PSY 201: Statistics in Psychology

Lecture 29

Hypothesis testing for dependent proportions

What do people think about death?

Greg Francis

Purdue University

Fall 2023

DEPENDENT SAMPLES

- when the samples are not independent, hypothesis testing of proportions becomes a bit more complicated
- samples are dependent when each score in one sample is paired with a score in the other sample
- just like dependent samples for the mean, the problem is that the samples are not independent (not truly random) and we need to take that into account
  - This can be a good thing from a statistical point of view
  - We can remove some variability

EXAMPLE

- Testing the difference of proportions of individuals who pass each of two similar items on a test. (e.g. comparing pass/fail for two sets of students who get better than 600 SAT)
- Test the difference in proportions of individuals who support something before and after discussion.
- Comparing proportions of husbands and wives on an issue.

HYPOTHESES

- for dependent samples we set our hypotheses as

$$H_0 : P_1 - P_2 = 0$$

$$H_a : P_1 - P_2 \neq 0$$

SAMPLING DISTRIBUTION

- we need to know the sampling distribution and the standard error
- but first we need to design a contingency table that shows disagreement or dissimilarity in responses

		Group 2		
		NO	YES	
Group 1	YES	A	B	$A+B$
	NO	C	D	$C+D$
		$A+C$	$B+D$	$A+B+C+D=n$

- A is the number of scores that are "no" in group 2 and "yes" in group 1
- B is the number of scores that are "yes" in group 2 and "yes" in group 1
- C is the number of scores that are "'no" in group 2 and "no" in group 1
- D is the number of scores that are "yes" in group 2 and "no" in group 1

CONTINGENCY TABLE

- we then convert these to proportions

	Group 2			
		NO	YES	
Group 1	YES	a	b	$a+b$
	NO	c	d	$c+d$
		$a+c$	$b+d$	$a+b+c+d=1.0$

- $a = A/n$ is the proportion of scores that are "no" in group 2 and "yes" in group 1
- $b = B/n$ is the proportion of scores that are "yes" in group 2 and "yes" in group 1
- $c = C/n$ is the proportion of scores that are "'no" in group 2 and "no" in group 1
- $d = D/n$ is the proportion of scores that are "yes" in group 2 and "no" in group 1

PROPORTIONS

- from the contingency table we can get the proportions of scores with the trait we are interested in
- this is what we need for our statistic

$$p_1 = b + d = \frac{B + D}{n}$$

$$p_2 = a + b = \frac{A + B}{n}$$

- but we need the contingency table for other reasons!

CONTINGENCY TABLES

- the sampling distribution is approximately normal with a mean of $P_1 - P_2$ if

$$A + D > 10$$

or

$$B + C > 10$$

- if not, do not use this test
- moreover, our estimate of standard error of the difference between dependent proportions is

$$s_{p_1 - p_2} = \sqrt{\frac{a+d}{n}} = \sqrt{\frac{p_d}{n}} = \sqrt{\frac{(A+D)/n}{n}}$$

- which we get from the contingency table

HYPOTHESIS TESTING

- so to actually carry out the test, we compute the test statistic

$$z = \frac{(p_1 - p_2) - (P_1 - P_2)}{s_{p_1 - p_2}}$$

- or, since our null hypothesis is that $(P_1 - P_2) = 0$

$$z = \frac{(p_1 - p_2)}{s_{p_1 - p_2}}$$

- and then look up a *p*-value from the normal distribution calculator

EXAMPLE

- I took the Exam 1 grades and the Homework grades and for each student computed:
  - Trait 1: Grade on Exam 1 is ≥ 70 (C range)
  - Trait 2: Grade on Homework is ≥ 70 (C range)

Exam 1	Homework Grade
0	1
1	1
0	1
1	1
1	1
1	1
0	1
1	1
1	1
0	1
0	1
0	1
1	1
0	1
0	1
1	1
1	1
1	1
0	1
1	0
1	1
0	0
0	1
0	1
0	1
0	1
1	0
1	0

EXAMPLE

- We can test for a difference in proportions using the on-line calculator:

		Has Trait 2?	
		No	Yes
Has Trait 1?	Yes	$A =$ 3	$B =$ 11
	No	$C =$ 1	$D =$ 13

Specify hypotheses:

$H_0 : P_1 - P_2 =$ 0

$H_a :$ Two-tails

$\alpha =$ 0.05

Run Test

Test summary	
Null hypothesis	$H_0 : P_1 - P_2 = 0$
Alternative hypothesis	$H_a : P_1 - P_2 \neq 0$
Type I error rate	$\alpha = 0.05$
Sample size	$n = 28$
Sample proportion for group 1	$p_1 = 0.5000$
Sample proportion for group 2	$p_2 = 0.8571$
Disagreement proportion	$p_d = 0.5714$
Standard error	$s_{p_1 - p_2} = 0.142857$
Test statistic	$z = -2.500000$
p value	$p = 0.012419$
Decision	Reject the null hypothesis
Confidence interval critical value	$z_{cv} = 1.960395$
Confidence interval	$CI_{95} = (-0.637199, -0.077086)$

ANOTHER EXAMPLE

- Suppose we want to know if there is a difference in the proportion of students that oppose the death penalty and the proportion of students that support gun control.
- Raise your hand if (feel free to lie if you do not want others to know your true opinions)
  - A: You support gun control, but do not oppose the death penalty.
  - B: You support gun control and oppose the death penalty.
  - C: You do not support gun control and do not oppose the death penalty.
  - D: You do not support gun control, but do oppose the death penalty.

CONTINGENCY TABLE

	OPPOSE DEATH PENALTY		
SUPPORT	NO	YES	
GUN CONTROL	YES NO		

- I want to test

$$H_0 : P_1 - P_2 = 0$$

$$H_a : P_1 - P_2 \neq 0$$

- In words: the proportion of people supporting gun control is the same as the proportion of people who oppose the death penalty (individuals are always pro-life or pro-death)
- I will use $\alpha = 0.05$
- Note: Group 1 is the set of responses to the question about opposition to the death penalty
- Group 2 is the set of responses to the question about support of gun control

CRITERION

- I need to check if I can use the normal approximation to the sampling distribution
- check if

$$A + D > 10$$

  or

$$B + C > 10$$

- if not, do not use this test
- We use the on-line calculator

INTERPRETATION

- If we reject H_0, that indicates the probability of getting the observed difference of proportions, or bigger difference, when the population parameters were equal is less than 0.05. We interpret that as meaning the population parameters are different.
- If we fail to reject H_0, that indicates the probability of getting the observed difference of proportions, or bigger, when the population parameters were equal is greater than 0.05. We do not have strong enough evidence to conclude that the population parameters are different.

CONFIDENCE INTERVAL

- easy to create confidence intervals too
- the general formula is

$$CI = \text{statistic} \pm (\text{critical value }) \times (\text{standard error})$$

- for the difference of dependent proportions it becomes

$$CI = (p_1 - p_2) \pm (z_{cv})(s_{p_1 - p_2})$$

POWER

- Computing power (and estimating minimum sample sizes) feels a bit awkward for dependent proportions
- Although you are testing the differences in proportions that *have* traits, the needed information is the proportions about **disagreements** across traits

Specify the population characteristics:

$H_0 : P_1 - P_2 =$ 0

H_a : Enter the *proportions* for the disagreement cells of the contingency table for the alternative hypothesis.

		Has Trait 2?	
		No	Yes
Has Trait 1?	Yes	$A/n =$	$B/n =$ NA
	No	$C/n =$ NA	$D/n =$

$H_a : P_{a1} - P_{a2} =$

Specify the properties of the test:

Type of test: Two-tails

Type I error rate, $\alpha =$ 0.05

Power= 0.8

Sample size, $n =$ 100

Calculate minimum sample size

Calculate power

- Let's look at an example

RETRIEVAL PRACTICE

- Studies have found that a good way to improve memory is to actively retrieve information from memory
  - practice test instead of study
- A common study goes like: Each subject reads two paragraphs about different topics (e.g., photosynthesis or leukemia). After reading:
  - For one paragraph the subject takes a practice test that requires them to recall information from the paragraph (e.g., "How does photosynthesis directly benefit our environment?")
  - For the other paragraph, the subject reads the paragraph a second time.
- A week later, subjects are tested on both paragraphs (new questions)

RETRIEVAL PRACTICE

- Typical results for correctly answering the final questions are something like:
- Retrieval practice: $p_1 = 0.44$
- Study: $p_2 = 0.28$
- From the raw data (you typically cannot get this information from what is published in scientific papers)

		Has Trait 2?	
		No	Yes
Has Trait 1?	Yes	$A =$ 6	$B =$ 10
	No	$C =$ 20	$D =$ 0

RETRIEVAL PRACTICE

- Suppose you want to explore retrieval practice in a new setting (statistics-related questions)
- The null hypothesis is no difference in proportions for retrieval versus study conditions

$$H_0 : P_1 - P_2 = 0$$

- the alternative hypothesis is that there is a some difference

$$H_a : P_1 - P_2 \neq 0$$

- You need a specific alternative hypothesis, and using the data from the previous study is a good starting point

$$H_a : P_{1a} - P_{a2} = 0.44 - 0.28 = 0.16$$

- but you specify it by identifying the disagreements in responses

RETRIEVAL PRACTICE

- We need

$$a = \frac{A}{n} = \frac{6}{36} = 0.1667$$

$$d = \frac{D}{n} = \frac{0}{36} = 0$$

- Suppose you want 0.9 for power

Specify the population characteristics:

$H_0 : P_1 - P_2 =$ 0

H_a : Enter the *proportions* for the disagreement cells of the contingency table for the alternative hypothesis.

		Has Trait 2?	
		No	Yes
Has Trait 1?	Yes	$A/n =$ 0.1667	$B/n =$ NA
	No	$C/n =$ NA	$D/n =$ 0

$H_a : P_{a1} - P_{a2} =$ 0.1667 - 0. = 0.1667

Specify the properties of the test:

Type of test: Two-tails

Type I error rate, $\alpha =$ 0.05

Power= 0.9

Sample size, $n =$ 64

Calculate minimum sample size

Calculate power

RETRIEVAL PRACTICE

- You might argue that a one-tailed test

$$H_0 : P_1 - P_2 > 0$$

- is appropriate because you know retrieval practice helps in most settings
- Then, to have 0.9 power:

Specify the population characteristics:

$H_0 : P_1 - P_2 =$ 0

H_a : Enter the *proportions* for the disagreement cells of the contingency table for the alternative hypothesis.

		Has Trait 2?	
		No	Yes
Has Trait 1?	Yes	$A/n =$ 0.1667	$B/n =$ NA
	No	$C/n =$ NA	$D/n =$ 0

$H_a : P_{a1} - P_{a2} =$ 0.1667 - 0. = 0.1667

Specify the properties of the test:

Type of test: Positive one-tail

Type I error rate, $\alpha =$ 0.05

Power= 0.9

Sample size, $n =$ 52

Calculate minimum sample size

Calculate power

CONCLUSIONS

- two-sample case
- dependent proportions
- confidence interval
- power

NEXT TIME

- Comparing two sample correlations
- Power

How careful are students?