Exclusivity Principle Determines the Correlations Monogamy

Zhih-Ahn Jia(贾治安)
Key Laboratory of Quantum Information,
University of Science and Technology of China
QCQMB, Prague, June 5, 2017

Outline

- ☐ INTERPRETATION OF QUANTUM MECHANICS
- ☐ GENERALIZED PROBABILITY THEORY
- ☐ EXCLUSIVITY PRINCIPLE(EP)
- ☐ APPLICATIONS OF EP (ESPECICALLY ON MONOGAMY RELATIONS)

Main refereces:

[1] Zhih-Ahn Jia, Yu-Chun Wu, and Guang-Can Guo, Monogamy relations in no-disturbance theories, PhysRevA.94.012111;

[2] Zhih-Ahn Jia, Gao-Di Cai, Yu-Chun Wu, Guang-Can Guo and Adán Cabello, Exclusivity principle determines the correlation monogamy, preprint.

Outline

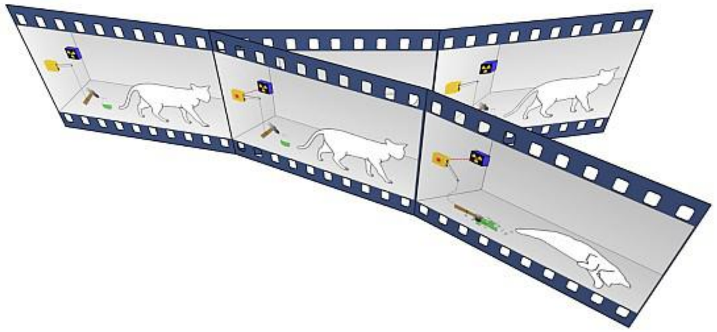
- ■ Interpretation of quantum mechanics
- □ Generalized probability theory
- □ Exclusivity principle(EP)
- □ Applications of EP (especically on monogamy relations)

Interpretation of quantum mechanics

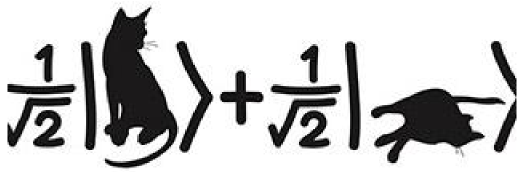
- The Copenhagen interpretation 
- Many worlds interpretation 
- De Broglie–Bohm interpretation 
- Decoherent histories interpretation 
- Quantum Darwinism 
- Quantum Bayesianism 

Interpretation of quantum mechanics

- Search for suit physical principles



- Measurement problems
- Interprete the statistics, Born's rule, Lüder's rule
- Interprete the reality of state



- Emerge classical world from quantum mechanics

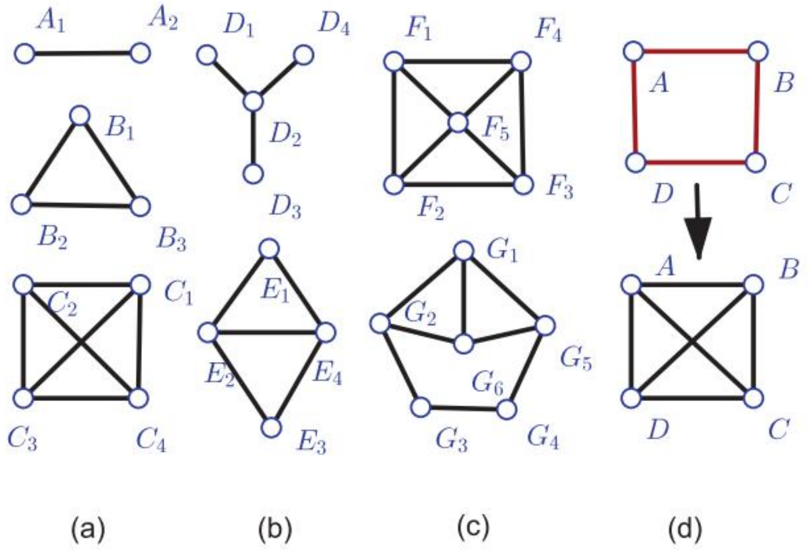
Outline

- ☐ INTERPRETATION OF QUANTUM MECHANICS
- ■ **GENERALIZED PROBABILITY THEORY**
- ☐ EXCLUSIVITY PRINCIPLE(EP)
- ☐ APPLICATIONS OF EP (ESPECICALLY ON MONOGAMY RELATIONS)

Generalized probability theory

- From an experimental point of view, to implement an experiment E, we can prepare a large number of indentical states ρ, and we choose to measure a set of measurements $\mathrm{M} = \{\mathrm{A}_1, \quad , \mathrm{A}_n\}$, by repeating the experiments many times, all we can get are some distributions for compatible measurements

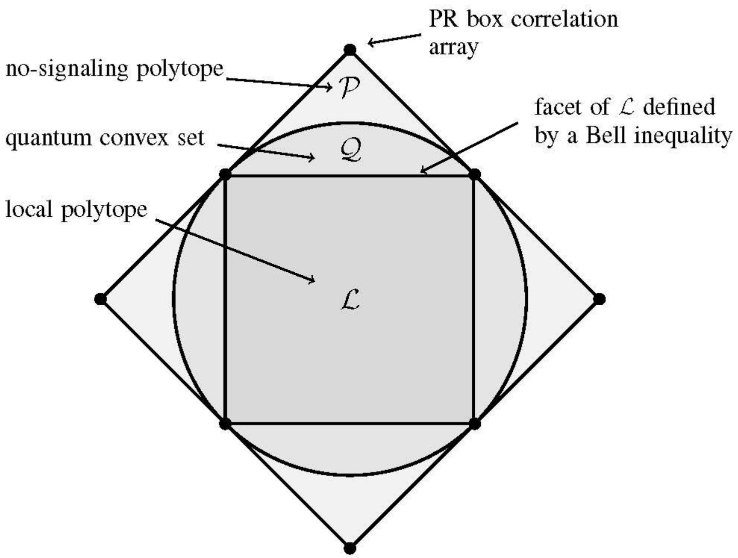
$$\mathcal{P} = \{Pr(a_{i_1}, \cdots, a_{i_k})\}_{(i_1 \cdots i_k) \subseteq (1 \cdots n)}$$

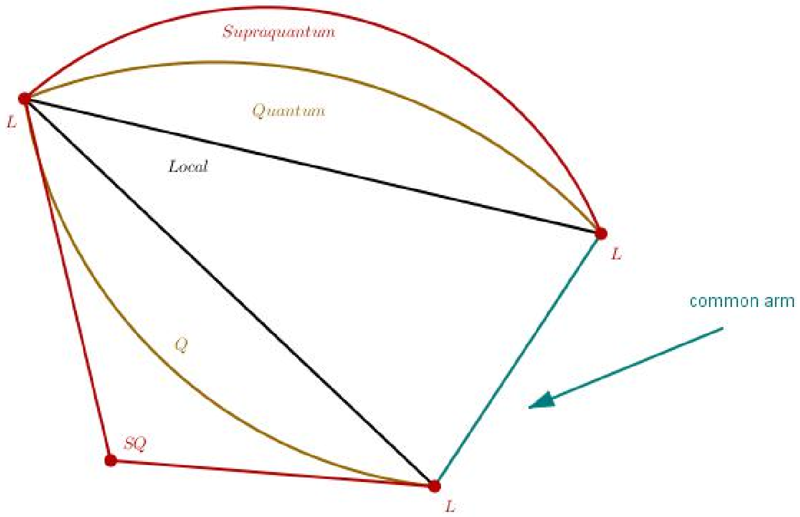


Zhih-Ahn Jia et al., PhysRevA.94.012111

Generalized probability theory

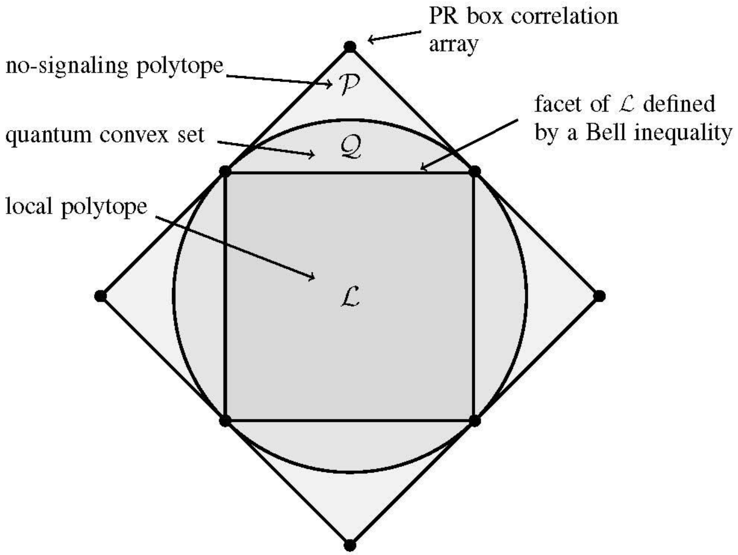
- bounding the set of quantum correlations in terms of simple physical principles
- different principle corresponds to different polytope

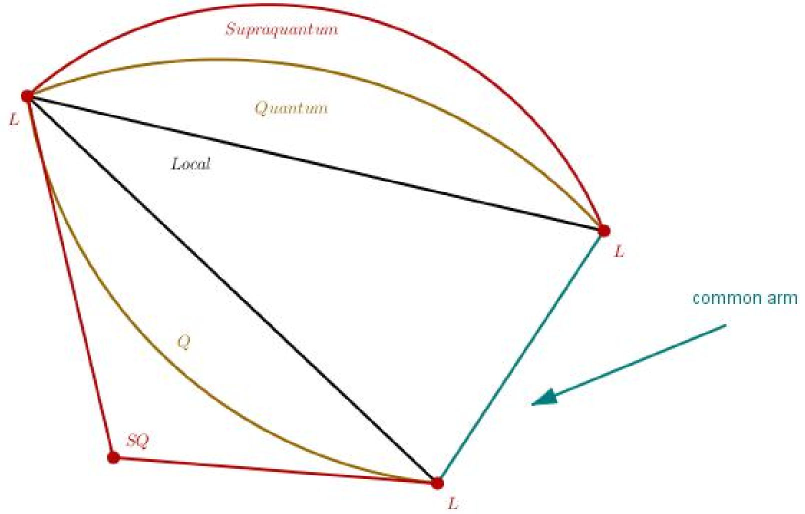




Generalized probability theory

- finite geometry
- convex geometry
- NP problem





Generalized probability theory

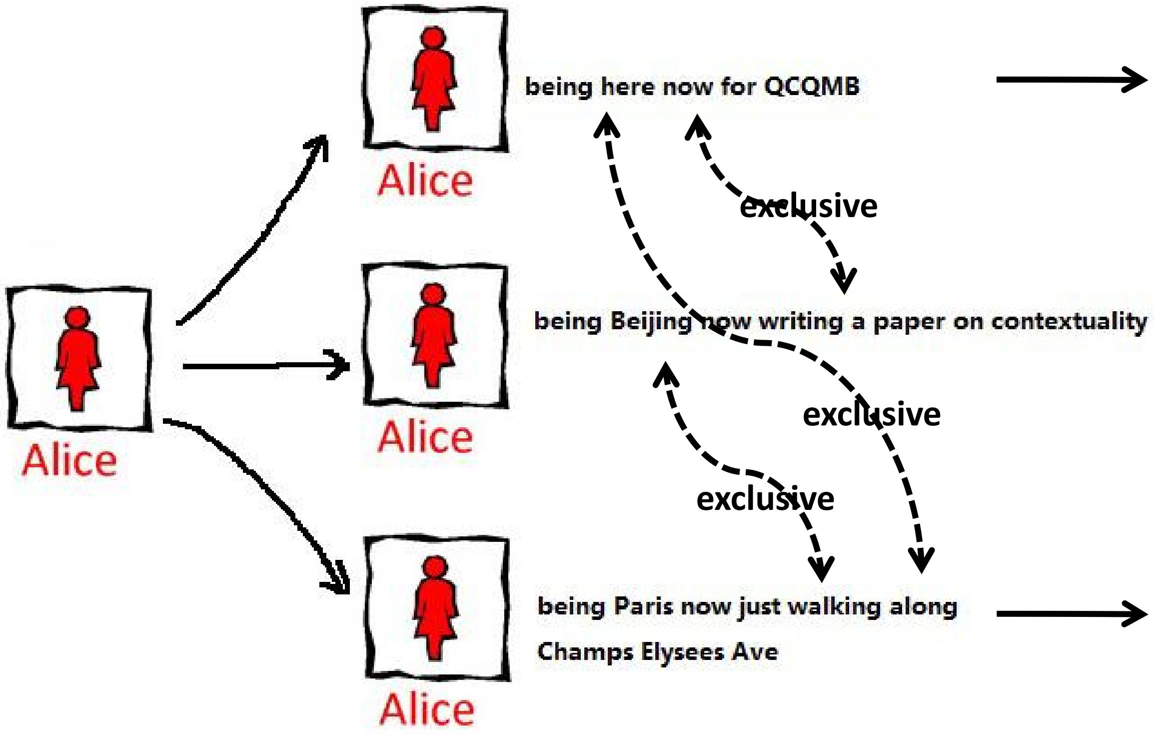
- No-signaling principle(no-disturbance principle)
- Information causuality
- Communication complexity
- Negativity
- Measurement sharpness
- Exclusivity
- et cetera

Outline

- □ Interpretation of Quantum Mechanics
- □ Generalized Probability Theory
- ■ **Exclusivity Principle(EP)**
- □ Applications of EP (especically on monogamy relations)

Exclusivity principle

Exclusive relations



event 1 with probability p(1)

event 2 with probability p(2)

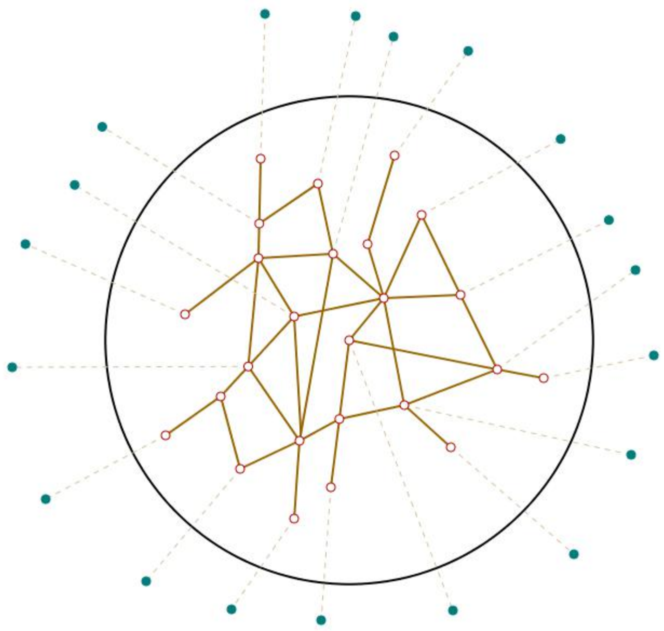
event 3 with probability p(3)

$$p(1)+p(2)+p(3)\leq 1$$

Exclusivity principle

Exclusivity principle(EP): the sum of probabilities of pairwise exclusive events cannot exceed 1.

- strong EP
- weak EP

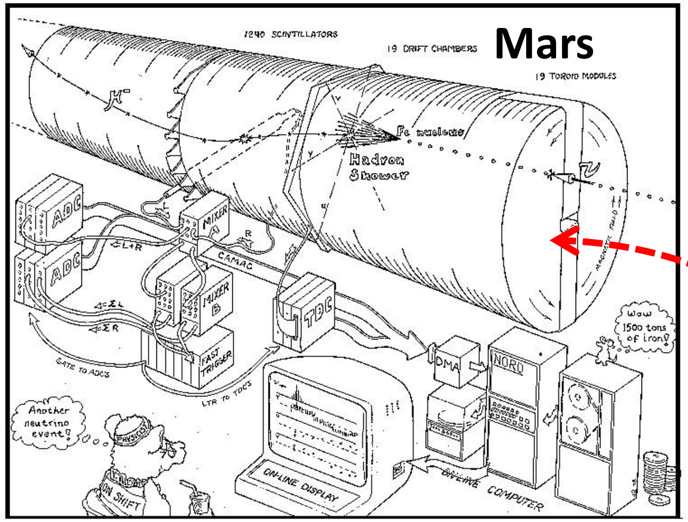


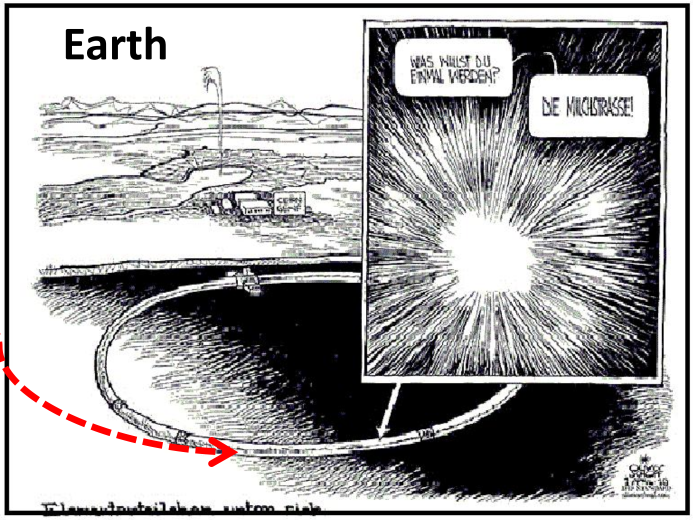
sharp measurement:
(i) Every measurement is sharp;
(ii) Tensor product of sharp measurements is sharp;
(iii) Coase-graining of sharp measurements are still sharp.

Exclusivity principle

Exclusivity principle(EP): the sum of probabilities of pairwise exclusive events cannot exceed 1.

- strong EP
- weak EP

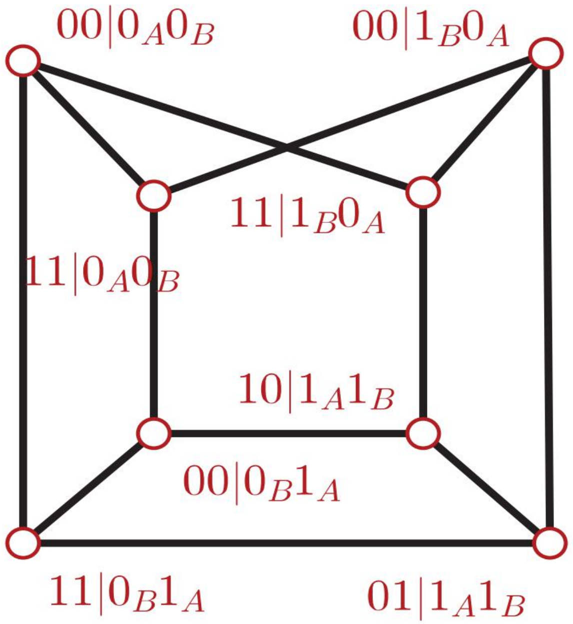




there may exist some exclusive pair of events between two very far laboratories!

Exclusivity principle

- Bell experiment



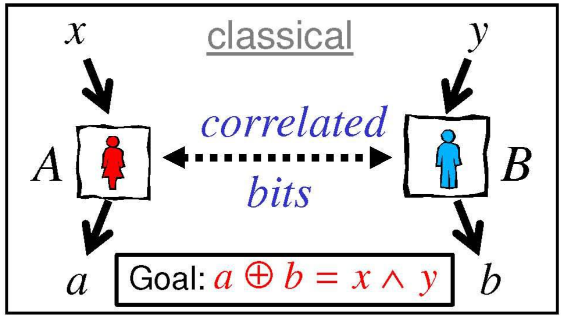
A. Cabello, S. Severini, and A. Winter, prl 112,040401, A. Cabello, prl 110,060402, B. Yan, prl 110,260406

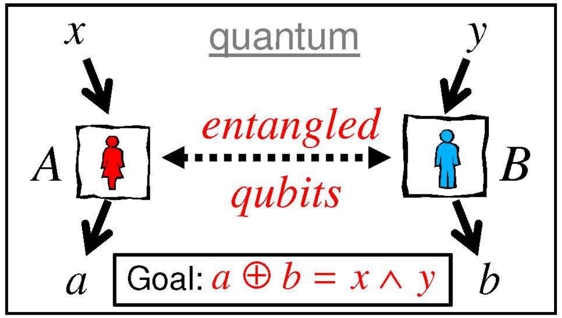
Experimental set $\mathcal{E}_{Bell}$ for Bell experiment

$00\|0_A0_B$	$11\|0_A0_B$	$01\|0_A0_B$	$10\|0_A0_B$
$00\|1_A0_B$	$11\|1_A0_B$	$01\|1_A0_B$	$10\|1_A0_B$
$00\|1_A1_B$	$11\|1_A1_B$	$01\|1_A1_B$	$10\|1_A1_B$
$00\|0_A1_B$	$11\|0_A1_B$	$01\|0_A1_B$	$10\|0_A1_B$

$$\mathcal{E}_{CHSH} = \{ab|xy : p(ab|xy)\}$$

$$\sum_{a\oplus b=AB}\sum_{A,B=0,1} p(a,b|A,B) \overset{C}{\leq} 3 \overset{Q}{\leq} 2+\sqrt{2} \overset{E}{\leq} 4$$



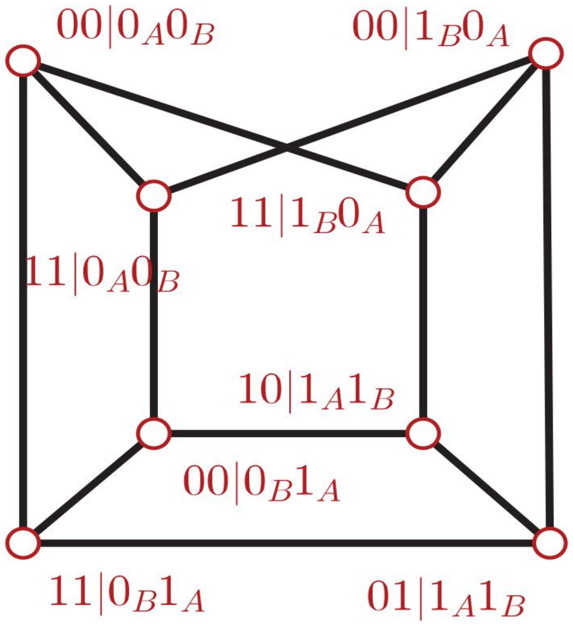


Exclusivity principle

- Bell experiment

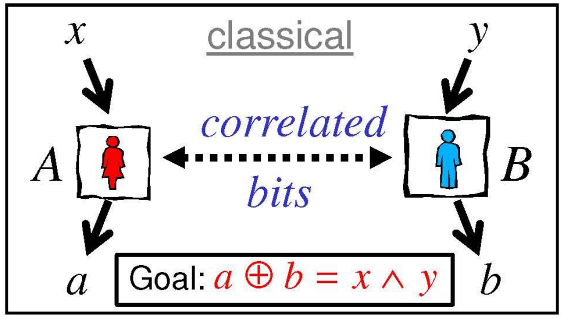
$$\mathcal{E}_{CHSH} = \{ab|xy : p(ab|xy)\}$$

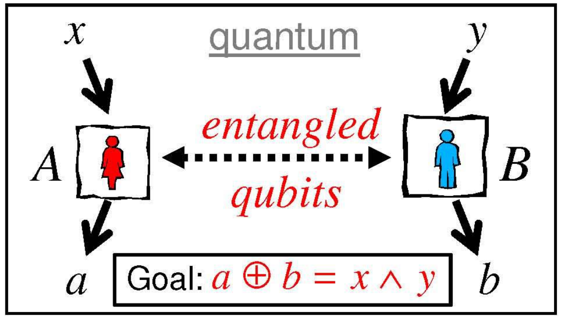
$$\sum_{a\oplus b=AB}\sum_{A,B=0,1} p(a,b|A,B) \overset{C}{\leq} 3 \overset{Q}{\leq} 2+\sqrt{2} \overset{E}{\leq} 4$$



graph parameter →

independent number: $\alpha(G_{CHSH})=3$;
Lovász number: $\vartheta(G_{CHSH}) = 2+\sqrt{2}$;
packing number: $\alpha^*(G_{CHSH}) = 4$.

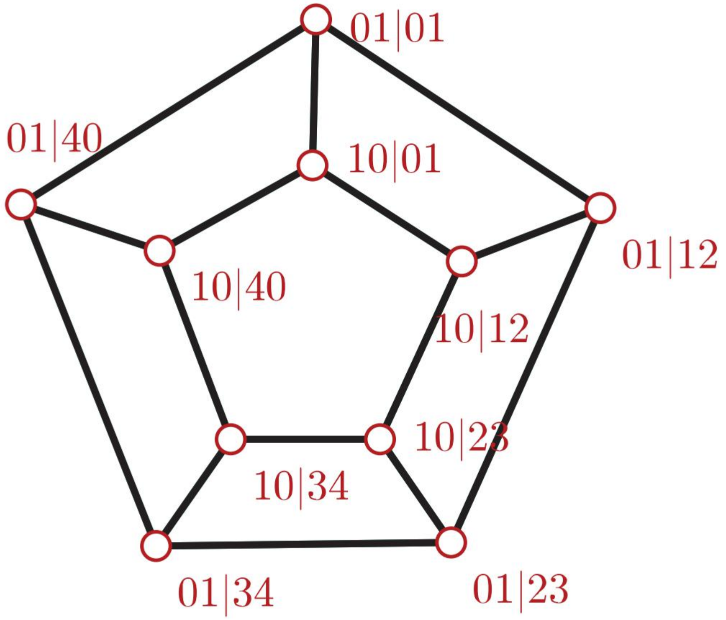




A. Cabello, S. Severini, and A. Winter, prl 112,040401, A. Cabello, prl 110,060402, B. Yan, prl 110,260406

Exclusivity principle

- KCBS experiment



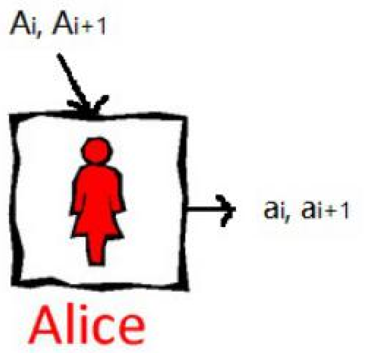
A. Cabello, S. Severini, and A. Winter, prl 112,040401, A. Cabello, prl 110,060402, B. Yan, prl 110,260406

Experiment set $\mathcal{E}_{KCBS}$ for KCBS experiment

00|01	11|01	01|01	10|01
00|12	11|12	01|12	10|12
00|23	11|23	01|23	10|23
00|34	11|34	01|34	10|34
00|40	11|40	01|40	10|40

$$\mathcal{E}_{KCBS} = \{a_i a_{i+1} | A_i A_{i+1} : p(a_i a_{i+1} | A_i A_{i+1})\}$$

$$\sum_{a_i \oplus a_{i+1} = 1} \sum_{i=0}^{4} p(a_i a_{i+1} | A_i A_{i+1}) \overset{C}{\leq} 4 \overset{Q}{\leq} 2\sqrt{5} \overset{E}{\leq} 5$$

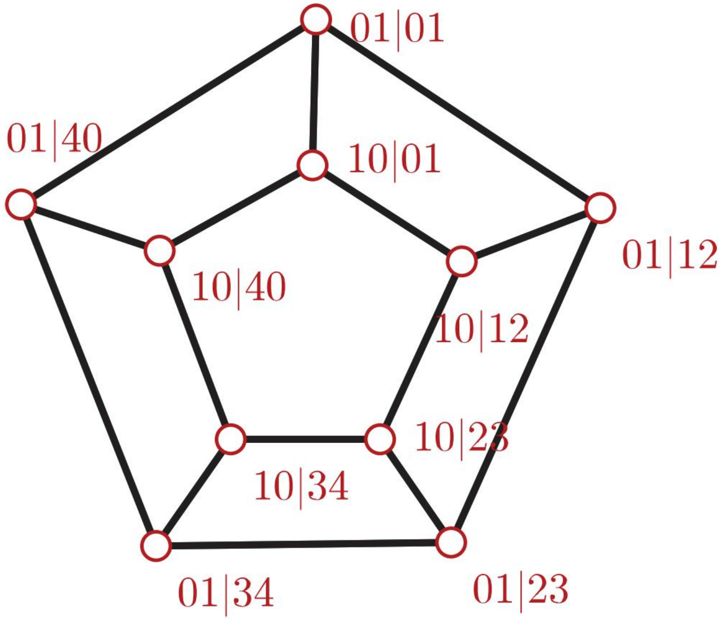


Exclusivity principle

- KCBS experiment

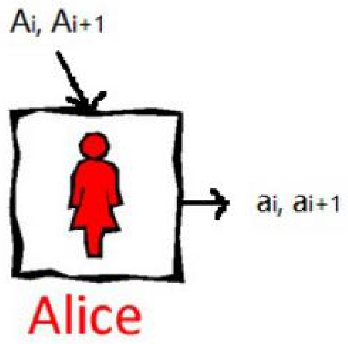
$$\mathcal{E}_{KCBS} = \{a_i a_{i+1} | A_i A_{i+1} : p(a_i a_{i+1} | A_i A_{i+1})\}$$

$$\sum_{a_i \oplus a_{i+1} = 1} \sum_{i=0}^{4} p(a_i a_{i+1} | A_i A_{i+1}) \overset{C}{\leq} 4 \overset{Q}{\leq} 2\sqrt{5} \overset{E}{\leq} 5$$



graph parameter $\longrightarrow$

independent number: $\alpha(G_{KCBS}) = 4$;
Lovász number: $\vartheta(G_{KCBS}) = 2\sqrt{5}$;
packing number: $\alpha^*(G_{KCBS}) = 5$.

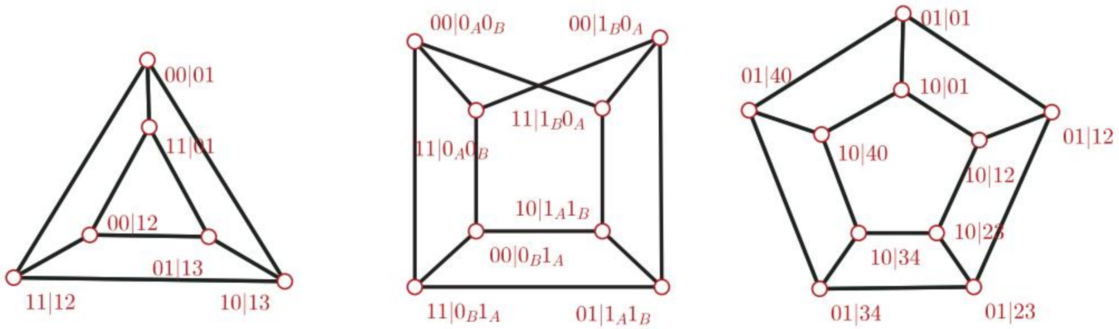


A. Cabello, S. Severini, and A. Winter, prl 112,040401, A. Cabello, prl 110,060402, B. Yan, prl 110,260406

Exclusivity principle

- some test parameters

- Non-locality test: CHSH inequality (two parties Alice and Bob with $\dim \mathcal{H}_A, \dim \mathcal{H}_B \geq 2$)
  $|\langle \mathcal{I}_{AB}^{CHSH} \rangle| = |\langle A_1 B_1 + B_1 A_2 + A_2 B_2 - B_2 A_1 \rangle| \leq 2$
  [J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969)];
- Contextuality test: KCBS inequality (one party Alice with $\dim \mathcal{H}_A \geq 3$)
  $\langle \mathcal{I}_A^{KCBS} \rangle = \langle A_1 A_2 + A_2 A_3 + A_3 A_4 + A_4 A_5 + A_5 A_1 \rangle \geq -3$
  [A. A. Klyachko, M. A. Can, S. Binicioğlu, and A. S. Shumovsky, Phys. Rev. Lett. 101, 020403 (2008)].
- Leggett-Garg test: $K = C_{12} + C_{23} - C_{13} \leq 1$
  [A. J. Leggett and Anupam Garg. Phys. Rev. Lett. 54, 857 (1985)]



(a) Y_3 (b) M_4 (c) Y_5

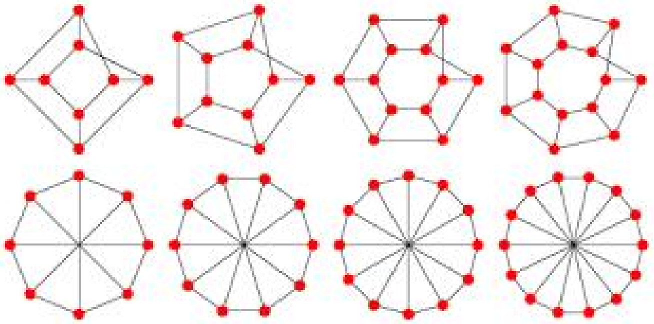
Exclusivity principle

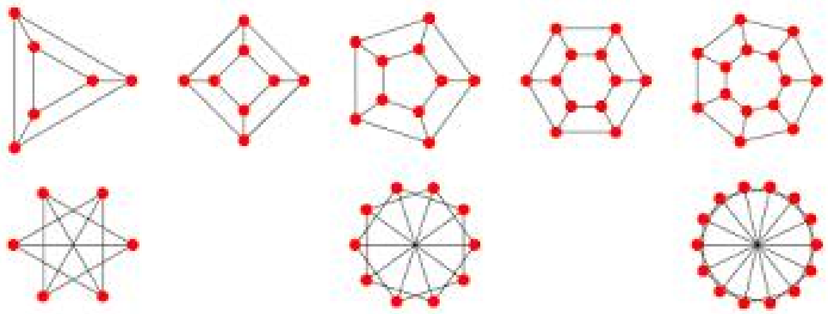
- A unified n-cycle form

- ► Genelized CHSH inequalities: $\mathcal{B}(2m) = A_1B_1 + B_1A_2 + A_2B_2 + \cdots + A_{m-1}B_m - B_mA_1 \le 2m-2$;
- ► KCBS inequality: $\mathcal{C}(5) = A_1A_2 + A_2A_3 + \cdots + A_5A_1 \ge -3$;
- ► Unified as a n-cycle non-contextual inequality:

$$
\begin{aligned}
\langle \mathcal{C}(n) \rangle \quad &= \quad \sum_{i=1}^{n} \gamma_i \langle A_i A_{i+1} \rangle, \\
&\le_C \quad n-2, \\
&\le_Q \quad \begin{cases} \frac{3n\cos(\frac{\pi}{n})-n}{1+\cos(\frac{\pi}{n})}, n \in 2\mathbb{N}+1 \\ n\cos(\frac{\pi}{n}), n \in 2\mathbb{N} \end{cases}, \\
&\le_{NS} \quad n.
\end{aligned}
$$

Phys. Rev. A 88, 022118 (2013)





Exclusivity principle

- A experiment $\mathcal{E} = \{e_i | p(e_i)\}$
- Correlation or behavior $\mathcal{C} = \{p(e_i)\}$
- Test parameter $\mathcal{I}_{\mathcal{E}} = \sum_i w(e_i) p(e_i) \overset{C}{\leq} R_C \overset{Q}{\leq} R_Q \overset{SQ}{\leq} R_{SQ}$
- Graph $G_{\mathcal{E}}$

$$\begin{array}{lcl} \text{classical bound: } R_C & \leftrightarrow & \text{independent number: } \alpha(G_{\mathcal{E}}); \\ \text{quantum bound } R_Q & \leftrightarrow & \text{Lovász number: } \vartheta(G_{\mathcal{E}}); \\ \text{exclusive bound } R_E & \leftrightarrow & \text{fractional packing number: } \alpha^*(G_{\mathcal{E}}). \end{array}$$

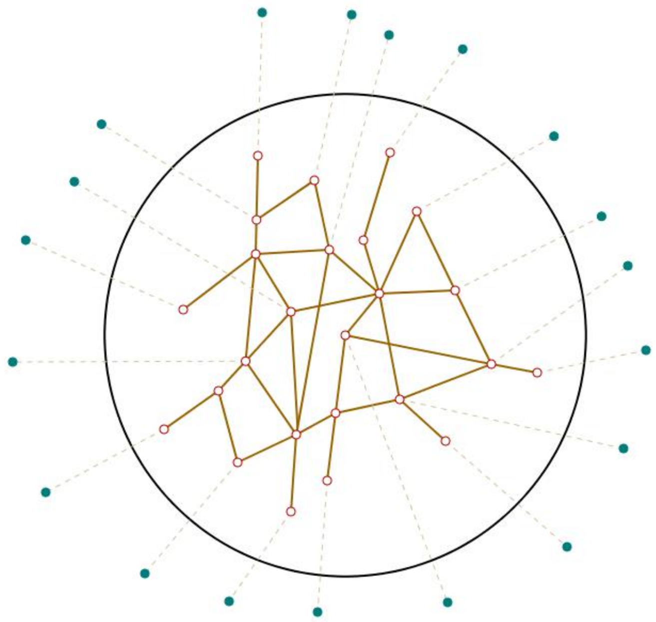
A. Cabello, S. Severini, and A. Winter, prl 112,040401, A. Cabello, prl 110,060402, B. Yan, prl 110,260406

Exclusivity principle

- Dictionary between physical terms and graph terms

$$
\begin{array}{rccccc}
\text{inequality } \mathcal{I}_{\mathcal{E}}: & R_C & \leq & R_Q & \leq & R_{SQ} \\
\uparrow & \uparrow & & \uparrow & & \uparrow \\
\text{experiment } \mathcal{E} \mid & \mid & & \mid & & \mid \\
\downarrow & \downarrow & & \downarrow & & \downarrow \\
\text{graph } G_{\mathcal{E}}: & \alpha(G_{\mathcal{E}}) & \leq & \vartheta(G_{\mathcal{E}}) & \leq & \alpha^*(G_{\mathcal{E}})
\end{array}
$$

$$\vartheta(G_{\mathcal{E}}) = \max \sum_{e_i \in V(G_{\mathcal{E}})} |\langle \phi | v_i \rangle|^2$$



Exclusivity principle

- Effective test parameter and effective exclusive graph

$$
\begin{array}{lccccc}
\text{inequality } \mathcal{I}_{\mathcal{E}}: & R_C & \leq & R_Q & \leq & R_{SQ} \\
\uparrow & \uparrow & & \uparrow & & \uparrow \\
\text{experiment } \mathcal{E} \,| & | & & | & & | \\
\downarrow & \downarrow & & \downarrow & & \downarrow \\
\text{graph } G_{\mathcal{E}}: & \alpha(G_{\mathcal{E}}) & \leq & \vartheta(G_{\mathcal{E}}) & \leq & \alpha^*(G_{\mathcal{E}})
\end{array}
$$

the graph of the exclusivity set has a distinct independence number, Lovász number and fractional packing number, and each graph theoretical term coincides with the physical bound of corresponding testing parameter, this is equivalent to say that the exclusivity graph is a pefect graph,i.e., contains, as induced subgraphs, odd cycles on five or more vertices and/or their complements

Annals of Mathematics, **164** (2006), 51–229

The strong perfect graph theorem

By Maria Chudnovsky, Neil Robertson,* Paul Seymour,** and Robin Thomas***

Abstract

A graph G is *perfect* if for every induced subgraph H, the chromatic number of H equals the size of the largest complete subgraph of H, and G is *Berge* if no induced subgraph of G is an odd cycle of length at least five or the complement of one.

The "strong perfect graph conjecture" (Berge, 1961) asserts that a graph is perfect if and only if it is Berge. A stronger conjecture was made recently by Conforti, Cornuéjols and Vušković — that every Berge graph either falls into one of a few basic classes, or admits one of a few kinds of separation (designed so that a minimum counterexample to Berge's conjecture cannot have either of these properties).

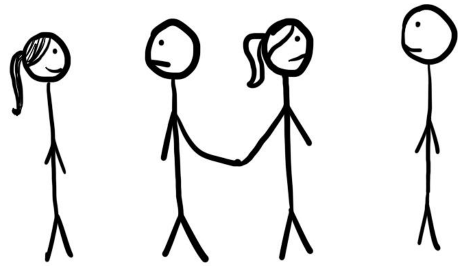
In this paper we prove both of these conjectures.

Outline

- □ INTERPRETATION OF QUANTUM MECHANICS
- □ GENERALIZED PROBABILITY THEORY
- □ EXCLUSIVITY PRINCIPLE(EP)
- ■ APPLICATIONS OF EP (ESPECICALLY ON MONOGAMY RELATIONS)

Monogamy relations of some physical proper P

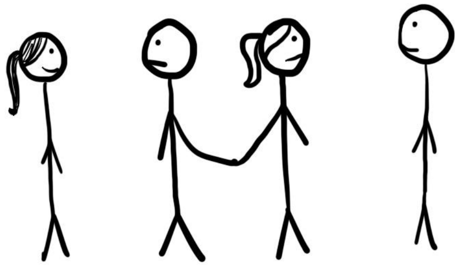
P property test experiment $\mathcal{I}_P \in \mathcal{R}_P = [r_P, R_P]$;
P property range $\mathcal{R}_P$;
corresponding graph: G_P.



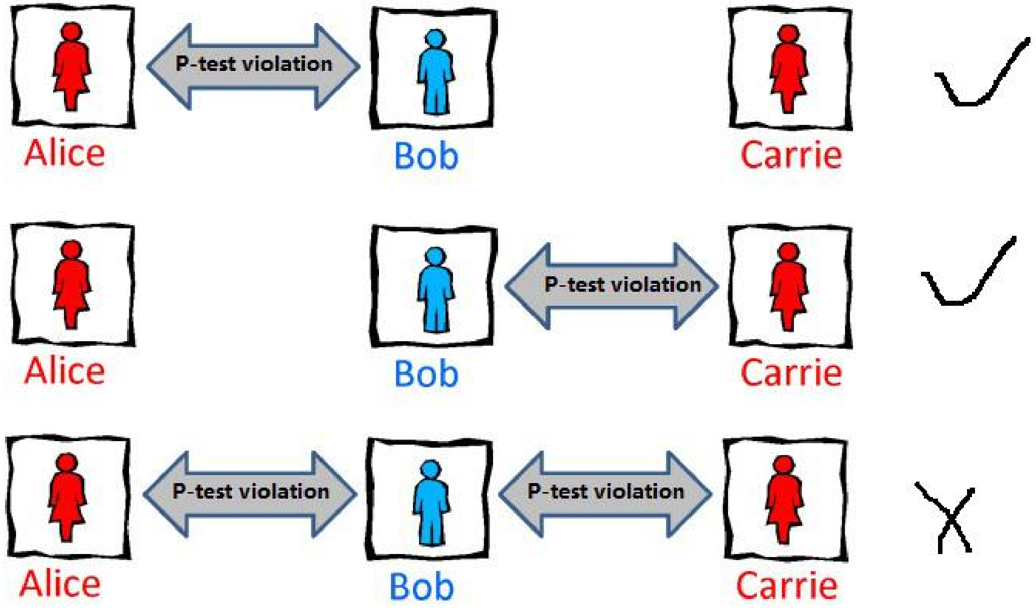
A physical proper P can not be shared by many parties, we call P monogamous among these N parties

$$\mathcal{M}(\mathcal{I}_1, \mathcal{I}_2) \overset{\mathcal{T}}{\in} [m_P, M_P] \subseteq [\mathcal{M}(r_P^1, r_P^2), \mathcal{M}(R_P^1, R_P^2)]$$

Monogamy relations of some physical proper P



A physical proper P can not be shared by many parties, we call P monogamous among these N parties

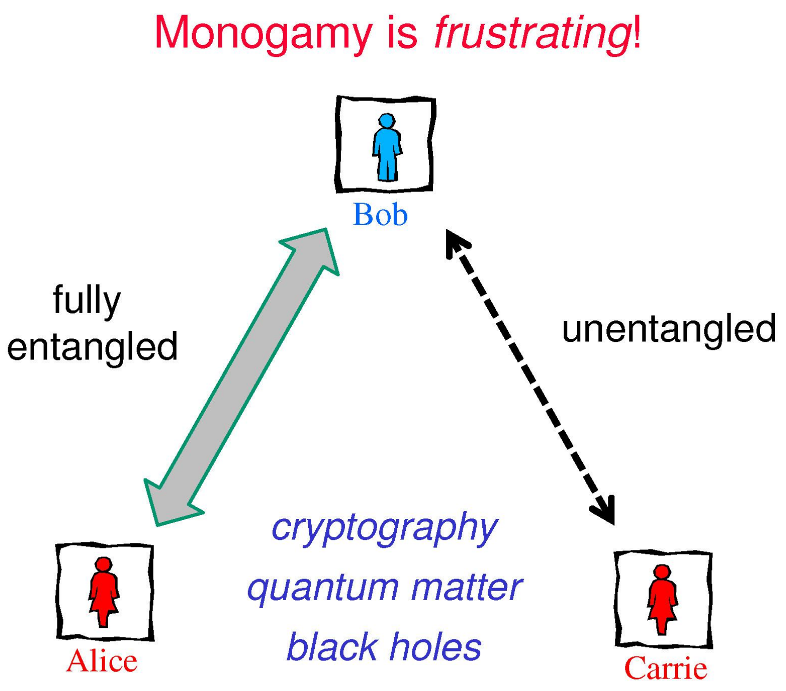


$$\mathcal{M}(\mathcal{I}_1,\mathcal{I}_2) \overset{\mathcal{T}}{\in} [m_P, M_P] \subseteq [\mathcal{M}(r_P^1, r_P^2), \mathcal{M}(R_P^1, R_P^2)]$$

Monogamy of maximal entanglement

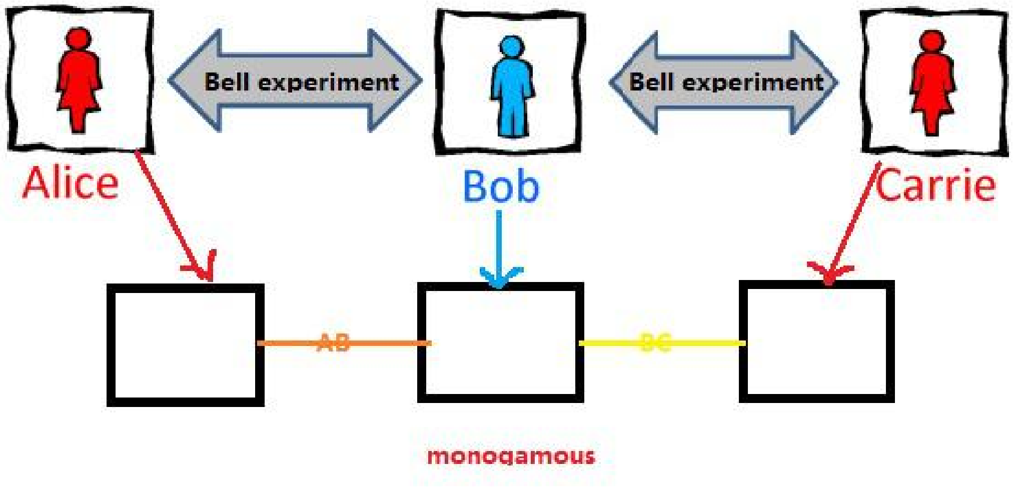
- (Coffman-Kundu-Wootters (CKW) monogamy inequality) If Alice and Bob is fully entangled then Carrie and Bob must be non-correlated:[V. Coffman, J. Kundu, and W. K. Wootters,PhysRevA.61.052306]

$$\mathcal{C}_{AB}^2+\mathcal{C}_{AC}^2\leq\mathcal{C}_{A(BC)}^2$$



Monogamy relations of nonlocality

- CHSH: $\langle \mathcal{I}_{AB}^{CHSH} \rangle + \langle \mathcal{I}_{AC}^{CHSH} \rangle \leq 4$ [V. Scarani and N. Gisin, Phys. Rev. Lett. 87, 117901];
- General Bell inequality: $\sum_{m=1}^{n} \mathcal{B}(A, B_m) \leq nR$ [M. Pawlowski and Č. Brukner, Phys. Rev. Lett. 102, 030403 (2009)]



(1)Loop-type many-party monogamy relation:

$$\mathcal{B}_{1,2} + \mathcal{B}_{2,3} + \cdots + \mathcal{B}_{N-1,N} + \mathcal{B}_{N,1} \leq \sum_{i=1}^{N} R_{L_{i,i+1}},$$

where $N+1=1$, each $R_{L_{i,i+1}}$ is the local bound of generalized CHSH inequality $\mathcal{B}_{i,i+1}$.

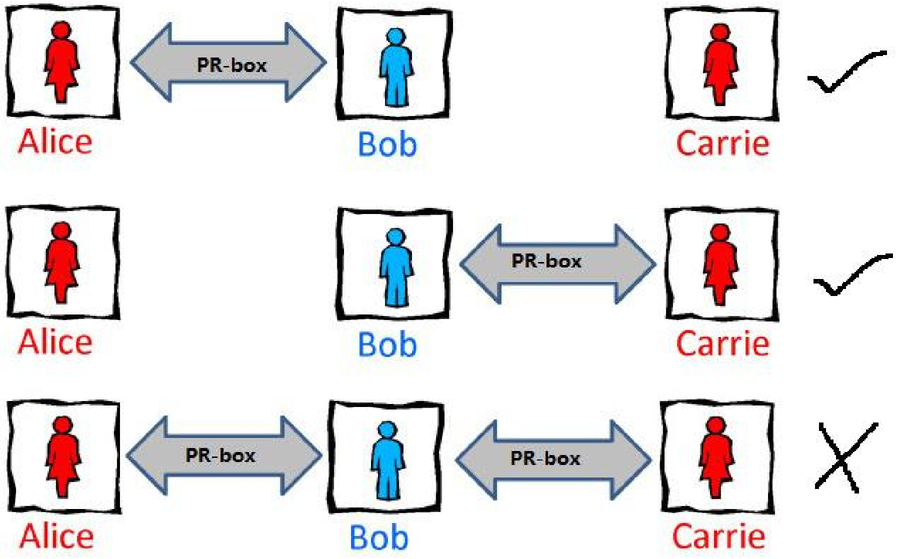
(2)Chain-type many-party monogamy relation:

$$\begin{aligned} CHSH_{1,2} + \cdots + CHSH_{2n,2n+1} &\leq R_L, \\ &= R_Q, \\ &= R_{NS} = 4n \end{aligned}$$

Zhih-Ahn Jia et.al. PhysRevA.94.012111

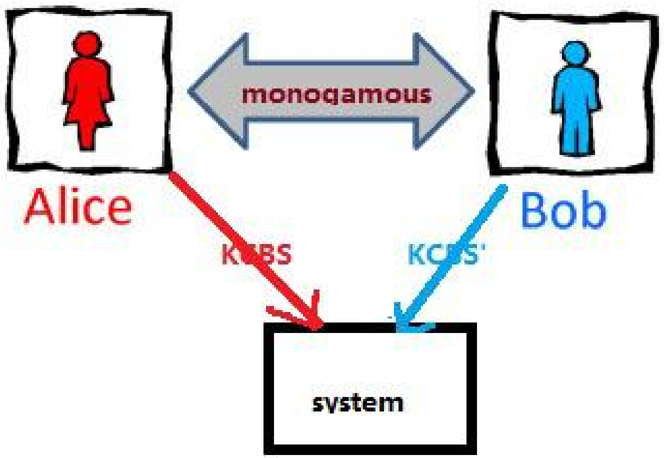
Monogamy of Popescu-Rohrlich(PR)-box

- Popescu-Rohrlich boxes: for PR-boxes $p(a,b|A,B)$ and $p(a,c|A,C)$, there is no joint probability distribution $p(a,b,c|A,B,C)$ which can reduce to these boxes [Zhih-Ahn Jia, Yu-Chun Wu, and Guang-Can Guo, Phys. Rev. A 94, 012111];



Monogamy relations of contextuality

- Conteuality and nonlocality: $\langle \mathcal{I}_{AB}^{CHSH} \rangle + \langle \mathcal{I}_{A}^{KCBS} \rangle \geq -5$ [PawełKurzyski, Adn Cabello, and Dagomir Kaszlikowski Phys. Rev. Lett. 112, 100401];
- $\sum_{i=1}^{k} \mathcal{C}(N_i) \leq \sum_{i=1}^{k} R_{C_i}$.
  [Zhih-Ahn Jia, Yu-Chun Wu, and Guang-Can Guo, Phys. Rev. A 94, 012111]



Applications of monogamy relations

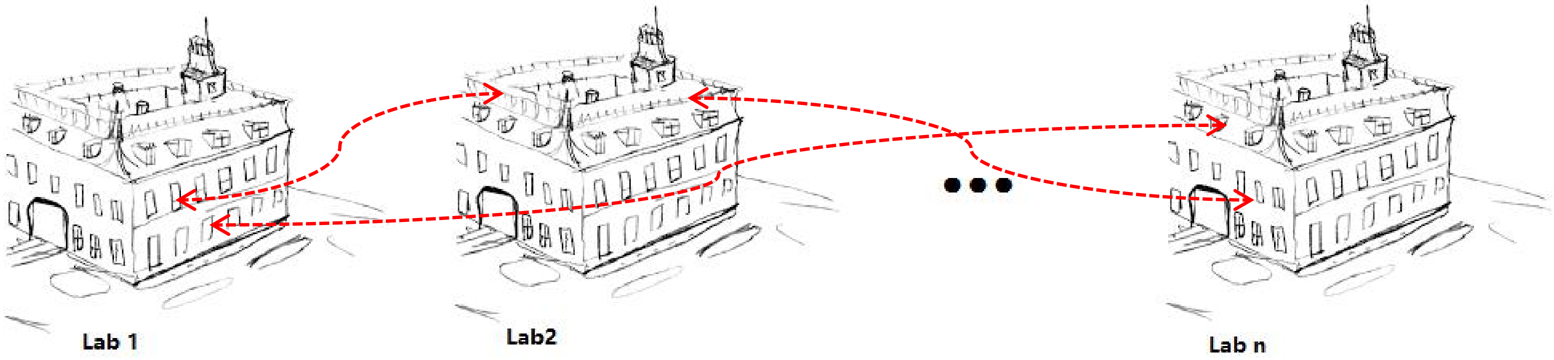
- Monogamy of Bells inequality violations, which is strictly weaker condition than the no-signaling principle is enough to prove **security of quantum key distribution**.
- **Condensed matter physics**:[X.-s. Ma, B. Dakic, W. Naylor, A. Zeilinger, and P. Walther, Nat. Phys. 7, 399 (2011); A. Garca-Sez and J. I. Latorre, Phys. Rev. B 87, 085130 (2013); K. Meichanetzidis, J. Eisert, M. Cirio, V. Lahtinen, and J. K. Pachos, Phys Rev Lett.116.130501(2016)]
- **Black hole**: [L. Susskind, arXiv:1301.4505; S. Lloyd and J. Preskill, J. High Energy Phys. 08 (2014) 126.]
- **Statistical mechanics**: [C. H. Bennett, in Proceedings of the FQXi 4th International Conference, Vieques Island, Puerto Rico, 2014]

EP determines the monogamy relations

Theorem Given several disjoint experimental event sets $\mathcal{E}_1, \cdots, \mathcal{E}_n$ with exclusive graphs $G_{\mathcal{E}_1}, \cdots, G_{\mathcal{E}_n}$ respectively, we can make them into an assemblage of events $\mathcal{E} = \sqcup_{i=1}^n \mathcal{E}_i$ with exclusive graph $G_{\mathcal{E}}$, if we have the relation

$$\alpha^*(G_{\mathcal{E}}) \leq \sum_{i=1}^{n} \alpha(G_{\mathcal{E}_i}),$$

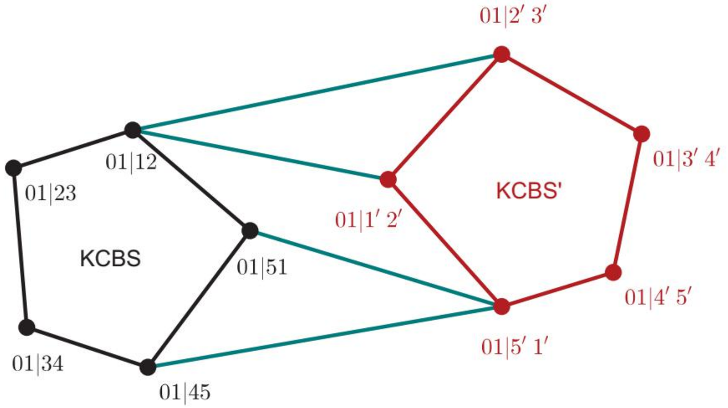
then these experiments are monogamous.



EP determines the monogamy relations

- Example： Monogamy of two KCBS experiments

assume that the triple 1,1',2' are exclusive, viz., they can not get 0 outcomes simultaneously, as is also the triple 4,5,5'.



$$\mathcal{E}_{\mathcal{I}_{KCBS}} = \{01|12, 01|23, 01|34, 01|45, 01|51\}$$

$$\mathcal{E}_{\mathcal{I}'_{KCBS}} = \{01|1'2', 01|2'3', 01|3'4', 01|4'5', 01|5'1'\}$$

$$\mathcal{E} = \mathcal{E}_{\mathcal{I}_{KCBS}} \sqcup \mathcal{E}_{\mathcal{I}'_{KCBS}}$$

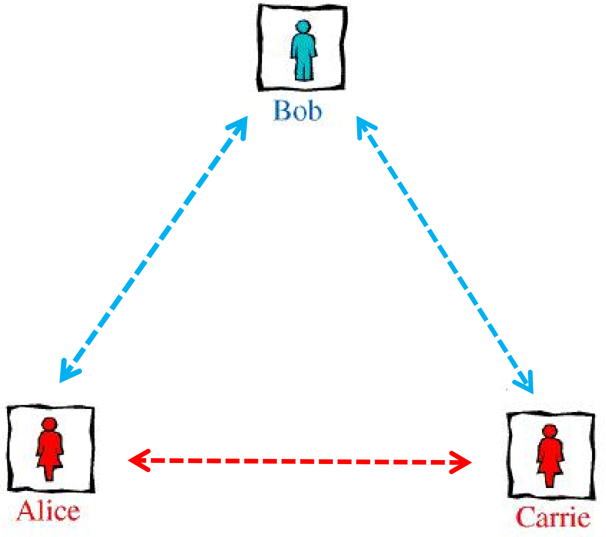
$$\alpha^*(G_{\mathcal{E}}) = 4 = R_C + R'_C \quad \Longrightarrow \quad \mathcal{I}_{KCBS} + \mathcal{I}'_{KCBS} \overset{E}{\leq} 4 = R_C + R'_C$$

EP determines the monogamy relations

- Example： Monogamy of three Bell experiments

$$\begin{aligned}\mathcal{E}_{AB} = \; & \{00|0_A0_B,\ 00|0_B1_A,\ 01|1_A1_B,\ 00|1_B0_A, \\ & 11|0_A0_B,\ 11|0_B1_A,\ 10|1_A1_B,\ 11|1_B0_A\} \\ \mathcal{E}_{BC} = \; & \{01|0_A0_C,\ 00|0_C1_A,\ 00|1_A1_C,\ 00|1_C0_A, \\ & 10|0_A0_C,\ 11|0_C1_A,\ 11|1_A1_C,\ 11|1_C0_A\} \\ \mathcal{E}_{CA} = \; & \{00|0_C0_B,\ 01|0_C1_A,\ 00|1_A1_C,\ 00|1_B0_C, \\ & 11|0_C0_B,\ 10|0_C1_A,\ 11|1_A1_C,\ 11|1_B0_C\}\end{aligned}$$

$$\mathcal{E} = \mathcal{E}_{AB} \sqcup \mathcal{E}_{BC} \sqcup \mathcal{E}_{CA}$$



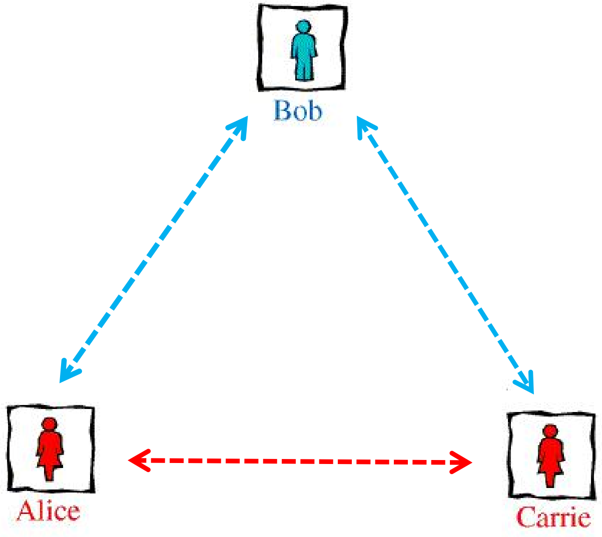
$$\Longrightarrow \quad \mathcal{I}_{AB} + \mathcal{I}_{BC} + \mathcal{I}_{CA} \overset{E}{\leq} 9 = R_C^{AB} + R_C^{BC} + R_C^{CA}$$

EP determines the monogamy relations

- Example: Monogamy of three Swetlichny experiments

$$
\begin{aligned}
\mathcal{I}_4 = & \textstyle\sum_{a\oplus b\oplus c\oplus d=0} p(abcd|1111) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=0} p(abcd|1101) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=0} p(abcd|0111) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=0} p(abcd|1011) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=0} p(abcd|0000) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=0} p(abcd|0010) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=0} p(abcd|1000) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=0} p(abcd|0100) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=1} p(abcd|0001) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=1} p(abcd|0011) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=1} p(abcd|1001) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=1} p(abcd|0101) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=1} p(abcd|1110) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=1} p(abcd|1100) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=1} p(abcd|0110) \\
& + \textstyle\sum_{a\oplus b\oplus c\oplus d=1} p(abcd|1010).
\end{aligned}
$$

$$
\left\{
\begin{array}{llll}
abcd|1111, & abcd|1101, & abcd|0111 & abcd|1011, \\
cdef|1111, & cdef|0101, & cdef|1110 & cdef|1100, \\
efab|1111, & efab|0111, & efab|1001 & efab|0010, \\
abcd|0001, & abcd|0011, & abcd|1001 & abcd|0101, \\
cdef|0111, & cdef|1101, & cdef|0110 & cdef|0100, \\
efab|1100, & efab|0100, & efab|1010 & efab|0001, \\
abcd|0000, & abcd|0010, & abcd|1000 & abcd|0100, \\
cdef|0000, & cdef|1010, & cdef|0011 & cdef|0001, \\
efab|0000, & efab|1000, & efab|1110 & efab|0101, \\
abcd|1110, & abcd|1100, & abcd|0110 & abcd|1010 \\
cdef|1000, & cdef|0010, & cdef|1011 & cdef|1001 \\
efab|0011, & efab|1011, & efab|1101 & efab|0110
\end{array}
\right\}.
$$



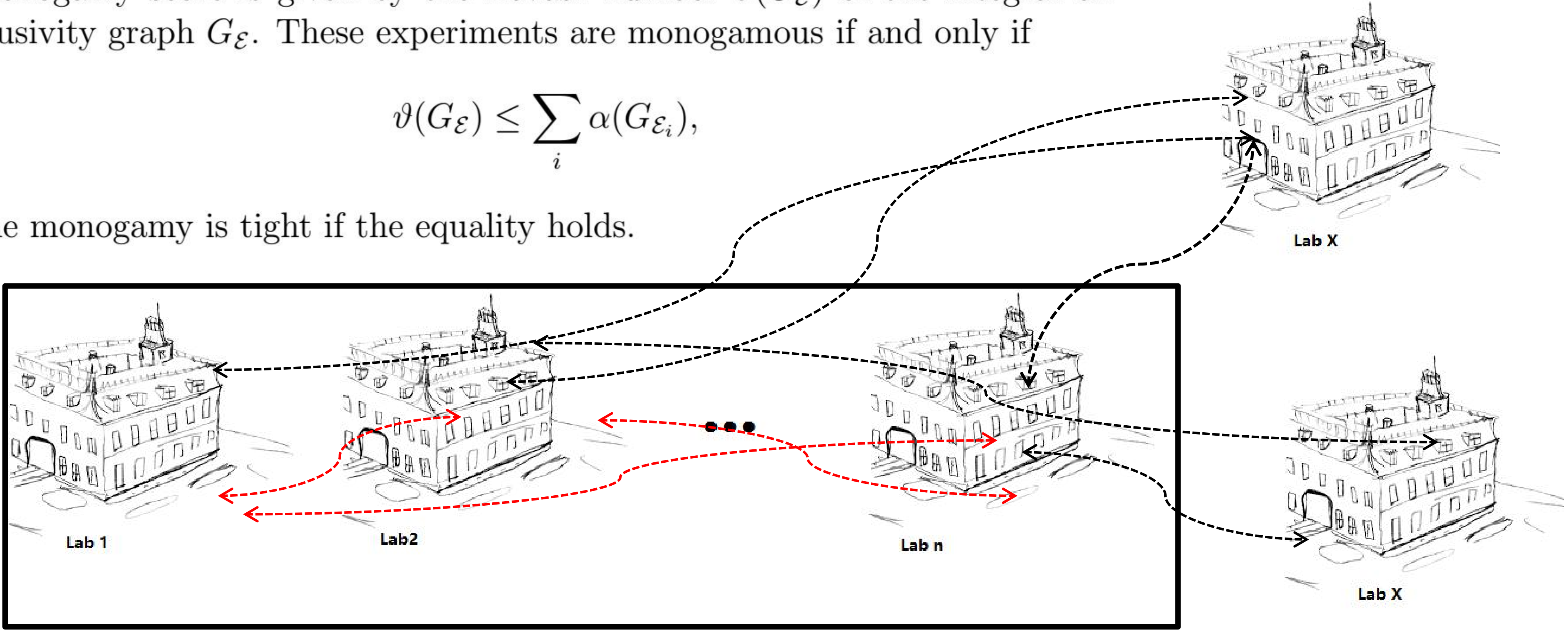
$$\Longrightarrow \quad \mathcal{I}_4^{ABCD} + \mathcal{I}_4^{CDEF} + \mathcal{I}_4^{EFAB} \overset{E}{\leq} 32 < 36 = R_C^{ABCD} + R_C^{CDEF} + R_C^{EFAB}$$

EP determines the monogamy relations

Theorem Let $\mathcal{E}_1, \cdots, \mathcal{E}_n$ be several disjoint experimental event sets, the integral event set $\mathcal{E} = \sqcup_{i=1}^{n} \mathcal{E}_i$ is the disjoint union of these sets. Their monogamy score is given by the Lovász number $\vartheta(G_\mathcal{E})$ of the integral exclusivity graph $G_\mathcal{E}$. These experiments are monogamous if and only if

$$\vartheta(G_\mathcal{E}) \leq \sum_i \alpha(G_{\mathcal{E}_i}),$$

the monogamy is tight if the equality holds.



EP determines the monogamy relations

$$\mathrm{EP} \quad \Rightarrow \quad \begin{cases} \text{nonlocality monogamy} \\ \text{genuine nonlocality monogamy} \\ \text{contextuality monogamy} \\ \text{nonlocality} - \text{contextuality monogamy} \end{cases}$$

Thank You for Your Attentions