Contextuality and indistinguishability

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Quantum Contextuality in Quantum Mechanics and Beyond

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Contextuality - single system

Goal: lack of explanation within NCHV theories
Contextuality - many systems

Goals: - find possible measurements
       - lack of explanation within NCHV theories
Distinguishability

Bell scenario, …, two independent single-system experiments
Indistinguishability

limited set of measurements

Measurement

001101101110111...

Does not reveal individual properties

Example:

\[ A_1 = 0, 1 \quad A_2 = 0, 1 \]

But we can only measure:

\[ A_1 + A_2 = 0, 1, 2 \]
Contextuality as lack of conservation

Two particles

- Distinguishable
- Identical

State independent contextually

Fermions vs Bosons

<table>
<thead>
<tr>
<th>N</th>
<th>Fermions</th>
<th>Bosons</th>
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<tbody>
<tr>
<td>0</td>
<td>No</td>
<td>No</td>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
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</tr>
<tr>
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</tr>
<tr>
<td>5</td>
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<td>Yes</td>
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<tr>
<td>...</td>
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Parity argument for bosons - M. Karczewski
Hardy-type contextuality - fermions

\[ |\psi\rangle = f_{67}^\dagger f_{69}^\dagger |0\rangle \]

\[ |\psi\rangle = \left( \frac{f_{39}^\dagger f_{23}^\dagger}{2\sqrt{2}} + \frac{f_{37}^\dagger f_{23}^\dagger}{4} - \frac{f_{37}^\dagger f_{39}^\dagger}{4} \right. \]

\[ - \left. \frac{3f_{34}^\dagger f_{23}^\dagger}{4} + \frac{f_{34}^\dagger f_{39}^\dagger}{4} - \frac{f_{34}^\dagger f_{37}^\dagger}{2\sqrt{2}} \right) |0\rangle \]
Hardy-type contextuality - bosons

\[ |\psi\rangle = \frac{b_{16}^\dagger}{\sqrt{2}} |0\rangle \]

\[ |\psi\rangle = \left( \frac{b_{45}^\dagger}{4\sqrt{2}} + \frac{b_{48}^\dagger}{4\sqrt{2}} - \frac{b_{47}^\dagger}{2\sqrt{2}} + \frac{b_{45}^\dagger b_{48}^\dagger}{4\sqrt{2}} - \frac{b_{45}^\dagger b_{47}^\dagger}{4} - \frac{b_{47}^\dagger b_{48}^\dagger}{4} \right) |0\rangle \]
Conclusions and open problems

• Indistinguishability restricts the set of measurements
• For more than one particle contextuality can be weaker
• In case of fermions the particle-hole symmetry is important

• Other dimensions and different number of modes
• Can one find examples for which there is no contextuality for $N=1$, but contextuality for $N>1$?
• What is the minimal contextual system for a given $N$?
• ...