Context-Dependence and Contextuality

Ehtibar N. Dzhafarov

Supported by AFOSR grant FA9550-14-1-0318

QCQMB 2017, Prague, Czech Republic

Acknowledging collaboration with

Janne V. Kujala, Matt Jones,

Victor Cervantes, Ru Zhang,

Acacio de Barros, Gary Oas

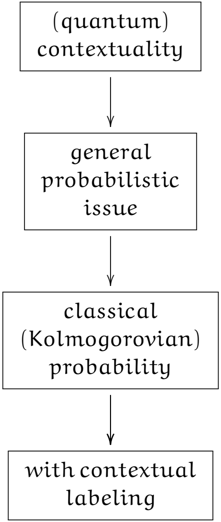
Benefited from discussions with

Samson Abramsky, Rui Soares Barbosa,

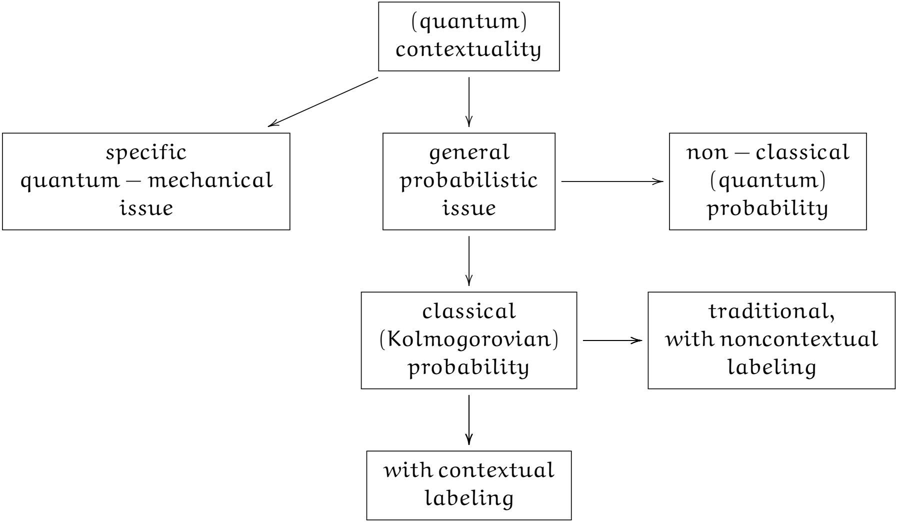
Pawel Kurzynski, Sam Staton,

Andrei Khrennikov, Federico Holic

Contextuality: My approach



Contextuality: Other Approaches



Contextuality-by-Default Theory: Three Forms of Context-Dependence

Contextuality-by-Default Theory: Three Forms of Context-Dependence

1. Contextual labeling (identity). Fundamental and universal.

Contextuality-by-Default Theory: Three Forms of Context-Dependence

1. Contextual labeling (identity). Fundamental and universal.
2. Direct (cross-) influences (inconsistent connectedness). Can be present or absent.

Contextuality-by-Default Theory: Three Forms of Context-Dependence

1. Contextual labeling (identity). Fundamental and universal.
2. Direct (cross-) influences (inconsistent connectedness). Can be present or absent.
3. Contextuality proper. Can be present or absent.

Contextuality-by-Default Theory: Contextual Labeling

1. Random variables are contextually labeled (i.e., their context is part of their identity).

R_1^1	R_2^1	$\cdot$	R_4^1	c_1
R_1^2	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	R_2^3	R_3^3	R_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{R}$

Contextuality-by-Default Theory: Contextual Labeling

1. Random variables are contextually labeled (i.e., their context is part of their identity).

R_1^1	R_2^1	$\cdot$	R_4^1	c_1
R_1^2	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	R_2^3	R_3^3	R_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{R}$

This implies that the sets of random variables in different contexts are disjoint.

Contextuality-by-Default Theory: Direct Influences

2. The distributions of two connected random variables (measuring the same content in different contexts) may be different.

Contextuality-by-Default Theory: Direct Influences

2. The distributions of two connected random variables (measuring the same content in different contexts) may be different.

R_1^1	R_2^1	$\cdot$	R_4^1	c_1
R_1^2	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	R_2^3	R_3^3	R_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{R}$

Contextuality-by-Default Theory: Direct Influences

2. The distributions of two connected random variables (measuring the same content in different contexts) may be different.

R_1^1	R_2^1	$\cdot$	R_4^1	c_1
R_1^2	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	R_2^3	R_3^3	R_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{R}$

Contextuality-by-Default Theory: Direct Influences

2. The distributions of two connected random variables (measuring the same content in different contexts) may be different.

R_1^1	R_2^1	$\cdot$	R_4^1	c_1
R_1^2	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	R_2^3	R_3^3	R_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{R}$

The interpretation is: "Direct" influences (cross-influences), Signaling, Disturbance, Context-dependent biases, etc.

Contextuality-by-Default Theory: Contextuality proper

3. Contextuality is present if the joint distributions within contexts are incompatible with certain joint distributions imposed on the content-sharing random variables.

Contextuality-by-Default Theory: Contextuality proper

3. Contextuality is present if the joint distributions within contexts are incompatible with certain joint distributions imposed on the content-sharing random variables.

R_1^1	R_2^1	$\cdot$	R_4^1	c_1
R_1^2	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	R_2^3	R_3^3	R_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{R}$

3. Contextuality is incompatibility of joint distributions of the bunched random variables with certain joint distributions imposed on the content-sharing random variables.

R_1^1	R_2^1	$\cdot$	R_4^1	c_1
R_1^2	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	R_2^3	R_3^3	R_4^3	c_3
q_1	q_2	q_3	q_4	$\boxed{\mathcal{R}}$

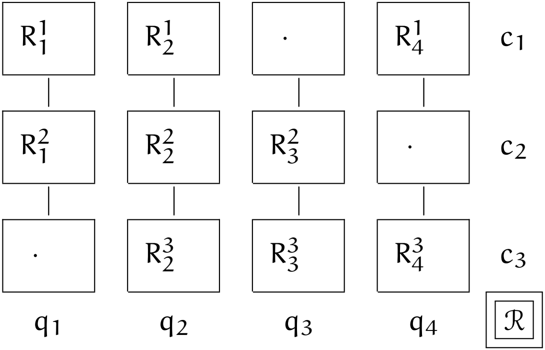
Contextuality-by-Default Theory: Contextuality proper

3. Contextuality is incompatibility of joint distributions of the bunched random variables with certain joint distributions imposed on the connected random variables.

R_1^1	R_2^1	$\cdot$	R_4^1	c_1
R_1^2	R_2^2	R_3^2	$\cdot$	c_2
$\cdot$	R_2^3	R_3^3	R_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{R}$

Contextuality-by-Default Theory: Contextuality proper

3. Contextuality is incompatibility of joint distributions of the bunched random variables with certain joint distributions imposed on the connected random variables.



Contextuality-by-Default Theory: Contextuality proper

3.Contextuality is incompatibility of joint distributions of the bunched random variables with certain couplings imposed on the connected random variables.

T_1^1	T_2^1	$\cdot$	T_4^1	c_1
T_1^2	T_2^2	T_3^2	$\cdot$	c_2
$\cdot$	T_2^3	T_3^3	T_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{T}$

Contextuality-by-Default Theory: Contextuality proper

3.Contextuality is incompatibility of joint distributions of the bunched random variables with certain couplings imposed on the connected random variables.

T_1^1	T_2^1	$\cdot$	T_4^1	c_1
T_1^2	T_2^2	T_3^2	$\cdot$	c_2
$\cdot$	T_2^3	T_3^3	T_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{T}$

Contextuality-by-Default Theory: Contextuality proper

3.Contextuality is incompatibility of joint distributions of the bunched random variables with certain couplings imposed on the connected random variables.

R_1^1	R_2^1	$\cdot$	R_4^1	c_1	
R_1^2	R_2^2	R_3^2	$\cdot$	c_2	
$\cdot$	R_2^3	R_3^3	R_4^3	c_3	
q_1	q_2	q_3	q_4		$\boxed{\mathcal{R}}$

vs

T_1^1	T_2^1	$\cdot$	T_4^1	c_1	
T_1^2	T_2^2	T_3^2	$\cdot$	c_2	
$\cdot$	T_2^3	T_3^3	T_4^3	c_3	
q_1	q_2	q_3	q_4		$\boxed{\mathcal{T}}$

Contextuality-by-Default Theory: Contextuality proper

3.Contextuality is incompatibility of joint distributions of the bunched random variables with certain couplings imposed on the connected random variables.

S_1^1	S_2^1	$\cdot$	S_4^1	c_1
S_1^2	S_2^2	S_3^2	$\cdot$	c_2
$\cdot$	S_2^3	S_3^3	S_4^3	c_3
q_1	q_2	q_3	q_4	$\boxed{\mathcal{S}}$

and

S_1^1	S_2^1	$\cdot$	S_4^1	c_1
S_1^2	S_2^2	S_3^2	$\cdot$	c_2
$\cdot$	S_2^3	S_3^3	S_4^3	c_3
q_1	q_2	q_3	q_4	$\boxed{\mathcal{S}}$

Contextuality-by-Default Theory: Contextuality proper

3. Contextuality is incompatibility of joint distributions of the bunched random variables with certain couplings imposed on the connected random variables.

S_1^1	S_2^1	$\cdot$	S_4^1	c_1
S_1^2	S_2^2	S_3^2	$\cdot$	c_2
$\cdot$	S_2^3	S_3^3	S_4^3	c_3
q_1	q_2	q_3	q_4	$\boxed{\boxed{S}}$

Contextuality-by-Default Theory: Contextuality proper

3. Contextuality is incompatibility of joint distributions of the bunched random variables with certain couplings imposed on the connected random variables.

S_1^1	S_2^1	$\cdot$	S_4^1	c_1
S_1^2	S_2^2	S_3^2	$\cdot$	c_2
$\cdot$	S_2^3	S_3^3	S_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{S}$

There is a universal measure of contextuality involving signed measures.

Contextuality-by-Default Theory: Multimaximal Couplings

T_1^1	T_2^1	$\cdot$	T_4^1	c_1
T_1^2	T_2^2	T_3^2	$\cdot$	c_2
$\cdot$	T_2^3	T_3^3	T_4^3	c_3
q_1	q_2	q_3	q_4	$\boxed{\mathcal{T}}$

Contextuality-by-Default Theory: Multimaximal Couplings

q_1	q_2	q_3	q_4	
T_1^1	T_2^1	$\cdot$	T_4^1	c_1
T_1^2	T_2^2	T_3^2	$\cdot$	c_2
$\cdot$	T_2^3	T_3^3	T_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{T}$

Couplings are imposed so that any two connected random variables are equal to each other with maximal possible probability. This is called a multi-maximal coupling. (*Can be generalized.*)

Necessity of Contextual Labeling: Traditional View is Contradictory

Noncontextual labeling is not an option, even in the absence of direct influences:

R_1	R_2	$\cdot$	R_4

R_1	R_2	R_3	$\cdot$

$\cdot$	R_2	R_3	R_4

?

$\Longrightarrow$

R_1	R_2	$\cdot$	R_4
R_1	R_2	R_3	$\cdot$
$\cdot$	R_2	R_3	R_4

?

Contextuality versus Inconsistent Connectedness

1. (Non)Contextuality may exist with or without direct influences.

Contextuality versus Inconsistent Connectedness

1. (Non)Contextuality may exist with or without direct influences.
2. Direct influences are causal, contextuality is correlational:

Contextuality versus Inconsistent Connectedness

1. (Non)Contextuality may exist with or without direct influences.
2. Direct influences are causal, contextuality is correlational:
    1. direct influences are manifested in changes of marginal distributions, contextuality is revealed on the level of joint distributions;

Contextuality versus Inconsistent Connectedness

1. (Non)Contextuality may exist with or without direct influences.
2. Direct influences are causal, contextuality is correlational:
    1. direct influences are manifested in changes of marginal distributions, contextuality is revealed on the level of joint distributions;
    2. contextuality can relate spacelike-separated measurements;

Contextuality versus Inconsistent Connectedness

1. (Non)Contextuality may exist with or without direct influences.
2. Direct influences are causal, contextuality is correlational:
   1. direct influences are manifested in changes of marginal distributions, contextuality is revealed on the level of joint distributions;
   2. contextuality can relate spacelike-separated measurements;
   3. with timelike separation, a future measurement can create an effective context for a past one.

Contextuality-by-Default Theory: Multimaximal Couplings

T_1^1	T_2^1	$\cdot$	T_4^1	c_1
T_1^2	T_2^2	T_3^2	$\cdot$	c_2
$\cdot$	T_2^3	T_3^3	T_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{T}$

Couplings are imposed so that any two connected random variables are equal to each other with maximal possible probability. This is called a multi-maximal coupling. (*Can be generalized.*)

Contextuality-by-Default Theory: Multimaximal Couplings

	q_1	q_2	q_3	q_4
c_1	T_1^1	T_2^1	$\cdot$	T_4^1
c_2	T_1^2	T_2^2	T_3^2	$\cdot$
c_3	$\cdot$	T_2^3	T_3^3	T_4^3

$\boxed{\mathcal{T}}$

Contextuality-by-Default Theory: Multimaximal Couplings

T_1^1	T_2^1	$\cdot$	T_4^1	c_1
T_1^2	T_2^2	T_3^2	$\cdot$	c_2
$\cdot$	T_2^3	T_3^3	T_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{T}$

In a well-designed theory a multi-maximal coupling always exists and is unique.

Contextuality-by-Default Theory: Multimaximal Couplings

T_1^1	T_2^1	$\cdot$	T_4^1	c_1
T_1^2	T_2^2	T_3^2	$\cdot$	c_2
$\cdot$	T_2^3	T_3^3	T_4^3	c_3
q_1	q_2	q_3	q_4	$\boxed{\boxed{\mathcal{T}}}$

In a well-designed theory a multi-maximal coupling always exists and is unique.

1. This is true if all random variables are binary (dichotomous).

Contextuality-by-Default Theory: Multimaximal Couplings

T_1^1	T_2^1	$\cdot$	T_4^1	c_1
T_1^2	T_2^2	T_3^2	$\cdot$	c_2
$\cdot$	T_2^3	T_3^3	T_4^3	c_3
q_1	q_2	q_3	q_4	$\mathcal{T}$

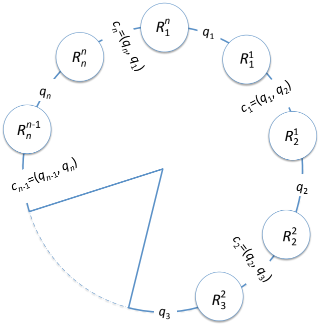
In a well-designed theory a multi-maximal coupling always exists and is unique.

1. This is true if all random variables are binary (dichotomous).
2. This is not generally true.

Example: Cyclic systems of binary random variables

R_1^1	R_2^1	$\cdot$	$\cdot$	$\cdots$	$\cdot$	$\cdot$	c_1
$\cdot$	R_2^2	R_3^2	$\cdot$	$\cdots$	$\cdot$	$\cdot$	c_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\iddots$	$\vdots$	$\vdots$	$\vdots$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdots$	R_{n-1}^{n-1}	R_n^{n-1}	c_{n-1}
R_1^n	$\cdot$	$\cdot$	$\cdot$	$\cdots$	$\cdot$	R_n^n	c_n
q_1	q_2	q_3	q_4	$\cdots$	q_{n-1}	q_n	$\boxed{\mathcal{CYC}_n}$

Example: Cyclic systems of binary random variables



- $n > 5$ investigated in psychophysics (Ru Zhang, Cervantes)
- $n = 5$— Klyachko-Can-Binicoglu-Shumovsky-type system
- $n = 4$— Einstein-Podolsky-Rosen/Bohm-Bell-type system
- $n = 3$— Suppes-Zanotti-Leggett-Garg-type system
- $n = 2$— question order (Moore-Wang-Busemeyer) type system

Example: Cyclic systems of binary random variables

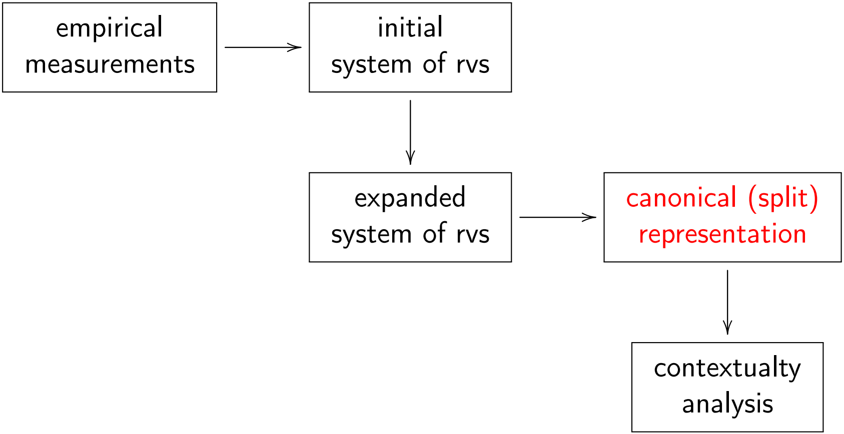
Theorem

A cyclic system of binary random variables is contextual if and only if

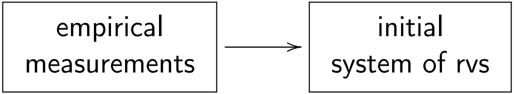
$$\max_{(\iota_1,\ldots,\iota_k)\in\{-1,1\}^n:\prod_{i=1}^n \iota_i=-1} \sum_{i=1}^{n} \iota_i \left\langle R_i^i R_{i\oplus 1}^i \right\rangle > n-2+\sum_{i=1}^{n} \left| \left\langle R_i^i \right\rangle - \left\langle R_i^{i\ominus 1} \right\rangle \right|.$$

Flow Chart of Contextuality Analysis

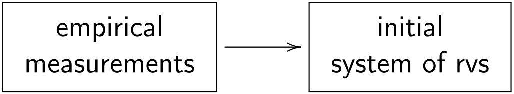
Flow Chart of Contextuality Analysis



Flow Chart of Contextuality Analysis

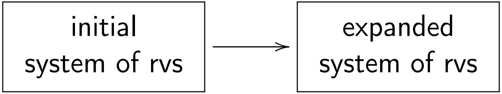


Flow Chart of Contextuality Analysis

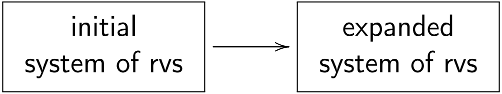


Ontic-epistemic interplay: the representation is non-unique (in many ways).

Flow Chart of Contextuality Analysis: Expansion through Joining



Flow Chart of Contextuality Analysis: Expansion through Joining



Joining:

R_1^1	R_2^1	$\cdot$	$c = 1$
R_1^2	R_2^2	$\cdot$	2
R_1^3	$\cdot$	R_3^3	3
$q = 1$	2	3	$\boxed{\mathcal{R}}$

$\Longrightarrow$

R_1^1	R_2^1	$\cdot$	(R_1^1, R_2^1)	$c = 1$
R_1^2	R_2^2	$\cdot$	(R_1^2, R_2^2)	2
R_1^3	$\cdot$	R_3^3	$\cdot$	3
$q = 1$	2	3	12	$\boxed{\mathcal{R}^*}$

Flow Chart of Contextuality Analysis: Expansion through Joining

initial system of rvs $\longrightarrow$ expanded system of rvs

Joining:

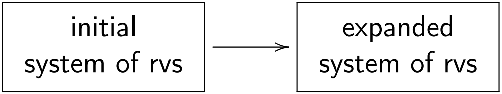
R_1^1	R_2^1	$\cdot$	$c = 1$	
R_1^2	R_2^2	$\cdot$	2	
R_1^3	$\cdot$	R_3^3	3	
$q = 1$	2	3	$\mathcal{R}$	

$\Longrightarrow$

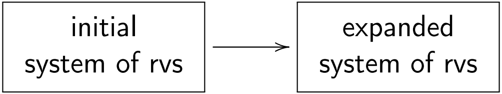
R_1^1	R_2^1	$\cdot$	(R_1^1, R_2^1)	$c = 1$
R_1^2	R_2^2	$\cdot$	(R_1^2, R_2^2)	2
R_1^3	$\cdot$	R_3^3	$\cdot$	3
$q = 1$	2	3	12	$\mathcal{R}^*$

Ontic-epistemic interplay: joining is non-unique and selective.
There are cases (e.g., question-order effect) when it is unwarranted.

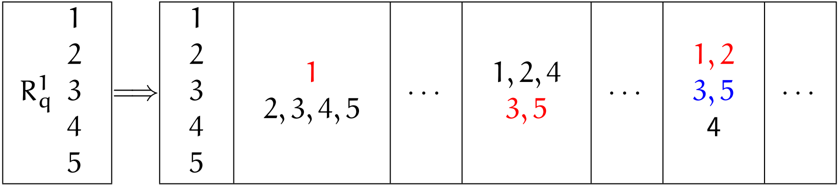
Flow Chart of Contextuality Analysis: Expansion through Coarsening



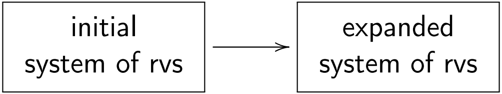
Flow Chart of Contextuality Analysis: Expansion through Coarsening



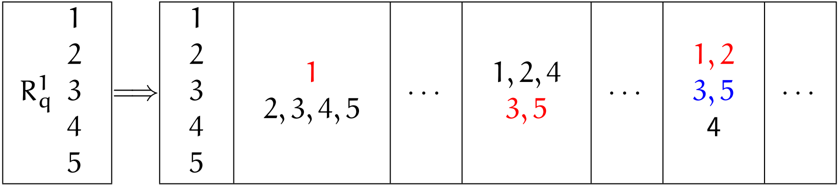
Coarsening (coarse-graining):



Flow Chart of Contextuality Analysis: Expansion through Coarsening



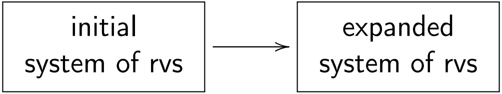
Coarsening (coarse-graining):



Ontic-epistemic interplay: coarsening is non-unique and selective.

Not all possible coarsenings may be of interest (depends on the internal structure, e.g., linear ordering or metric, of the space of possible values).

Flow Chart of Contextuality Analysis: Expansion through Coarsening

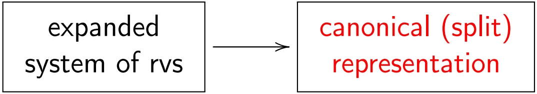


Coarsening (coarse-graining):

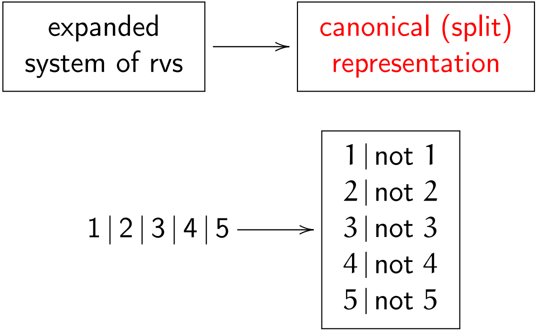
R_q^1								
1	$\Longrightarrow$	1						
2		2					1, 2	
3		3	1	$\dots$	1, 2, 3	$\dots$	3, 4	$\dots$
4		4	2, 3, 4, 5		4, 5		5	
5		5						

Ontic-epistemic interplay: coarsening is non-unique and selective.
Not all possible coarsenings may be of interest (depends on the internal structure, e.g., linear ordering or metric, of the space of possible values).

Flow Chart of Contextuality Analysis: Canonical Representation

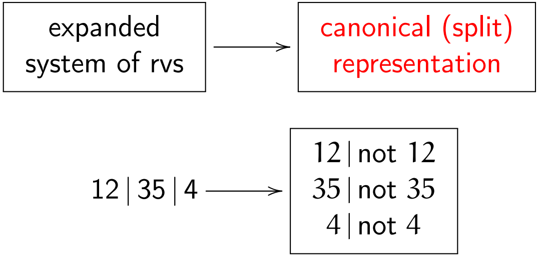


Flow Chart of Contextuality Analysis: Canonical Representation



Each random variable in the expanded system is replaced with its split representation, a binary random variable isolating one of the values from the rest (detecting this value).

Flow Chart of Contextuality Analysis: Canonical Representation



Each random variable in the expanded system is replaced with its split representation, a binary random variable isolating one of the values from the rest (detecting this value).

Flow Chart of Contextuality Analysis: Canonical Representation

R_1^1	$c = 1$
R_1^2	$c = 2$
$q = 1$	$\boxed{\mathcal{R}}$

$$\Longrightarrow$$

D_{W1}^1	D_{W2}^1		$D_{W(2^{k-1}-1)}^1$	$c = 1$
D_{W1}^2	D_{W2}^2	$\cdots$	$D_{W(2^{k-1}-1)}^2$	$c = 2$
$q = W1$	$W2$	$\cdots$	$W\left(2^{k-1}-1\right)$	$\boxed{\mathcal{D}}$

Flow Chart of Contextuality Analysis: Canonical Representation

R_1^1	$c = 1$
R_1^2	$c = 2$
$q = 1$	$\boxed{\mathcal{R}}$

$$\Longrightarrow$$

D^1_{W1}	D^1_{W2}		$D^1_{W(2^{k-1}-1)}$	$c = 1$
D^2_{W1}	D^2_{W2}	$\cdots$	$D^2_{W(2^{k-1}-1)}$	$c = 2$
$q = W1$	$W2$	$\cdots$	$W\left(2^{k-1}-1\right)$	$\boxed{\mathcal{D}}$

The support of the system does not change.

Flow Chart of Contextuality Analysis: Canonical Representation

R_1^1	$c = 1$
R_1^2	$c = 2$
$q = 1$	$\boxed{\mathcal{R}}$

$\Longrightarrow$

D_{W1}^1	D_{W2}^1		$D_{W(2^{k-1}-1)}^1$	$c = 1$
D_{W1}^2	D_{W2}^2	$\cdots$	$D_{W(2^{k-1}-1)}^2$	$c = 2$
$q = W1$	$W2$	$\cdots$	$W\left(2^{k-1} - 1\right)$	$\boxed{\mathcal{D}}$

Theorem

The system $\mathcal{D}$ is noncontextual if and only if one of the R_1^1 and R_1^2 nominally dominates the other.

Flow Chart of Contextuality Analysis: Canonical Representation

R_1^1	$c = 1$		D_{W1}^1	D_{W2}^1		$D_{W(2^{k-1}-1)}^1$	$c = 1$
R_1^2	$c = 2$	$\Longrightarrow$	D_{W1}^2	D_{W2}^2	$\cdots$	$D_{W(2^{k-1}-1)}^2$	$c = 2$
$q = 1$	$\boxed{\mathcal{R}}$		$q = W1$	$W2$	$\cdots$	$W\left(2^{k-1}-1\right)$	$\boxed{\mathcal{D}}$

Theorem

The system $\mathcal{D}$ is noncontextual if and only if one of the R_1^1 and R_1^2 nominally dominates the other.

Definition

R_1^1 *nominally dominates* R_1^2 if $\Pr\left[R_1^1 = i\right] < \Pr\left[R_1^2 = i\right]$ for no more than one value of $i = 1, \ldots, k$.

Flow Chart of Contextuality Analysis: Canonical Representation

R_1^1	$c=1$		D^1_{W1}	D^1_{W2}		$D^1_{W(2^{k-1}-1)}$	$c=1$
R_1^2	$c=2$	$\Longrightarrow$	D^2_{W1}	D^2_{W2}	$\cdots$	$D^2_{W(2^{k-1}-1)}$	$c=2$
$q=1$	$\boxed{\mathcal{R}}$		$q=W1$	$W2$	$\cdots$	$W\left(2^{k-1}-1\right)$	$\boxed{\mathcal{D}}$

Theorem

The system $\mathcal{D}$ is noncontextual if and only if one of the R_1^1 and R_1^2 nominally dominates the other.

Example

x:	1	2	3	4	5
$\Pr\left[R_1^1=x\right]$	.2	.3	.1	.3	.1
$\Pr\left[R_1^2=x\right]$	.1	.2	.4	.3	0

Flow Chart of Contextuality Analysis: Canonical Representation

R_1^1	$c = 1$		D_{W1}^1	D_{W2}^1		$D_{W(2^{k-1}-1)}^1$	$c = 1$
R_1^2	$c = 2$	$\Longrightarrow$	D_{W1}^2	D_{W2}^2	$\cdots$	$D_{W(2^{k-1}-1)}^2$	$c = 2$
$q = 1$	$\boxed{\mathcal{R}}$		$q = W1$	$W2$	$\cdots$	$W\left(2^{k-1}-1\right)$	$\boxed{\mathcal{D}}$

Theorem

The system $\mathcal{D}$ is noncontextual if and only if one of the R_1^1 and R_1^2 nominally dominates the other.

Example

x:	1	2	3	4	5
$\Pr\left[R_1^1 = x\right]$	.2	.3	.1	.3	.1
$\Pr\left[R_1^2 = x\right]$	.1	.2	.4	.3	0

Measure of contextuality: Quasi-couplings

D_1^1	D_2^1	$\cdot$	D_4^1

D_1^2	D_2^2	D_3^2	$\cdot$

$\cdot$	D_2^3	D_3^3	D_4^3

$\boxed{\mathcal{D}}$

$\Longrightarrow$

S_1^1	S_2^1		S_4^1
S_1^2	S_2^2	S_3^2	$\cdot$
$\cdot$	S_2^3	S_3^3	S_4^3

$\boxed{S}$

TOTAL VARIATION:

$$\sum \left| p\left(s_1^1,\, s_2^1,\, s_4^1,\, s_1^2,\, s_2^2,\, s_3^2,\, s_2^3,\, s_3^3,\, s_4^3 \right) \right| = V_T \geqslant 1$$

$V_T = 1$ if and only if the quasi-coupling is a proper coupling

Measure of contextuality

Theorem

Any system $\mathcal{R}$ of random variables has a quasi-coupling S such that
(i) any pair $\left(S_q^c, S_q^{c'}\right)$ *is properly distributed, and the probability of* $S_q^c = S_q^{c'}$ *has the maximal possible value;*
(ii) the value of total variation V_T *in* S *has the smallest possible value among all quasi-couplings satisfying (i).*

Measure of contextuality

Theorem

Any system $\mathcal{R}$ of random variables has a quasi-coupling S such that
(i) any pair $\left(S_q^c, S_q^{c'}\right)$ is properly distributed, and the probability of $S_q^c = S_q^{c'}$ has the maximal possible value;
(ii) the value of total variation V_T in S has the smallest possible value among all quasi-couplings satisfying (i).

The minimum value of $V_T - 1$ can be taken as a universal measure of contextuality in any system of random variables.

Its value is 0 if and only if the system is not contextual.

Dummy measurements

D_1^1	D_2^1	$\cdot$	D_4^1

D_1^2	D_2^2	D_3^2	$\cdot$

$\cdot$	D_2^3	D_3^3	D_4^3

$\boxed{\mathcal{D}}$

$\Longrightarrow$

D_1^1	D_2^1	C_3^1	D_4^1

D_1^2	D_2^2	D_3^2	C_4^2

C_1^3	D_2^3	D_3^3	D_4^3

$\boxed{\mathcal{D}'}$

Dummy measurements

D_1^1	D_2^1	$\cdot$	D_4^1
D_1^2	D_2^2	D_3^2	$\cdot$
$\cdot$	D_2^3	D_3^3	D_4^3

$\mathcal{D}$

$\Longrightarrow$

D_1^1	D_2^1	C_3^1	D_4^1
D_1^2	D_2^2	D_3^2	C_4^2
C_1^3	D_2^3	D_3^3	D_4^3

$\mathcal{D}'$

Theorem

Contextuality of a system does not change if every context-content pair associated with no measurement is assigned a deterministic random variable ("dummy measurement").