A SEPARABLE REPRESENTATION OF THE SINGLET BEYOND STANDARD QUANTUM MECHANICS

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Poster at

Quantum Contextuality in Quantum Mechanics and Beyond

Prague 2017

The two slit experiment and other representations beyond standard QM

It is possible to formulate Quantum Mechanics in extended phase spaces, with coordinates corresponding to non commuting operators. In the two slit experiment the slit variable $S \in \{L, R\}$ and the position q at the final screen do not commute. A distribution of amplitude $\Psi(S, q)$ defines the quantum state. IN practice, these $\Psi(S, q)$ are the ones computed. The distribution of probability is found in two steps, fist computing the **marginal** amplitude $\Psi(q) = \Psi(L, q) + \Psi(R, q)$ and second applying Born's rule $P(q) = |\Psi(q)|^2$, where the usual interference term gives account of the diffraction pattern

$$P(q) = |\Psi(L, q)|^2 + |\Psi(R, q)|^2 + \Psi^*(L, q)\Psi(R, q) + \Psi^*(R, q)\Psi(L, q)$$

Beyond standard QM there are subquantum variables describing the position of the corpuscular component, e.g. (L^*, q^*) (particle through the left slit arriving to the final position q^*). The amplitude of probability gives account of the correlation of the particle with the accompanying field, which goes through both slits and explains the diffraction pattern.

The path integral formalism can be understood as an extended phase space too, made of all paths, and a distribution of amplitude $\Psi(path) = exp(iS[path]/\hbar)$, with $S[path]$ the action integral along $path$. Again, the marginal amplitude along all paths **allowed by the context** with final position q defines the reduced (standard) distribution of amplitude

$$\Psi(q) = \sum_{path \to q} exp(iS[path]/\hbar)$$

It is also possible to consider partial marginals, $\Lambda(q, p) = \Sigma_{path \to (q,p)} exp(iS[path]/\hbar)$ with integral along all paths with endpoint q and final momentum p, and to develop a quantum formalism in the classical phase space $\{(q, p)\}$. $Q = i\hbar\partial_p$ and $P = p - i\hbar\partial_q$ fulfil the canonical commutation rules $[Q, P] = i\hbar$ [1,2,3].

Extended spin phase space

Similarly, an extended phase space for spin variables [4,5,6] allows a formulation beyond the standard two dimensional Hilbert space for spin. Let us consider values of spin in all directions (a finite number for easy) $(s_1, s_2, \dots, s_N)$, $s_j = \pm$ the spin in direction $\mathbf{n}_j$. In the corresponding phase space $\mathcal{H}_{Spin} = \{(s_1, s_2, \dots, s_N) | \forall s_j = \pm\}$ we can define distributions of amplitude of probability $Z(s_1, s_2, \dots, s_N)$. We define $Z(s_1, \dots, s_N)$ as sum of elementary amplitudes $s_j \mathbf{N}_j$, with $\mathbf{N}_j$ the imaginary quaternion associated to the unit vector $\mathbf{n}_j$.

$$Z(s_1, s_2, \dots, s_N) = \sum_j s_j \mathbf{N}_j$$

Marginal amplitudes towards standard QM

Every standard spin state, e.g. $|+_1>$, finds a representation in this framework. The distribution $Z_{+_1}(s_1, s_2, \dots, s_N)$ with $Z_{+_1}(+_1, s_2, \dots, s_N) = \mathbf{N}_1 + \sum_{j \geq 2} s_j \mathbf{N}_j$ and $Z_{+_1}(-_1, s_2, \dots, s_N) = 0$ determines, after computing the marginal amplitudes $Z_{+_1}(+_1) = \sum_{s_2, \dots, s_N} Z_{+_1}(+_1, s_2, \dots, s_N) = 2^{N-1}\mathbf{N}_1$, and $Z_{+_1}(-_1) = 0$, the standard distributions of probability for the spin state $|+_1>$: $P(+_1) = 1$, $P(-_1) = 0$.

Projections (marginal amplitudes) over the spin variable in another direction become

$$Z_{+_1}(s_2) = \sum_{s_3, \dots, s_N} Z(+_1, \dots s_N) = 2^{N-2} (\mathbf{N}_1 + s_2 \mathbf{N}_2)$$

with associated probabilities $P(s_2) = \frac{1}{2}(1 + s_2 \mathbf{n}_1 \cdot \mathbf{n}_2)$, the standard in QM.

A NEW isotropic spin state

If we consider the state defined by ALL

$$Z(s_1, s_2, \ldots, s_N) = \sum_j s_j \mathbf{N}_j$$

we find an isotropic state, that is, $Z(s_j) = 2^{N-1} s_j \mathbf{N}_j$, with associated probabilities $P(s_j) = \frac{1}{2}$. **This quantum state has no counterpart in the standard formalism**, where any spin state is eigenstate for the spin operator in some spatial direction. When a particle is generated without defined spin, as for example each particle of the singlet, we find in this framework a quantum state representing the isotropic spin distribution of probability for one individual particle.

The singlet revisited

The singlet spin state, entangled state of as composite of particles α and β

$$|+_1>^\alpha \otimes |-_1>^\beta - |-_1>^\alpha \otimes |+_1>^\beta$$

can be represented in this new framework in separable form

$$\left(\sum Z(s_1^\alpha, s_2^\alpha, \ldots, s_N^\alpha) |s_1, s_2, \ldots, s_N>^\alpha\right) \otimes$$
$$\left(\sum Z(s_1^\beta, s_2^\beta, \ldots, s_N^\beta) |s_1, s_2, \ldots, s_N>^\beta\right)$$

The perfect correlation is imposed over the subquantum states $s_j^{*\alpha} + s_j^{*\beta} = 0$ for all $1 \leq j \leq N$. The distribution of probability $P(s_1^\alpha, s_2^\beta)$ is obtained as follows:

Because of the perfect correlation, $s_2^{*\alpha} = -s_2^{*\beta}$, measurement of $s_2^{*\beta}$ allows to **infer** $s_2^{*\alpha}$. Now, $P(s_1^\alpha, s_2^\alpha)$ is obtained through the corresponding (partial) marginal amplitude

$$Z(s_1^\alpha, s_2^\alpha) = \sum_{s_3^\alpha,\ldots,s_N^\alpha} Z(s_1^\alpha, s_2^\alpha, \ldots, s_N^\alpha) = 2^{N-2}(s_1^\alpha \mathbf{N}_1 + s_2^\alpha \mathbf{N}_2)$$

which determines $P(s_1^\alpha, s_2^\alpha) = \frac{1}{2}(1 + s_1^\alpha s_2^\alpha \mathbf{n}_1 \cdot \mathbf{n}_2)$. Therefore, the observed distribution of probability becomes

$$P(s_1^\alpha, s_2^\beta \equiv -s_2^\alpha) = P(s_1^\alpha, s_2^\alpha) = \frac{1}{2}(1 - s_1^\alpha s_2^\beta \mathbf{n}_1 \cdot \mathbf{n}_2)$$

reproducing the standard result for the singlet.

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