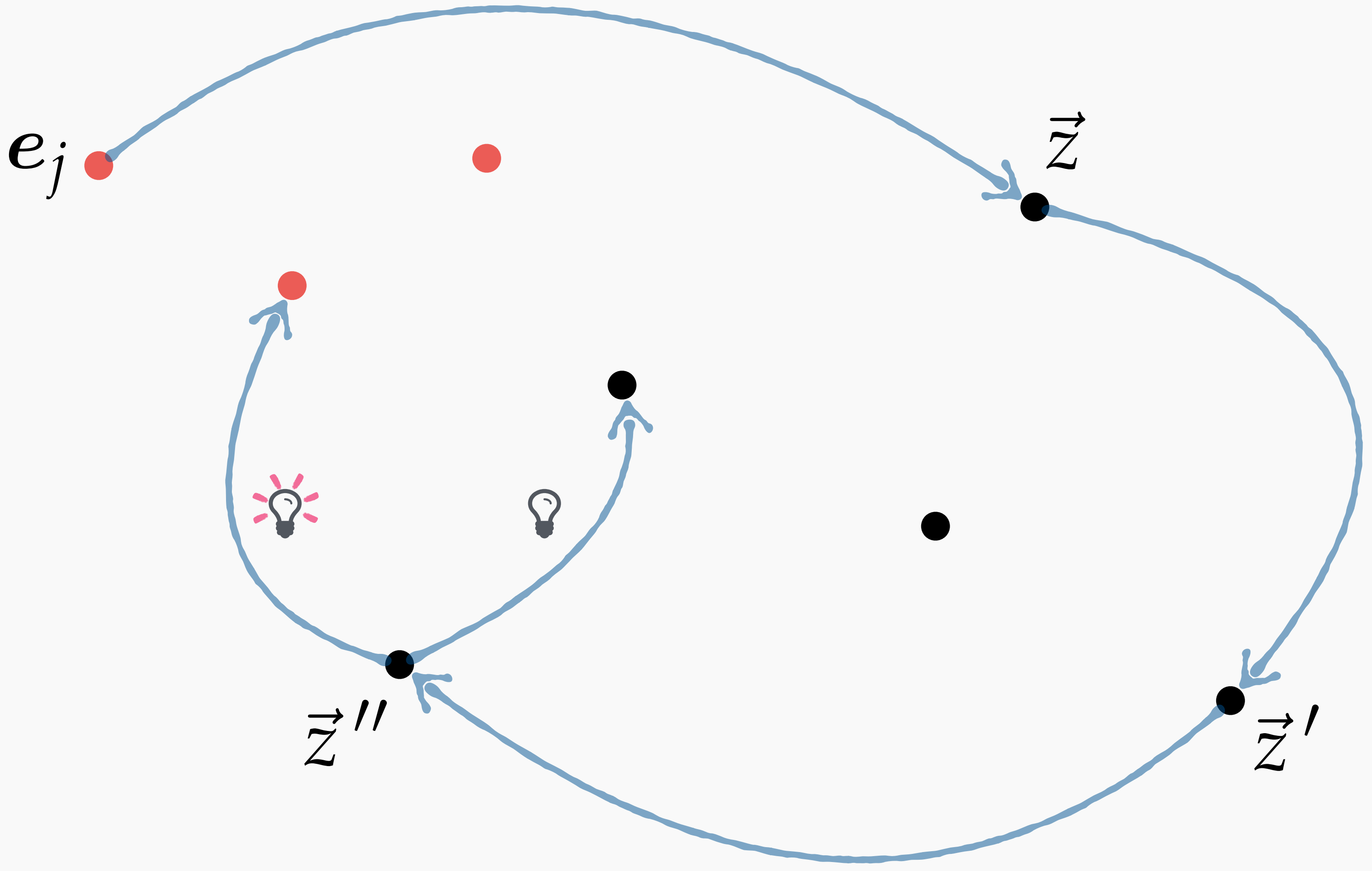
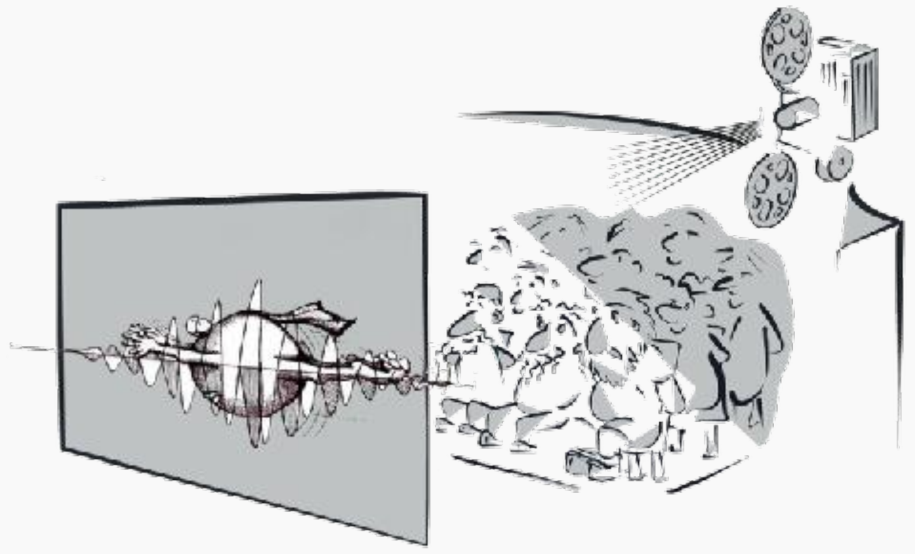
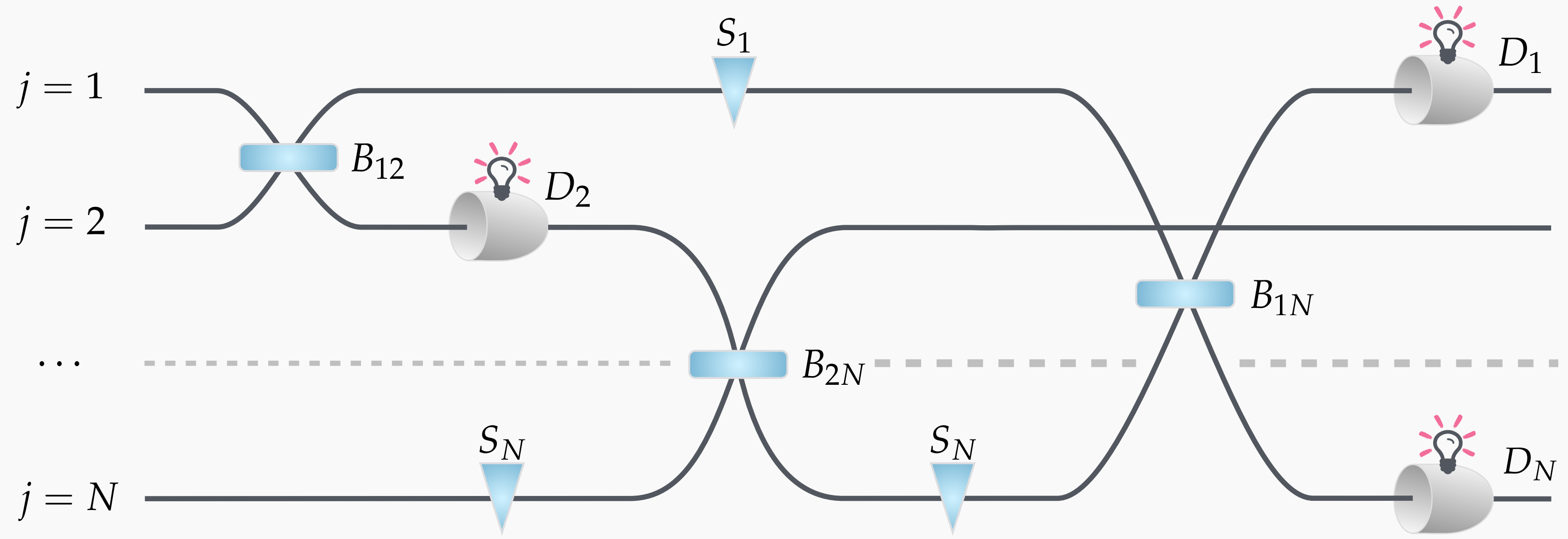


Operational *state space:* $\mathbb{C}^N$

Summary

Recovery of quantum predictions





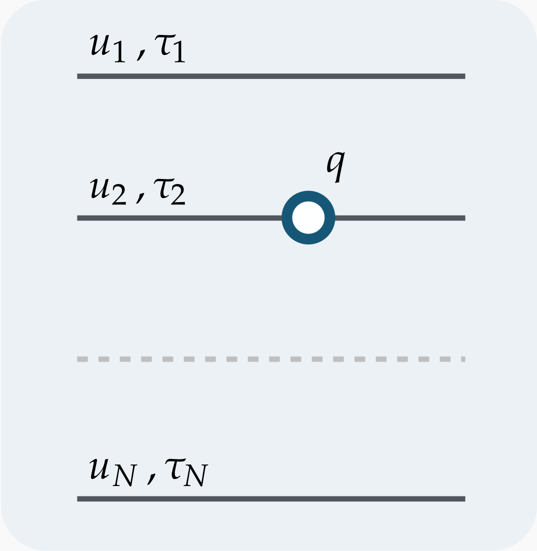
Interferometric circuits

Local ontological model

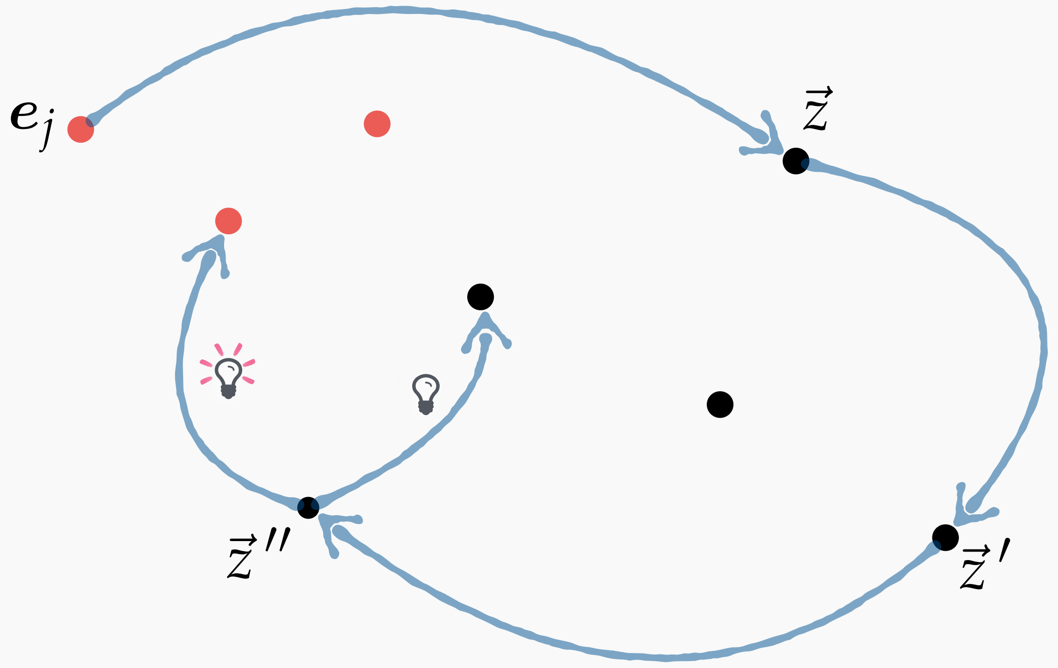
(hidden variables)

Quantum Mechanics

(single-particle framework)



Ontology



Ontology

$\vec{z} \leftrightsquigarrow \vec{\psi}$

Indistinguishable

Equivalent description !!
Same predictions !!

Operational description:

States: $[\vec{z}]$ s.t. $\vec{z} \in \mathbb{C}^N$

Transform.: $\vec{z} \longrightarrow \vec{z}\,'$ + Born's rule

Quantum description:

States: $|\vec{\psi}\rangle$ s.t. $\vec{\psi} \in \mathbb{C}^N$

Transform.: $\vec{\psi} \longrightarrow \vec{\psi}\,'$ + Born's rule

Conclusions

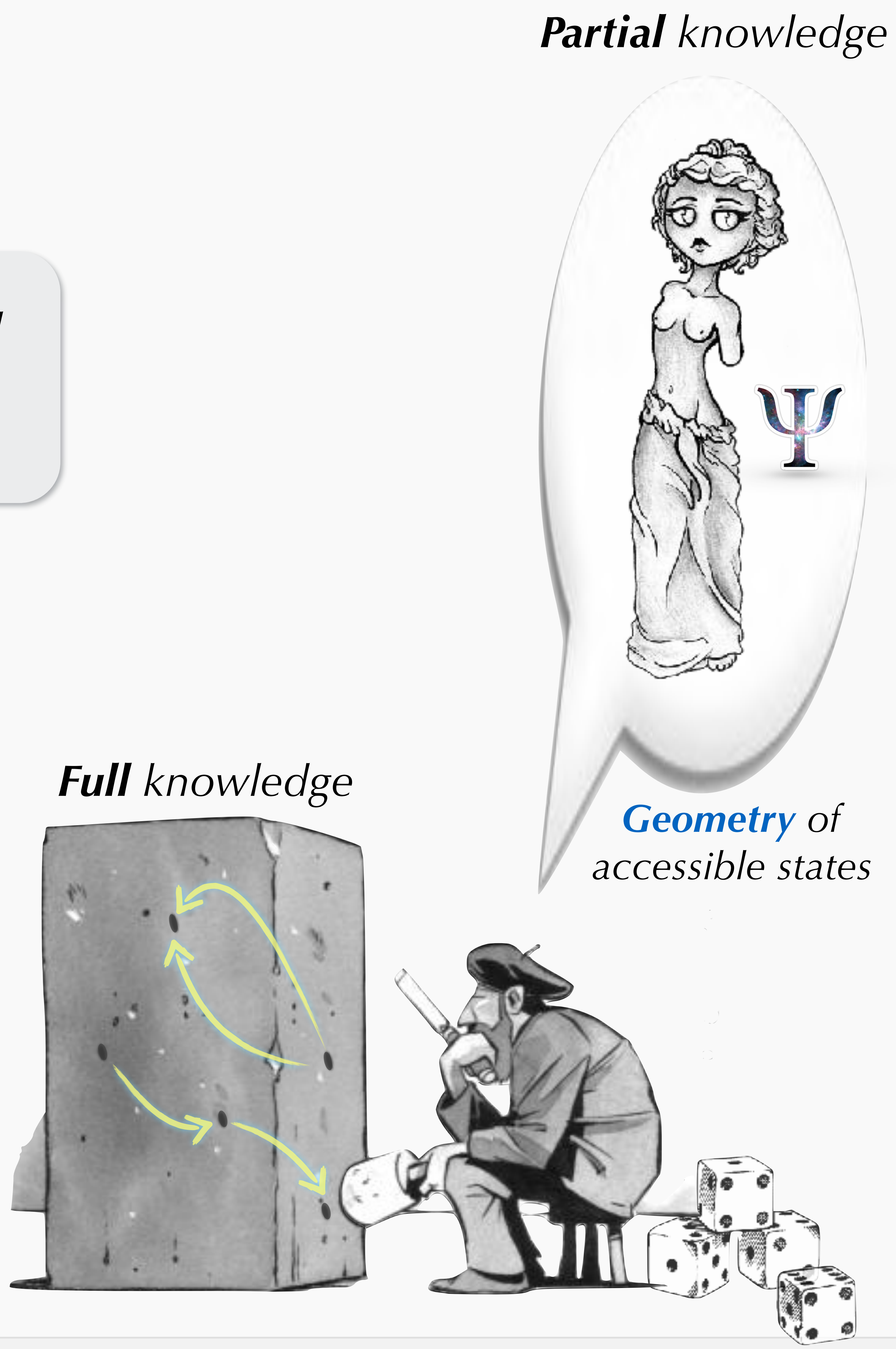
Take out message

- ***Restrictions on gaining knowledge*** ***add variety*** *to classical models*
  (perception of the system may dramatically change if resources are constrained).

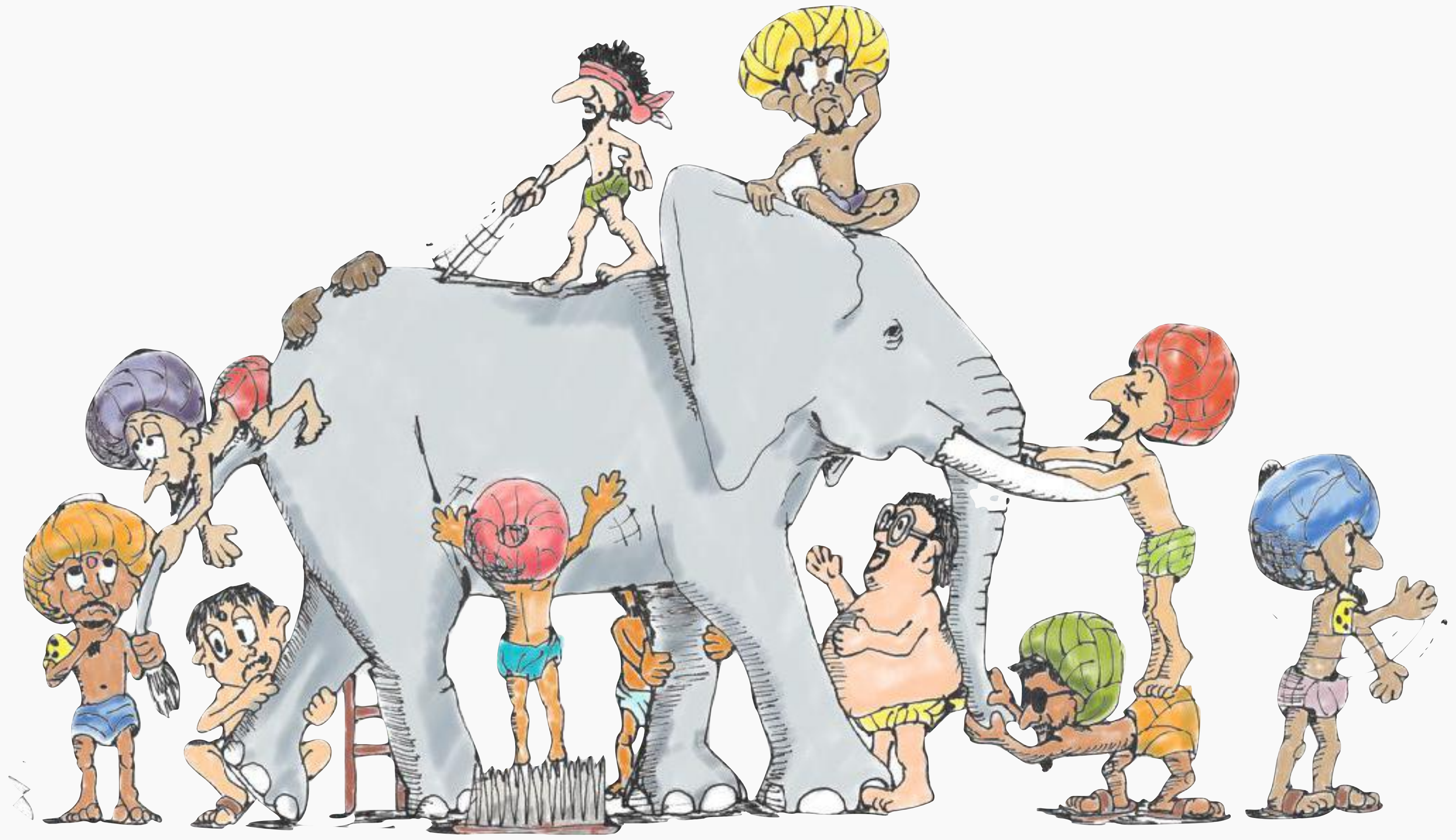
- ***Single-particle phenomena*** *are* ***not enough*** *to preclude local hidden variable model*
  (explicit ***counterexample*** *without spooky-action-at-a-distance).*

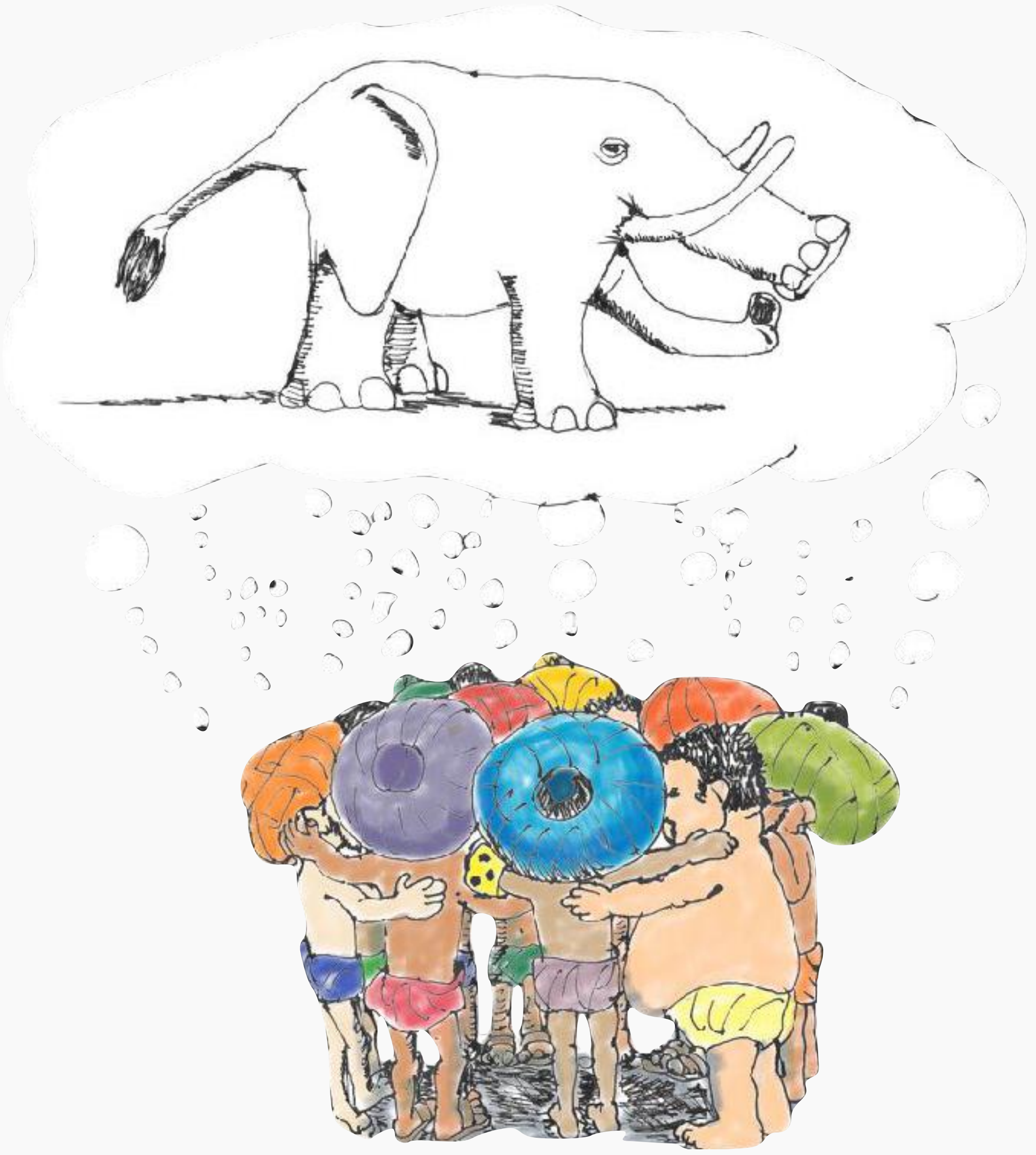
- ***Quantum interference, collapse of the wave function, contextuality, etc***
  have ***classical-like analogues*** *in models with epistemic constraints.*

- ***The real quantum mystery*** *should be sought in the* ***multi-particle*** *behaviour*
  (tensor products, entanglement, non-locality, Bell inequalities, etc…).



Thank you for your attention





*P. Blasiak “**Is single-particle interference spooky?**”*
arXiv:1701.02552 [quant-ph] (2017)

P. Blasiak “Ontological models with epistemic constraints: Local reconstruction of a dual-rail qubit”
Preprint (June 2017)

P. Blasiak “Local model of a qubit in the interferometric setup”
*New Journal of Physics **17** 113043 (2015)*

P. Blasiak “Classical systems can be contextual too: Analogue of the Mermin–Peres square”
*Annals of Physics **353** 326 (2015)*