Geometric probability theory in contextual probabilistic theories

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Outline

1. States

2. On a non-commutative geometric probability theory
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2 On a non-commutative geometric probability theory
We look for group actions and invariant states in (possibly) contextual probabilistic theories.

These notions are important in formulating symmetries, constrains in the MaxEnt principle and physical principles in general.

We look for a formal framework based on measure theory: after all, probabilities can be considered as measures over suitable algebraic structures.

We explore the possibility of developing a non-commutative version of geometric probability theory.

Joint work with Cesar Massri (CONICET) and Angel Plastino (CONICET).
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Classical probability

Probability measures

\[ \mu : \Sigma \rightarrow [0, 1] \]  \hspace{1cm} (1) 

such that:

1. \( \mu(\emptyset) = 0 \)
2. \( \mu(A^c) = 1 - \mu(A) \)
3. For each family of pairwise disjoint sets \( \{A_i\}_{i \in I} \)
   \[ \mu(\bigcup_{i \in I} A_i) = \sum_{i} \mu(A_i) \]

Classical case

\( \sigma : \Gamma \rightarrow [0; 1] \), such that \( \int_{\Gamma} \sigma(p, q)d^3pd^3q = 1 \)
Convex set of quantum states
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Figure: Geometric representation.
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Quantum probability (Born’s rule)

Probability measures

\[ s : \mathcal{L}_{vN} \rightarrow [0; 1] \]  

such that:

1. \( s(\mathbf{0}) = 0 \) (\( \mathbf{0} \) null subspace).
2. \( s(P^\perp) = 1 - s(P) \)
3. for each family of pairwise orthogonal projections \( (P_j) \),
   \[ s(\sum_j P_j) = \sum_j s(P_j) \]

Gleason’s theorem

\[ s_\rho(P) = tr(\rho P) \]  

Quantum probability (Born’s rule)

**Probability measures**

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**Gleason’s theorem**

*Gleason’s theorem assures that there exists a density matrix for each probability measure as defined above (\( \text{dim}(\mathcal{H}) \geq 3 \)).*

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More general theories

In a series of papers Murray and von Neumann searched for algebras more general than $\mathcal{B}(\mathcal{H})$.

The new algebras are known today as von Neumann algebras, and their elementary components can be classified as Type I, Type II and Type III factors.

It can be shown that, the projective elements of a factor form an orthomodular lattice. Classical models can be described as commutative algebras.

The models of standard quantum mechanics can be described by using Type I factors (Type $I_n$ for finite dimensional Hilbert spaces and Type $I_\infty$ for infinite dimensional models). These are algebras isomorphic to the set of bounded operators on a Hilbert space.
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More general theories

- Further work revealed that a rigorous approach to the study of quantum systems with infinite degrees of freedom needed the use of more general von Neumann algebras, as is the case in the axiomatic formulation of relativistic quantum mechanics. A similar situation holds in algebraic quantum statistical mechanics.

- In these models, States are described as complex functionals satisfying certain normalization conditions, and when restricted to the projective elements of the algebras, define measures over lattices which are not the same to those of standard quantum mechanics.
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von Neumann algebras

- Canonical example of a von Neumann algebra: $\mathcal{B}(H)$
  - The easiest way to define a von Neumann algebra regards it as a $\ast$-subalgebra $\mathcal{W}$ satisfying $\mathcal{W}'' = \mathcal{W}$, where given $S \subseteq \mathcal{B}(H)$, $S'$ is defined as
    \[ S' = \{ A \in \mathcal{B}(H) \mid AB - BA = 0 \forall B \in S \} \]
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- The collection of orthogonal projections of a von Neumann algebra $\mathcal{W}$ is an orthomodular lattice $\mathcal{P}(\mathcal{W})$. 
A state $\nu : \mathcal{W} \rightarrow \mathbb{C}$ is defined as a continuous positive linear functional such that $\nu(I) = 1$.

- Positivity means that $\nu(A^*A) \geq 0$ for all $A \in \mathcal{W}$ or, equivalently, that $\nu(A) \geq 0$ for all $A \geq 0$.
- Normal states can be defined as those states satisfying the condition $\nu(\sup_\alpha (a_\alpha)) = \sup_\alpha \nu(a_\alpha)$ for any uniformly bounded increasing net $a_\alpha$ of positive elements of $\mathcal{W}$ (equivalently, states satisfying $\nu(\sum_{i \in I} E_i) = \sum_{i \in I} \nu(E_i)$ for any countable and pairwise orthogonal family of events $\{E_i\}_{i \in I}$).
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Why orthomular?

Thus, normal states of physical theories define probabilities on orthomodular lattices satisfying the following properties:

Let $\mathcal{L}$ be an orthomodular lattice. Then, define

$$s : \mathcal{L} \rightarrow [0; 1],$$

($\mathcal{L}$ standing for the lattice of all events) such that:

$$s(0) = 0. \quad (4)$$

$$s(E^\perp) = 1 - s(E),$$

and, for a denumerable and pairwise orthogonal family of events $E_j$

$$s(\sum_j E_j) = \sum_j s(E_j).$$

where $\mathcal{L}$ is a general orthomodular lattice (with $\mathcal{L} = \Sigma$ and $\mathcal{L} = \mathcal{P}(\mathcal{H})$ for the Kolmogorovian and quantum cases respectively).
Maximal Boolean subalgebras

- An orthomodular lattice $\mathcal{L}$ can be described as a pasting of Boolean algebras:

$$\mathcal{L} = \bigvee_{B \in \mathcal{B}} B$$

(where $\mathcal{B}$ is the set of maximal Boolean algebras of $\mathcal{L}$).

- A state $s$ of $\mathcal{L}$ defines a classical probability on each classical Boolean subalgebra $B$. In other words: $s_B(\ldots) := s|_B(\ldots)$ is a Kolmogorovian measure over $B$. 
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But then...

- One can think about much more general theories.
- In fact, more general non-Kolmogorovian structures have been found associated to problems in biology, cognition and computer science.
- This has direct implications for information theory: F. Holik, G. M. Bosyk and G. Bellomo, “Quantum Information as a Non-Kolmogorovian Generalization of Shannon’s Theory”, *Entropy* 2015, 17 (11), 7349-7373.
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Axioms of geometric probability theory

**Axiom 1**

\[ \mu(\emptyset) = 0 \]

**Axiom 2**

If \( A \) and \( B \) are measurable sets:

\[ \mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B) \]

which (for Boolean algebras) is equivalent to:

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for disjoint \( A \) and \( B \).
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Axioms of geometric probability theory

The following axiom reflects the action of a group that leaves the measure invariant:

**Axiom 3**

The volume of a set $A$ does not depend on the position of $A$; in other words, if $A$ can be rigidly transformed in $B$, then, the volumes (measures) of $B$ and $A$ are equal.

**Axioma**

Given a parallelotope $P$, with orthogonal sides $x_1, \ldots, x_n$, we impose the normalization condition: $\mu(P) = x_1 x_2 \cdots x_n$
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Symmetric polynomials

\[ e_1(x_1, x_2, \ldots, x_n) = x_1 + x_2 + \ldots + x_n \quad (6a) \]

\[ e_2(x_1, x_2, \ldots, x_n) = x_1x_2 + x_1x_3 + \ldots + x_{n-1}x_n \quad (6b) \]

\[ e_{n-1}(x_1, x_2, \ldots, x_n) = x_2x_3 \cdots x_n + x_1x_3x_4\ldots x_n + \ldots + x_1x_2 \cdots x_{n-1} \quad (6c) \]

\[ e_n(x_1, x_2, \ldots, x_n) = x_1x_2\ldots x_n \quad (6d) \]

Each one of these polynomials gives a different invariant measure.
Generalization

Consider a function:

\[ s : \mathcal{L} \to [0; 1], \]

(7)

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There exists a group of automorphisms $\mathcal{F}$ such that for all $g \in \mathcal{F}$ and all $E \in \mathcal{L}$

$$s(g \cdot E) = s(E)$$

**Axiom 4**

The normalization condition has the form

$$e_i(s(E_1), s(E_2), \ldots) = 0$$

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Generalization

Maximization process

- These axioms determine a convex set $C$ in a univocal way and axiom 3 determines a variety $M$.
- The set of states of a concrete physical system can be described as $C \cap M$.
- We compute the measurement entropy on this set:

$$H_E(s) := - \sum_{x \in E} s(x) \ln(s(x))$$

$$H(s) := \inf_{E \in \mathcal{L}} H_E(s)$$
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Consider the conditions:

\[ \langle R_1 \rangle = r_1 \]
\[ \langle R_2 \rangle = r_2 \]
\[ \vdots \]
\[ \langle R_n \rangle = r_n, \quad (8) \]

We want to determine the least biased probability distribution satisfying these constrains.
MaxEnt

MaxEnt tell us that:

\[ \rho_{\text{Max-ent}} = \exp^{-\lambda_0 1 - \lambda_1 R_1 - \cdots - \lambda_n R_n}, \quad (9) \]

where the \( \lambda \)s are Lagrange multipliers satisfying:

\[ r_i = -\frac{\partial}{\partial \lambda_i} \ln Z, \quad (10) \]

and

\[ Z(\lambda_1 \cdots \lambda_n) = \text{tr}[\exp^{-\lambda_1 R_1 - \cdots - \lambda_n R_n}], \quad (11) \]

The normalization condition reads:

\[ \lambda_0 = \ln Z. \quad (12) \]
Given an effect $E$, let us consider the set of states:

$$C_{(E,\lambda)} := \{ \rho \in \mathcal{C} \mid \text{tr}(\rho E) = \lambda, \; \lambda \in [0, 1] \}. \quad (13)$$

It is a convex set and there exists $S$ (a real subspace in $\mathcal{A}$) such that:

$$C_{(E,\lambda)} = S \cap \mathcal{C}, \quad (14)$$
In a generalized model

In general, an equation of the form:

\[ \langle R \rangle = r, \quad (15) \]

Geometric characterization

...can be expressed as subspace intersected with the convex set of states:

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\[ C_{\text{max-ent}} := \bigcap_{i} C_{R_i} = \bigwedge_{i} C_{R_i}. \]  \hspace{1cm} (16)

Given a series of conditions represented by convex sets \( C_i \), one should maximize entropy in \( C_{\text{max-ent}} = \bigwedge_{i} C_i \).

Let $\mathcal{L}$ be an orthocomplemented lattice. Then, there exists an abelian group $M = M(\mathcal{L})$ such that the functor $\mathcal{M}(\mathcal{L}; -)$ satisfies,

$$\mathcal{M}(\mathcal{L}; -) = \text{Hom}_{\mathbb{Z}}(M, -)$$

This means that a measure in $\mathcal{L}$ valued in $A$ is equivalent to a $\mathbb{Z}$-linear map from $M(\mathcal{L})$ to $A$. 
Measures factorize

Let $\mathcal{L}$ be an orthocomplemented lattice and assume that a group $G$ acts by automorphism. Let $A$ be an abelian group where $G$ acts trivially.

Let $\nu (\text{resp. } \nu')$ be a measure (resp. invariant measure) and $\bar{\nu}, \bar{\nu}'$ are linear maps. The commutativity means that $\nu = \bar{\nu}\pi$, $\nu' = \bar{\nu}'\pi_G$. 
[Non-Boolean Groemer’s integral theorem] Let $\mathcal{L}$ be an orthocomplemented lattice where a group $G$ acts. Let $B$ be an orthogonal generating set for the action of $G$. Then, invariant measures on $\mathcal{L}$ are in bijection with $N_G(B)$-invariant functions on $B$, $\nu$, such that

$$\nu(b_1 \lor b_2) = \nu(b_1) + \nu(b_2), \quad \forall b_1, b_2, b_1 \lor b_2 \in B, \quad b_1 \perp b_2.$$ 

Conclusions

- We study invariant measures in a general framework that includes many contextual theories of interest.
- We approach the problem from the perspective of measure theory. More precisely, we present a non-commutative version of geometric probability theory. Formulating the problem in terms of invariant measures allows link states and group actions in a natural way.
- We give conditions for the solution of the MaxEnt maximization problem with very general constrains.
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Some references

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