Contextuality and Realism Need Not Be Incompatible: The Process Algebra Approach

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Abstract

- **The Kochen-Specker theorem** in 1967 represented a serious challenge to any realist model of quantum mechanics **stating that it was impossible for a quantum system to realize all possible properties simultaneously** – properties and measurement were shown to be fundamentally contextual in nature
- However in biological, psychological and social systems **contextuality takes many forms** and is an aspect of both the classical as well as the quantum mechanical world
- **Khrennikov has shown the existence of non-Kolmogorov probability** structures at the classical level and developed a contextual probability theory
- The **process algebra** approach seeks to apply insights from process theory, complex adaptive systems theory, interpolation theory, combinatorial game theory and ideas of non-Kolmogorov probability structures to the study of fundamental processes
- It takes a generative, emergentist, approach to space-time and to the fundamental entities existing within it – **probabilities are also emergent and contextual** (non-Kolmogorov)
- It is directed towards developing a realist, **maximally local** (quasi-non-locality) and **minimally contextual** (quasi-contextuality) model of fundamental systems, free of paradoxes and divergences
- To date, it has been applied with some success to non-relativistic quantum mechanics and relativistic quantum mechanics in the setting of a semi-classical approach to quantum electrodynamics

The Key Idea

- The standard approach in quantum mechanics is to postulate a wave function in a Hilbert space from which can be derived a probability distribution function. Measurement is represented through the action of self adjoint operators and the wave function is decomposed into a sum of eigenfunctions of the measurement operator yielding the spectrum of measurement values and their associated probability distribution functions.

$$f(x) = \sum c_n \Psi_n(x)$$

- In contrast, the process algebra approach starts with the recognition that the Hilbert space of quantum mechanics is a reproducing kernel Hilbert space, so that the wave function may be decomposed into a sum of local interpolation functions (local wavelets) where the coefficients represent local strengths (actual local values)

$$f(x) = \sum_{n} f(x_n) T_{x_n} g(x) = \sum_{n,i} c_i \Psi_i(x_{n,i}) T_{x_{n,i}} g(x)$$

- Conjecture: Reality is fundamentally discrete – the appearance of continuity is an artefact of observation (Kempf, ArXiv: 10104354.v1 (2010)). Physical entities emerge from informationally coherent collections of fundamental entities ***generated*** by a process using ***causally local*** information and ***imparting*** a (reduced) set of properties to these local entities (including the local strength of the process attributed to the local entity (local value of wave function))

The Elements of Process Theory

- The theory of process was initially developed by A.N. Whitehead (Process and Reality, 1929)
- Its use here was inspired by ideas of Trofimova, Rosen, Shimony, Kempf, Sorkin, Smolin
- It originates in theories of organisms and of complex systems, and posits an emergent rather than a static reality. Process generates the fundamental elements of reality, *actual occasions*, which are both ontological and informational in character.
- Actual occasions are transient. They arise, propagate their information to form new informons, and then fade away. Reality is viewed as a compound present formed of the current generation of actual occasions, the generating process, and the elements of the next generation of actual occasions being constructed.
- Actual occasions are discrete, finite (though vast) in number, and possess a “fuzzy” extensionality. Their properties are inherited from the process that generates them.
- Information propagates from prior to nascent actual occasions as a discrete wave.
- Actual occasions are unobservable, yet the generation of an informon determines subsequent actions by, transformations of, and interactions among, processes. Only interactions give rise to observable events, particularly those specialized interactions termed “measurements”.
- Space-time and its fundamental entities do not exist a priori. Instead they are emergent constructs supervening on the succession of actual occasions generated. Processes generate space-time and so exist outside of space-time proper. The succession of processes forms proper time and the emergent space-time is four dimensional.

Actual Occasions are formally modeled as **Informons** which collectively form a **Causal Tapestry** (primitive space-time). They possess both *intrinsic components* (inherited from the generating process) and extrinsic components (acquired through observation)

Extrinsic features Intrinsic features

$$[n]<\mathbf{m}_n,\phi_n(\mathbf{z}),\Gamma_n,\mathbf{p}_n>\{G_n\}$$

Label

Properties

Causal Interpretation

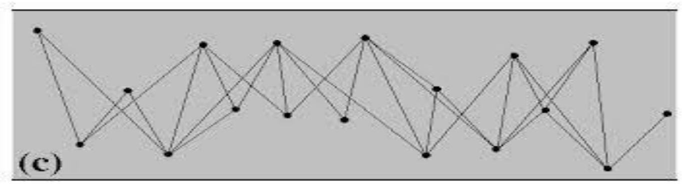
Local H(M) interpretation

Process Strength

Content

Intrinsic Components of Informons

- ***(Quasi-Non-Locality)*** The information *content* G_n of an informon consists of a *causally ordered* collection of informons from *prior* causal tapestries contributing information to the *current* informon, Usually equipped with a (spatial) metric d and a causal function D (i.e. causal distance)

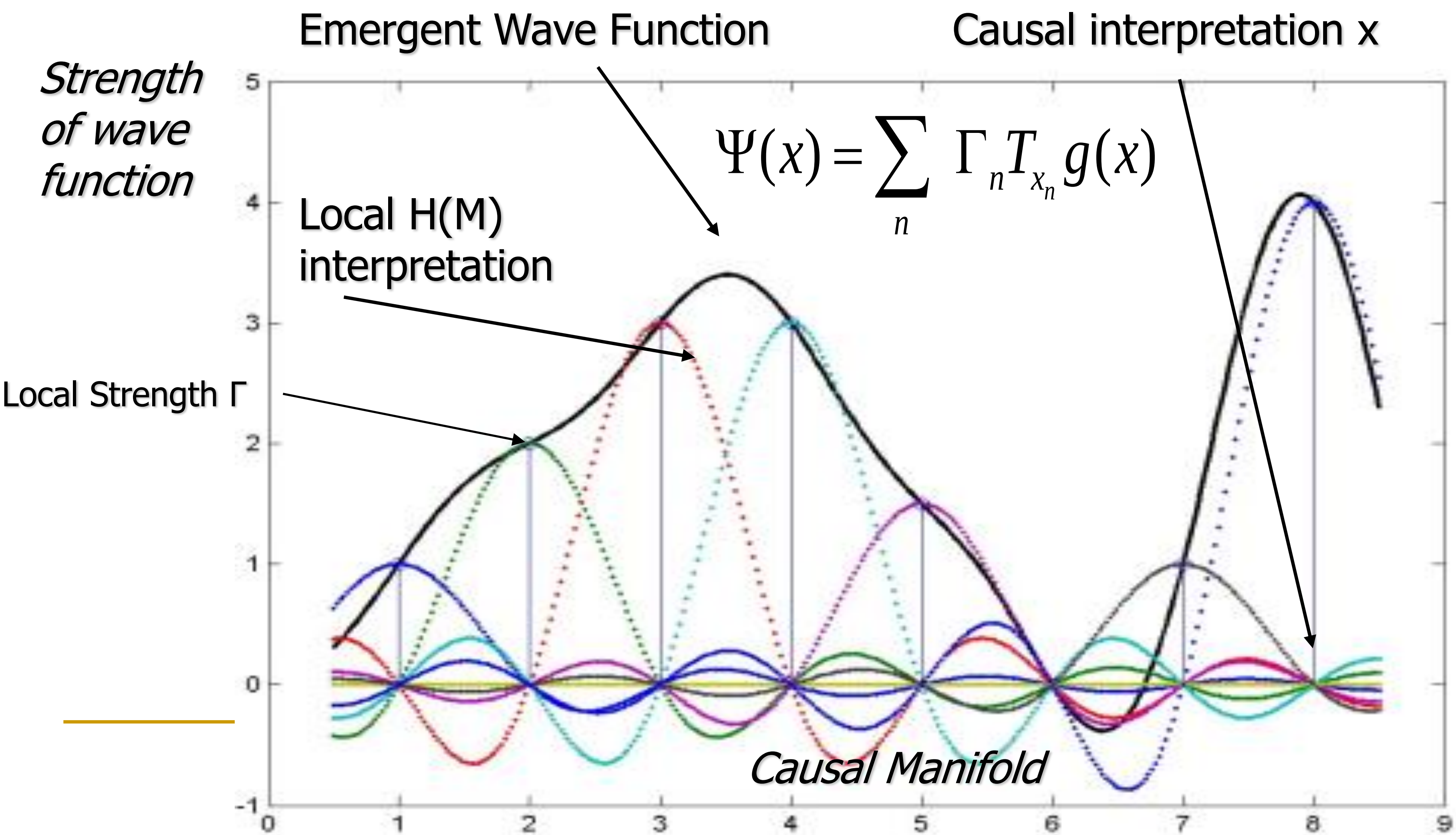


- ***(Quasi-Contextuality)*** P_n is a vector of different *properties*, inherited from the generating process, which may be Scalar (real, complex, quaternion), Vector, Spinor, Tensor
  - Distinct properties possess definite values
  - Not all properties are expressed – only those generated by its process
  - Properties reflect intrinsic and/or conserved characteristics
- Γ_n is the *local strength* of the generating process at the informon *n,* used to determine the compatibility or coupling effectiveness of the generating process to other active processes

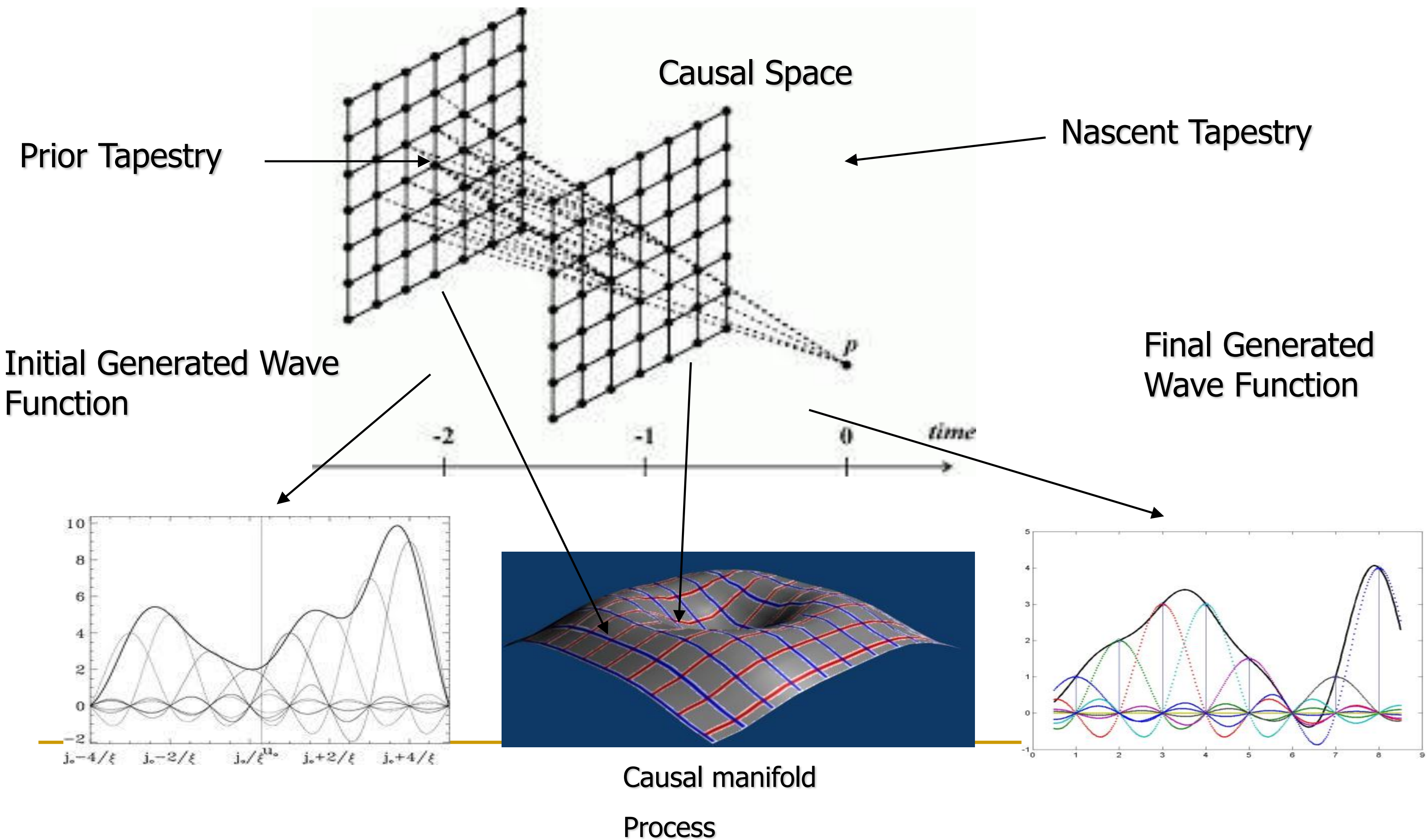
Notion of compatibility based upon Trofimova, I.N. (2002) in Nation, J., Rand, J, Trofimova, I, Sulis, W. Formal Descriptions of Developing Systems, Kluwer

Extrinsic Components of Informons

A causal tapestry is interpreted as an emergent wave function on a causal manifold



Information flows from prior to nascent tapestry under a process. Once all the information has been transferred, the prior tapestry fades from existence and a new compound present forms. Process can be interpreted as an operator on the space of global H(M) interpretations



Process Algebra describes how processes combine and interact. The Process algebra possesses a natural heuristic representation as an algebra of two player partizan combinatorial games

- Independent processes: p_1, p_2, ….
- Concatenation (non-commutative): $p_1 p_2$…
- 8 distinct sums and products (all commutative) determined by the manner in which the component processes interrelate:
  - Contribute only to different informons (exclusive) "no hat"
  - May contribute to the same informon (free) "hat"
  - Act sequentially (not necessarily in any fixed order) (sum)
  - Act concurrently (product)
  - Act independently of one another (independent) "circle"
  - Act in a coordinated, correlated or mutually constrained manner (interactive) "box"

Rules for Combining Processes

- Primitive processes generate single informons sequentially
- Multi-informon processes are formed algebraically from primitive processes
- Processes may be *active* or *inactive* (triggered by informons and interactions between processes)
- Concatenation represents the sequential activation (or inactivation) of processes, usually as a result of interactions
- Processes representing different states of single entities combine via the exclusive sum (e.g. mixed states)
- Processes representing identical states of single entities combine via the free sum (e.g. left/right components of two slit experiment)
- Processes representing different entities combine via products (bosonic-like is free, fermionic-like is exclusive)
- Interacting processes use the interactive product – in some cases this may have an algebraic form (e.g. entanglement

$$(P_{1a} \otimes P_{2b}) \oplus (P_{1b} \otimes P_{2a})$$

Measurement is defined as a specific interaction between a system and a dynamically specialized ***measurement*** process. It consists of a sequence of transformations and interactions between a system and a measurement process ultimately culminating in a state of the measurement process corresponding to a *measurement value*. The probability associated with measurement is emergent and non-Kolmogorov

- Stage 1
  - Free coupling of system P, measurement M

$$\mathsf{P} \otimes \mathsf{M} = \mathsf{P} \otimes (\square_j \mathsf{M}_j(\mu_j))$$

- Stage2
  - Transformation of P to an epistemologically equivalent process induced by measurement apparatus M (boundary effect) – informational only, no exchange of energy. Only eigenprocesses couple to M

$$\mathsf{M}(\mathsf{P}) = \oplus_j w_j \mathsf{P}_j(\lambda_j)$$

- Stage 3
  - Following the generation of a P, M informon pair, an interaction between P and M is initiated dependent upon local strengths of system process (via compatibility)

$$w_i \mathsf{P}_i(\lambda_i) \leftrightarrow w_j \mathsf{P}_j(\mu_j)$$

$$Prob \propto l_P^3 \Gamma_n^* \Gamma_n$$

- Stage 4
  - Interaction results in a new system-measurement process leading to an observable.

$$k w_i \mathsf{P}_i(\lambda_i) \otimes \mathsf{M}'$$

Compatibility of Quasi-Contextuality and Kochen-Specker Type Theorems

- Let us examine how the process algebra model generates a mixed state

$$\frac{1}{\sqrt{2}}(\Psi_1(x)\oplus\Psi_2(x))=\frac{1}{\sqrt{2}}(\sum_{x_n\in l_1}\Psi_1(x_n)T_{x_n}g(x)+\frac{1}{\sqrt{2}}\sum_{x_m\in l_2}\Psi_2(x_m)T_{x_m}g(x))$$

- Consider a mixed state primitive process described by the exclusive sum of two primitive processes. Informons are thus generated sequentially with no sharing of information. The informons of each subprocess are organized into disjoint lattices – thus no fundamental element of space-time ever possesses inconsistent properties
- The interpolation procedure creates global Hilbert space interpretations for each subprocess on each lattice whose sum forms the global Hilbert space interpretation for the combined process – the subprocesses are compartmentalized in the causal tapestry even though the final Hilbert space interpretation is global on the causal manifold – operators on the global interpretation miss the local structure
- Measurement, by contrast, is triggered by a *local* interaction between the measurement apparatus and process informons, resulting in a measured value corresponding only to either process 1 or process 2 – never both together
- There is no global measured valued associated with the combined process
- No process can generate all measurement values – process concatenation is non-commutative so there is only quasi-contextuality, not non-contextuality

Process Covering Map (for primitive processes)

- The link to standard quantum mechanics is via the process covering map
- In the absence of interaction, the activation of a process P on an initial causal tapestry I results in the generation of a nascent causal tapestry I'.
- The informons of I' will constitute a single path along the process sequence tree Σ
- Each such path contributes a global Hilbert space interpretation

$$\Psi(\mathbf{x}) = \Sigma_{n\in I}\phi_n(\mathbf{x}) = \Sigma_{n\in I}\Gamma_n T_{\mathbf{m_n}} sinc(\mathbf{x})$$

- The collection of all such global Hilbert space interpretations results in a set valued map, the process covering map of P

$$\mathsf{P}_I(\mathsf{P}) = \{\Psi_k(\mathbf{x})\}$$

- In the non-relativistic case it has been shown that in the asymptotic limit as the number of informons and amount of information transferred increases to infinity, the process covering map converges to a single valued map, which approximates the standard quantum mechanical wave function

$$\mathsf{P}_I(\mathsf{P}) \rightarrow \{\Phi^{t_P, l_P}(z)\}$$

- The standard NRQM wave function arises in the asymptotic limit as t_P, l_P tend to 0
- The process covering map defines a functor from the space of processes to the space of operators on the Hilbert space over the causal manifold
- The process covering map provides an ontological description, representing a single complete action of process to generate a single complete causal tapestry. In order to carry out correlational calculations, especially in the case of multiple processes, a configuration space process covering map must be defined because a single evolution need not express all possible correlations induced by the process. The configuration space process covering map is not ontological. It is constructed on the entire process sequence tree and incorporates information along multiple distinct paths (analogous to multiple versions of reality).

Configuration Space Process Covering Map

The problem for forming a product representation lies in the fact that the global Hilbert space interpretation is generated by a single path along the process sequence space consisting of correlated sets of informons $A_i = (n_i^1, \ldots, n_i^n)$ yielding

$$\hat{\Psi}_p(\mathbf{z}_1, \ldots, \mathbf{z}_n) = \sum_{(n_i^1, \ldots, n_i^n) \in \mathrm{I}'} \Gamma_{n_i^1} \cdots \Gamma_{n_i^n} T_{\mathbf{m}_{n_i^1}} g(\mathbf{z}_1) \cdots T_{\mathbf{m}_{n_i^n}} g(\mathbf{z}_n)$$

and a single path need not express all possible correlations. Instead it is necessary to construct a configuration sequence space $\Sigma^C(\mathsf{K}, \otimes_i \mathsf{P}_i)$ and a configuration process covering map. To do so one artificially extends each path of the sequence space by appending compatible informons from other paths until a maximal such path is created. The PCM is formed by constructing global Hilbert space interpretations based upon these maximal paths. Let $\Im^M_{\Sigma^C(\mathsf{K}, \mathsf{P})}$ denote the set of all of its maximal causal tapestries. We define the configuration process covering map or $\mathsf{P}^C(\otimes_i \mathsf{P}_i))$ to be

$$\mathsf{P}^C_{\mathsf{K}}(\otimes_i \mathsf{P}_i) = \{\Phi^j(\mathbf{z}) = \sum_{(n_k^1, \ldots, n_k^n) \in \mathsf{K}'} \Gamma_{n_k^1} \cdots \Gamma_{n_k^n} T_{\mathbf{m}_{n_k^1}} g(\mathbf{z}_1) \cdots T_{\mathbf{m}_{n_k^n}} g(\mathbf{z}_n) \,|\, \mathsf{K}' \in \mathsf{J}^M_{\Sigma^C(\mathsf{K}, \otimes_i \mathsf{P}_i)}\}$$

Clearly this map is not ontological but merely heuristic.

A Sample Process Model: Semiclassical QED

- The informons of our semi-classical strategy take the following form:
  - The causal manifold is Minkowski 4-space (content embeds as a causal subspace
  - The Hilbert space consists of all 4-vector valued functions on the causal manifold
  - The properties are energy, momentum, helicity
  - The causal interpretations are 4-vectors within the causal manifold

$$\mathbf{m_n} = (\mathbf{nt_P}, \mathbf{ml_P}, \mathbf{rl_P}, \mathbf{sl_P})$$

  - The strengths are real 4-vectors (ϕ, A_x, A_y, A_z)
  - The local Hilbert space interpretations take the form

$$(\phi, A_x, A_y, A_z)\ T_{\mathbf{m}_{n'}} sinc_{t_P, l_P}(\mathbf{z}) = sinc\left(\frac{\pi(t - nt_P)}{t_P}\right) sinc\left(\frac{\pi(x - ml_P)}{l_P}\right) sinc\left(\frac{\pi(y - rl_P)}{l_P}\right) sinc\left(\frac{\pi(z - sl_P)}{l_P}\right)$$

  - Note the absence of relativistic invariance. These interpretations are not ontological but merely heuristic since the ontology rests with the causal tapestry and so they need not in themselves be invariant. Different observers would have different interpretations but the informons and their causal structure are relativistically invariant.

Process Strategy for Generating an Informon

- Player I selects an informon s from the prior tapestry whose information propagates
- Player II selects either the informon under construction or a new unused label n (arbitrary since the label is a mere heuristic tool)
- Player I updates the content set G_n of n
- Player II selects an element $\mathbf{m_n}$ of the causal manifold lying in the future cone of $\mathbf{m_s}$ and within a causal distance of ct_P
- Player I then propagates the information of s, namely its strength, forward as a discrete wave to n and updates the strength of n, i.e. where K(n,s) is the appropriate kernel propagator. In the case of photons one is propagating the 4-vector consisting of the scalar and vector potentials $\Gamma_n \to \Gamma_n + K(m,s)\Gamma_n$
- Player II forms the local Hilbert space interpretation which will take the form

$$(\phi_n, A_x^n, A_y^n, A_z^n)T_{\mathbf{m_n}}\, sinc_{t_P l_P}(\mathbf{z})$$

- The global Hilbert space interpretation is formed by taking the sum of the local Hilbert space contributions over all of the informons of the nascent causal tapestry

$$\Sigma_{n\in I}(\phi_n, A_x^n, A_y^n, A_z^n)T_{\mathbf{m_n}}\, sinc_{t_P l_P}(\mathbf{z})$$

- This is a discrete and finite approximation to the scalar and vector potentials of the E-M field. The approximation will improve if the number of informons and amount of information transferred grows arbitrarily large, which can happen if either N,R for the single photon process grow (unphysical) or the number of photons being generated grows (consistent with the ideas of collective electrodynamics (Mead, C. (2002) Collective electrodynamics, MIT)

Construction of Strength

- The individual process strength components are updated using discrete versions of the propagator formulas

$$A_i(r,t) = \int K(r,s,t) A_i(s,0) d^3 s + \int \tilde{K}(r,s,t) \dot{A}_i(s,0) d^3 s \quad + \text{(contributions from charges)}$$

- The propagators have the general form

$$K(r,s,t) = \frac{1}{4\pi d(r,s)} \text{(sum of } \delta \text{ functions)\$}$$

- So the local strength propagates only a short distance from the originating informon

Conclusions

- Previous work has shown that the process algebra model of non-relativistic quantum mechanics reproduces to a high degree of accuracy the computational results of the theory of non-relativistic scalar particles while providing an ontology in which:
  - Reality is generated, emergent, discrete, finite, forming a compound present.
  - Reality is quasi-non-local, quasi-contextual and information flows causally (therefore at no more than luminal speed).
  - A fundamental entity is represented as an ontologically existent (discrete) wave reflecting the outcome of information flow between instances of reality.
  - Computation requires the use of a purely heuristic, epistemological, process or configuration space process covering map
  - Standard NRQM arises as an effective theory under certain asymptotic limits
  - Quantum probability is an emergent, non-Kolmogorov consequence of measurement.

- A relativistic version of the model has been applied to a semi-classical version of quantum electrodynamics.
- There is conceptual clarity – for example while the standard wave function is ontological, in configuration space it is heuristic. Moreover, issues such as wave-particle duality, quantum paradoxes, divergences appear to be resolved